A Successive Integration Method for The Analysis of the Thermal Environment of Building

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This method is to calculate the room air temperature or the heating (cooling) load variations for each Δt step with repetition of simple multiplications and additions by utilizing both the nature of an exponential function which decreases by equal ratio for each Δt step and the fact the indicial response of the room, wall and/or the heating equipment to a thermal input of unit step function is approximate to the sum of exponential functions.

The method is effective not only for the ordinal transient heating (cooling) load calculations but for the simulations of such cases as ; when the system has multiple rooms of different conditions, when the ventilation rate of the room or the heat transfer coefficient of the wall varies, etc..

As another distinctive feature of this method, it is easy to change Δt in the way of calculation whenever it is necessary, therefore when the heat capacity of the building is quite large and the actual outdoor conditions (including solar radiation) should be considered, it is possible to calculate with fewer times of calculations with high accuracy by this method.

In the report the authors deal with the principle of this method, method to change Δt ,some considerations for setting the initial conditions to minimize the time of calculations and some examples of calculations.

Key Words : Thermal environment, Successive integration method, Indicial response, Temperature excitation, Heat flow response, Duhamel's integration formula, Heating load, Non-linear factor, Heat transfer coefficient, Radiation, Ventilation, Change of Δt .

1. Introduction

The factors related to precise analysis of the thermal environment of a building are numerous as follows.

- Outside conditions ; fluctuations of temperature, solar radiation, atmospheric radiation, wind velocity, movement of sunlit and shaded area, etc.
- Inside conditions ; regulation of temperature and zoning, intermittent heat supply, effect of unconditioned space, distribution of air temperature and radiant heat transfer in a room, rate of ventilation or infiltration and its change, etc.

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Others ; two or three dimensional heat flow at the beams, columns or wall corners, thermal capacity of furnitures and room furnishings, dynamic performance of heating equipment, etc.

Detailed studies on the weighting function for the analysis of thermal environment have been carried on for a long time by Dr. T. Maeda, Dr. S. Fujii, Dr. F. Hasegawa and others.

This successive integration method is one application of these earlier investigations, the intention is to make analysis more flexible and make calculation easier. This method would be applicable for many kinds of problems in which the factors listed above are concerned and depending on the purpose or the requisit accuracy of the analysis various conbinations and simplifications would be possible.

2. Duhamel's integration formula

Consider a system (such as a room, a wall or a piece of equipment) as illustrated in figure 1 and assume the heat flow response of the system to a unit step function of temperature $\theta_u(t)$ is given by $h(t)$, then the heat flow response consequent on the arbitrary temperature excitation $\theta(t)$ is obtained by the use of Duhamel's integration formula as follows ;

$$H(t) = \int_0^t \theta'(\tau) \cdot h(t-\tau)\, d\tau + \theta(t=0) \cdot h(t) \tag{1}$$

where τ : variable of integration

$\theta'(t)$: the first derivative of $\theta(t)$

$h(t)$: indicial response of heat flow to a unit step function of temperature

however when $t<0$ $\left.\begin{array}{l}\theta(t)=0\\ h(t)=0\end{array}\right\}$

For example, heat flow at the inner surface of the wall under the conditions of arbitrary inside and outside air temperature variations $\theta_i(t)$ and $\theta_o(t)$ are expressed by the sum of the responses which are excited by both excitation of $\theta_i(t)$ and $\theta_o(t)$ as follows ;

$$H(t) = H_i(t) - H_o(t)$$

$$= \int_0^t \theta_i'(\tau) \cdot h_i(t-\tau)\, d\tau + \theta_i(t=0) \cdot h_i(t) - \int_0^t \theta_o'(\tau)\, h_o(t-\tau)\, d\tau - \theta_o(t=0) \cdot h_o(t) \tag{2}$$

where $H_i(t)$: heat flow at the inner surface of the wall when the inside air temperature is $\theta_i(t)$ and the outside air temperature is kept at 0°C

$H_o(t)$: heat flow at the inner surface of the wall when the outside air temperature is $\theta_o(t)$ and the inside air temperature is kept at 0 °C

$h_i(t), h_o(t)$: indicial response of heat flow at the inner surface of the wall as seen in figure 2 (Watt deg^{-1})

3. Approximation of an indicial response of heat flow

The indicial response of heat flow of the system to a unit step function of temperature can be obtained by several ways, and in so far as the linear characteristics of the system are kept, it would be expressed by the sum of the infinite series of exponential functions. And in actual use it can be sufficiently approximated by several terms as follows ;

$$h(t) = B_o + \sum_{m=1}^{j} B_m \cdot e^{-\beta_m \cdot t} + g\ \delta(t) \tag{3}$$

B_o : the term for steady state heat flow ($Watt \cdot deg^{-1}$)
$\delta(t)$: delta function
g : imaginary thermal capacity of the system ($Watt \cdot h \cdot deg^{-1}$)
β_m : β_m becomes larger in order of suffix m

The imaginary thermal capacity g is the amount of heat to be supplied to the system instantly when the temperature of excitation is raised suddenly from 0 °C to 1 °C at $t = 0$. However, for the approximation of the indicial response of heat flow at the inner surface $h_o(t)$, it would be advisable to approach it so as to satisfy the following conditions from the nature of thermal response to an outside excitation.

$$h_o(t=0) = 0$$

$$\therefore \quad g = 0 \quad , \quad B_o + \sum_{m=1}^{j} B_m = 0$$

There are many studies concerning the approximation and simplification of the indicial response as shown in the reference. [4),5),6)]

4. Approximation of temperature variation by linear equation and the successive calculation method of heat flow

By substituting eq. (3) for (1), the heat flow response of the system $H(t)$ to an arbitrary temperature excitation $\theta(t)$ is expessed as ;

$$H(t) = \int_0^t \theta'(\tau) \cdot h(t-\tau) \; d\tau \; + \theta(t=0) \cdot h(t)$$

$$= \underbrace{B_o \, \theta(t)}_{Y(t)} + \underbrace{\sum_m \left[\int_0^t \theta'(\tau) \, B_m \cdot e^{-\beta_m (t-\tau)} d\tau + \theta(t=0) \cdot B_m \, e^{-\beta_m t} \right]}_{\Sigma Z_m(t)} + \underbrace{\int_0^t \theta'(\tau) \cdot g \cdot \delta(t-\tau) \cdot d\tau}_{D(t)} \tag{4}$$

The heat flow $H(t)$ is expressed as the sum of steady state term $Y(t)$, transient terms $Z_m(t)$ and impulsive term $D(t)$.

Now, let's assume the temperature variation $\theta(t)$ is approximated by a linear equation within the time of $t_n \leq t \leq t_n + \Delta t$ as seen in figure 3 and expressed as ;

$$\theta(t) = \theta_1(t) + \theta_2(t) \tag{5}$$

where
$$\left.\begin{aligned} \theta_1(t) &= \theta(t) \\ \theta_2(t) &= 0 \end{aligned}\right\} \; t \leq t_n$$

$$\left.\begin{aligned} \theta_1(t) &= \theta_n = \text{const.} \\ \theta_2(t) &= A_{(n+1)} \; (t - t_n) \end{aligned}\right\} \; t_n \leq t \leq t_n + \Delta t$$

$A_{(n+1)}$: temperature gradient ($deg \cdot h^{-1}$)

Substituting eq. (5) for (4) the heat flow at the time ($t_n + \Delta t$) will be ;

$$\begin{aligned} H(t_n+\Delta t) = \; & B_o \; \theta(t_n+\Delta t) + \sum_m \left[\int_0^{t_n+\Delta t} \theta_1'(\tau) \cdot B_m \cdot e^{-\beta_m (t_n+\Delta t-\tau)} d\tau \; + \theta(t=0) \cdot B_m \; e^{-\beta_m (t_n+\Delta t)} \right] \\ & + \sum_m \left[\int_0^{t_n+\Delta t} \theta_2'(\tau) \cdot B_m \cdot e^{-\beta_m \cdot (t_n+\Delta t-\tau)} \, d\tau \right] \\ & + \int_0^{t_n+\Delta t} \left\{ \theta_1'(\tau) + \theta_2'(\tau) \right\} \cdot g \cdot \delta(t_n+\Delta t-\tau) \cdot d\tau \end{aligned} \tag{6}$$

defined as follows ;

$$\theta_1'(t) = 0 \qquad t_n \leqq t \leqq t_n + \Delta t$$

$$\theta_2'(t) = \theta_2(t) = 0 \qquad t \leqq t_n$$

$$\int_0^\infty f(\tau)\cdot\delta(t-\tau)\,d\tau = f(t) \qquad t>0$$

Equation (6) becomes

$$H(t_n+\Delta t) = B_o\{\theta_{(n)} + A_{(n+1)}\cdot\Delta t\} + \sum_m \Big[\underbrace{\int_0^{t_n}\theta_1'(\tau)\cdot B_m\cdot e^{-\beta_m(t_n-\tau)}d\tau + \theta(t=0)\cdot B_m\cdot e^{-\beta_m\cdot t_n}}_{Z_{m(n)}}\Big]\cdot e^{-\beta_m\cdot\Delta t} + \sum_m\Big[\int_{t_n}^{t_n+\Delta t}A_{(n+1)}\cdot B_m\cdot e^{-\beta_m(t_n+\Delta t-\tau)}d\tau\Big] + A_{(n+1)}\cdot\gamma \tag{7}$$

In equation (7) the underlined portion is the same as the transient term Z_m at the time of t_n . Thus the following simple calculation method is obtained ;

$$H(t_n+\Delta t) = H_{(n+1)} = Y_{(n+1)} + \sum_m Z_{m(n+1)} + D_{(n+1)}$$

$$= B_o\cdot\theta_{(n)} + A_{(n+1)}\,B_o\cdot\Delta t + \sum_m\{Z_{m(n)}\cdot E_m + A_{(n+1)}\cdot X_m\} + A_{(n+1)}\cdot\gamma \tag{8}$$

where $E_m = e^{-\beta_m\cdot\Delta t} = \text{const.}$ (9)

$$X_m = \frac{B_m}{\beta_m}\cdot(1-e^{-\beta_m\cdot\Delta t}) = \text{const.} \tag{10}$$

If we take the time interval of each section Δt to be the same as seen in figure 4 then the coefficient of E_m and X_m becomes constant and the calculation of eq. (8) becomes very simple. It is also easy to change Δt by only changing E_m and X_m whenever necessary.

This eq. (8) gives the rate of heat flow at the end of each interval so that we are able to analyze the heating load or the temperature variations by substituting this for eq. (16) and it may be somewhat easier to understand but the following method (which uses the quantity of heat flow during the interval instead of the rate of heat flow at the end of the interval) would be more accurate in calculation.[2]

5. Integration of heat flow in each interval

The quantity of heat flow during an interval of $t_n \sim (t_n+\Delta t)$ can be obtained by integrating eq. (7) with respect to Δt as follows (by using a variable of integration ξ instead of Δt) ;

$$\Delta H_{(n+1)} = \int_{t_n}^{t_n+\Delta t} H(t)\,dt = \int_0^{\Delta t} H(t_n+\xi)\cdot d\xi$$

$$= \int_0^{\Delta t} B_o\{\theta_{(n)} + A_{(n+1)}\cdot\xi\}\,d\xi + \sum_m\Big[Z_{m(n)}\cdot\int_0^{\Delta t}e^{-\beta_m\cdot\xi}d\xi + A_{(n+1)}\cdot\frac{B_m}{\beta_m}\int_0^{\Delta t}(1-e^{-\beta_m\xi})\,d\xi\Big] + A_{(n+1)}\cdot\gamma\cdot\int_0^{\Delta t}d\xi$$

$$= B_o\cdot\theta_{(n)}\cdot\Delta t + \sum_m\Big[Z_{m(n)}\cdot\frac{1}{\beta_m}(1-e^{-\beta_m\cdot\Delta t})\Big] + A_{(n+1)}\cdot\Big[\frac{B_o\cdot\Delta t^2}{2} + \sum_m\frac{B_m}{\beta_m}\Big\{\Delta t - \frac{1}{\beta_m}(1-e^{-\beta_m\Delta t})\Big\} + \gamma\cdot\Delta t\Big]$$

$$= B_o\cdot\theta_{(n)}\cdot\Delta t + \sum_m\Delta Z_{m(n)} + A_{(n+1)}\cdot\Delta X_o \tag{11}$$

where $\Delta X_o = \Big[\frac{B_o\cdot\Delta t^2}{2} + \sum_m\frac{B_m}{\beta_m}\Big\{\Delta t - \frac{1}{\beta_m}(1-e^{-\beta_m\cdot\Delta t})\Big\} + \gamma\cdot\Delta t\Big] = \text{const.}$ (12)

$$\Delta Z_{m(n)} = Z_{m(n)}\cdot\frac{1}{\beta_m}(1-e^{-\beta_m\cdot\Delta t}) \tag{13}$$

If the time interval Δt is invariable after the time of t_n as seen in figure 4 then eq. (12) becomes constant and from eqs.(13) and (8) the following equation results ;

$$\Delta Z_{m(n+1)} = Z_{m(n+1)} \frac{1}{\beta_m}(1-e^{-\beta_m \cdot \Delta t})$$

$$= \left\{ Z_{m(n)} \cdot E_m + A_{(n+1)} \cdot X_m \right\} \cdot \frac{1}{\beta_m}(1-e^{-\beta_m \cdot \Delta t})$$

$$= \Delta Z_{m(n)} \cdot E_m + A_{(n+1)} \cdot \Delta X_m \tag{14}$$

where $$\Delta X_m = \frac{B_m}{\beta_m^2}(1-e^{-\beta_m \cdot \Delta t})^2 \tag{15}$$

$$= \text{const.}$$

If the temperature gradients $A_{(n+1)}$, $A_{(n+2)}$, ... are known, each $\Delta Z_{m(n+1)}$, $\Delta Z_{m(n+2)}$ .... can be easily calculated in succession by eq. (14), and each $\Delta H_{(n+1)}$, $\Delta H_{(n+2)}$, ... is also obtained from eq. (11) by the repetition of simple calculations.

6. Equation of heat balance

(when the heat transfer coefficients of the room do not change)

Consider room k which is adjacent to rooms $K = 1, 2, 3, \cdots\cdots$ where room air temperatures are different from each other as seen in figure 5. The following equation of heat balance can be given ;

$$Q_k \cdot \frac{d\theta_k(t)}{dt} = W(t) - H_k(t) + \sum_K \left[H_K(t) + C_p \cdot V_K(t) \cdot \left\{ \theta_K(t) - \theta_k(t) \right\} \right] \tag{16}$$

where

- Q_k : thermal capacity of the air of room k ($Watt \cdot h \cdot deg^{-1}$) including that of furnishings of which the temperature change is considered the same as that of the air temp.
- $\theta_k(t)$, $\theta_K(t)$: temperature of the room and adjacent rooms
- $W(t)$: heating rate supplied to room air (Watt) including auxiliary heat from human bodies and equipment
- $H_k(t)$: heatloss through the surrounding walls of the room, where the air temp. is $\theta_k(t)$ and the adjacent room air temp. is $\theta_K(t) = 0$
- $H_K(t)$: inflow of the heat from the inside surface of the partitions adjacent to the room K under the condition of $\theta_k(t) = 0$ adjacent room $\theta_K(t)$
- $V_K(t)$: air volume infiltrated from room K (outflow air is not related)
- C_p : specific heat of the air for unit volume ($Watt \cdot h \cdot deg^{-1} \cdot m^{-3}$)

Assume the divisions of time are the same as in figure 4 and assume the temperature variations within the time interval $t_n \sim (t_n + \Delta t)$ are as follows ;

$$\theta(t) = \theta_{(n)} + A_{(n+1)} \cdot (t - t_n)$$

$$\theta(t_n+\Delta t) = \theta_{(n+1)} = \theta_{(n)} + A_{(n+1)} \cdot \Delta t \tag{17}$$

And assume the heating rate $W(t)$ and the infiltration rate $V(t)$ are constant during the time interval of Δt . Then by integrating eq. (16) with respect to t from t_n to $(t_n+\Delta t)$ and substituting eq. (11), the following relation is obtained ;

$$\int_{t_n}^{t_n+\Delta t} \left[Q_k \cdot \frac{d\theta_k(t)}{dt} \right] dt = \int_{t_n}^{t_n+\Delta t} \left[W_k(t) - H_k(t) + \sum_K \left[H_K(t) + C_p \cdot V_K(t) \left\{ \theta_K(t) - \theta_k(t) \right\} \right] \right] dt$$

$$Q_k \cdot \Delta t \cdot A_{k(n+1)} = W_{k(n+1)} \cdot \Delta t - B_{ok} \cdot \theta_{k(n)} \cdot \Delta t - \sum_m \Delta Z_{mk(n)} - \Delta X_{ok} \cdot A_{k(n+1)}$$ (continue)

$$+\sum_{k}\left[B_{ok}\cdot\theta_{k(n)}\cdot\Delta t+\sum_{m}\Delta Z_{mk(n)}+\Delta X_{ok}\cdot A_{k(n+1)}+C_p\cdot V_{k(n+1)}\cdot\Delta t\cdot\left\{\theta_{k(n)}-\theta_{k(n)}+\frac{\Delta t}{2}(A_{k(n+1)}-A_{k(n+1)})\right\}\right] \tag{18}$$

6.1 When the temperature is known

In eq. (18) the temperature gradients $A_{k(n+1)}$ and $A_{k(n+1)}$ are known so the total heating load $W_{k(n+1)}$ is obtained easily. By subtracting the auxiliary heat from this result, the net heating load is obtained.

6.2 When the temperature is not known

As for unconditioned space or when intermittent heating or cooling is a factor, the temperature gradient $A_{k(n+1)}$ is obtained from the following equation.

$$A_{k(n+1)}=\left[W_{k(n+1)}\cdot\Delta t-B_{ok}\cdot\theta_{k(n)}\cdot\Delta t-\sum_{m}\Delta Z_{mk(n)}+\sum_{k}\left\{B_{ok}\cdot\theta_{k(n)}\cdot\Delta t+\sum_{m}\Delta Z_{mk(n)}+A_{k(n+1)}\cdot\Delta X_{ok}\right.\right.$$
$$\left.\left.+C_p\cdot\Delta t\cdot\left(\theta_{k(n)}-\theta_{k(n)}+\frac{\Delta t}{2}\cdot A_{k(n+1)}\right)\right\}\cdot V_{k(n+1)}\right]\div\left[\Delta t\cdot Q_k+\Delta X_{ok}+\frac{C_p\cdot\Delta t^2}{2}\cdot\sum_{k}V_{k(n+1)}\right] \tag{19}$$

6.3 When the temperatures of adjoining rooms are not known

When the building has many rooms or spaces of which temperature variations are not given, we have to solve equation (19) as a simultaneous equation of unknown temperature gradients.

However, from the nature of the thermal response to a outside excitation, the influence of the temperature variation of the adjoining room is gradual as seen in figure 2. Therefore the accuracy of the temperature gradient of the adjoining room $A_{k(n+1)}$ is not too important for the calculation of $A_{k(n+1)}$ and this is a very fortunate characteristic of this calculation method.

In this case it would be good to assume that the temperature variation of the adjoining room is the same as that at the time Δt before. That is to use $A_{k(n)}$ which was already calculated instead of the unknown quantity $A_{k(n+1)}$ in eq. (19).

7. When the heat transfer coefficient varies with time or temperature

Actually a fairly large part of the heat from a human body, electricity, radiators and window sunlight, transfers by radiation to the surrounding walls and moreover the convection heat transfer coefficient varies with the wind velocity or with the temperature difference between the wall surface and the air, so that for the precise analysis of the problem it is necessary to treat the heat transfer coefficient as a variable.

In this case, it is alse possible to use the same method as above by considering each surface of the wall, of which the heat transfer coefficient is a variable, as a kind of a room where the temperature is not known. The order of the calculation is as follows ;

Step 1. heat balance of the surface of the wall

An equation of heat balance of the surface of the wall at time t and during the time interval of Δt would be written as follows ;

$$\alpha_{k\ell}(t)\left\{\theta_k(t)-\theta_{k\ell}(t)\right\}-H_{k\ell}(t)+H_{k\ell}(t)+J_{k\ell}(t)=0 \tag{20}$$

$$\int_{t_n}^{t_n+\Delta t}\alpha_{k\ell(n+1)}\cdot\left\{\theta_k(t)-\theta_{k\ell}(t)\right\}dt-\Delta H_{k\ell(n+1)}+\Delta H_{k\ell(n+1)}+J_{k\ell(n+1)}\cdot\Delta t=0 \tag{21}$$

where $\alpha_{k\ell(n+1)}$: heat transfer coefficient of wall "ℓ" of room k and which is constant during the time of $t_n\sim(t_n+\Delta t)$

$J_{k\ell(n+1)}$: effective radiation to the surface of wall "ℓ"

$\theta_k(t)$: air temperature of room k

$\theta_{k\ell}(t)$: temperature of wall surface " ℓ " of room k

$$H_{k\ell}(t) = \int_0^t \theta'_{k\ell}(\tau)\cdot h_{k\ell}(t-\tau)\,d\tau \quad , \quad \Delta H_{k\ell(n+1)} = \int_{t_n}^{t_n+\Delta t} H_{k\ell}(t)\,dt$$

$$H_{k\ell}(t) = \int_0^t \theta'_{k\ell}(\tau)\cdot h_{k\ell}(t-\tau)\,d\tau \quad , \quad \Delta H_{k\ell(n+1)} = \int_{t_n}^{t_n+\Delta t} H_{k\ell}(t)\,dt$$

$h_{k\ell}(t)$: indicial response of heat flow of the inside surface of wall " ℓ " to a unit step function of temperature of the same surface

$h_{k\ell}(t)$: indicial responce of heat flow of the inside surface of wall " ℓ " to a unit step function of temperature of the outside surface

$\theta_{k\ell}(t)$: temperature of outside surface of wall " ℓ "

By substituting the following relations to the eq. (21)

$$\left.\begin{aligned}
\theta_{k\ell}(t) &= \theta_{k\ell(n)} + A_{k\ell(n+1)}\cdot(t-t_n) \\
\theta_k(t) &= \theta_{k(n)} + A_{k(n+1)}\cdot(t-t_n) \\
\theta_{k\ell}(t) &= \theta_{k\ell(n)} + A_{k\ell(n+1)}\cdot(t-t_n)
\end{aligned}\right\}$$

whereby
$$\alpha_{k\ell(n+1)}\cdot\left[\left\{\theta_{k(n)} - \theta_{k\ell(n)}\right\}\Delta t + \left\{A_{k(n+1)} - A_{k\ell(n+1)}\right\}\cdot\frac{\Delta t^2}{2}\right]$$
$$- \theta_{k\ell(n)}\cdot B_{ok\ell}\cdot\Delta t - \sum_m Z_{k\ell m(n)} - \Delta X_{ok\ell}\cdot A_{k\ell(n+1)}$$
$$+ \theta_{k\ell(n)}\cdot B_{ok\ell}\cdot\Delta t + \sum_m Z_{k\ell m(n)} + \Delta X_{ok\ell}\cdot A_{k\ell(n+1)} + J_{k\ell(n+1)}\cdot\Delta t = 0 \tag{22}$$

If the temperature gradients $A_{k(n+1)}$, $A_{k\ell(n+1)}$ are not known, assuming that $A_{k(n+1)}$ is $A_{k(n)}$, and $A_{k\ell(n+1)}$ is $A_{k\ell(n)}$, the temperautre gradient of the wall surface $A_{k\ell(n+1)}$ is obtained from eq. (22).

Step 2. Equation of heat balance of the room air

After the temperature gradients of the surrounding wall surfaces $A_{k\ell(n+1)}$ are obtained the heat balance of the room air will be expressed as follows ;

$$Q_k\cdot A_{k(n+1)}\cdot\Delta t = \sum_\ell S_\ell\cdot\int_{t_n}^{t_n+\Delta t}\alpha_{k\ell}(t)\cdot\left\{\theta_{k\ell}(t) - \theta_k(t)\right\}\,dt$$
$$+ \sum_{k}\cdot\int_{t_n}^{t_n+\Delta t} c_p\cdot V_k(t)\left\{\theta_k(t) - \theta_k(t)\right\}dt + W_{k(n+1)}\cdot\Delta t$$
$$= \sum_\ell S_\ell\cdot\alpha_{k\ell(n+1)}\cdot\Delta t\cdot\left\{\theta_{k\ell(n)} - \theta_{k(n)} + \frac{\Delta t}{2}\left(A_{k\ell(n+1)} - A_{k(n+1)}\right)\right\}$$
$$+ \sum_k\left[c_p\cdot V_{k(n+1)}\cdot\Delta t\left\{\theta_{k(n)} - \theta_{k(n)} + \frac{\Delta t}{2}\left(A_{k(n+1)} - A_{k(n+1)}\right)\right\}\right] + W_{k(n+1)}\cdot\Delta t \tag{23}$$

From eq. (23) the unknown value of the temperature gradient $A_{k(n+1)}$ or the heating load $W_{k(n+1)}$ is obtained.

8. Initial conditions of calculation

Figure 6 shows an example of the room air temperature variation of a flat house which is built with reinforced concrete and is heated intermittently. As seen in the example, if the thermal capacity of the system is large it will take many calculations to eliminate the influence of inadequate initial conditions. Therefore, to minimize the number of calculations, the setting of initial condition is important and the following methods can be used.

8.1 To assume a steady state condition

One of a simple method is to make the initial condition of temperature of each room as close to the mean daily temperature of each room as possible and assume a steady state as in case 2 in figure 6,then the initial value of $\Delta Z_{m(n)}$ becomes 0.

8.2 To assume periodical variation of temperature

When the temperature variation is approximated to a sine function as seen in figure 7 then the value of $\Delta Z_{m(n)}$ becomes as follows ;

$$Z_{m(n)} = \int_{-\infty}^{t_n} \theta'(\tau) \cdot B_m \cdot e^{-\beta_m (t-\tau)} d\tau$$

$$= \frac{\omega}{\beta_m^2 + \omega^2} \cdot B_m \cdot \theta_a \cdot \left[\beta_m \cdot \cos \omega \cdot t_n + \omega \cdot \sin \omega \cdot t_n \right] \tag{24}$$

$$\Delta Z_{m(n)} = \frac{1}{\beta_m} (1 - e^{-\beta_m \cdot \Delta t}) \cdot \frac{\omega}{\beta_m^2 + \omega^2} \cdot B_m \cdot \theta_a \cdot \left[\beta_m \cdot \cos \omega \cdot t_n + \omega \cdot \sin \omega \cdot t_n \right] \tag{25}$$

By using eq. (25) for the initial value of $\Delta Z_{m(n)}$ at the time of t_n , high accuracy of analysis can be obtained with fewer calculations.

8.3 To change the time interval of calculation

One of the distinctive features of this method is the easiness of changing Δt . Thus by at first using a large Δt and rough calculations, much detailed analysis can be done later as seen in figure 8.

To change the time interval Δt to $\Delta t'$, we have to recalculate the new constants $\Delta X_0'$, E_m' and X_m' and replace the terms $\Delta Z_{m(n)}$ with $\Delta Z_{m(n)}'$ as follows

$$\Delta Z_{m(n)}' = \Delta Z_{m(n)} \times \frac{(1 - e^{-\beta_m \cdot \Delta t'})}{(1 - e^{-\beta_m \cdot \Delta t})} \cdot \frac{E_m}{E_m'} + A_{(n)} \cdot \left\{ X_m \times \frac{(1 - e^{-\beta_m \cdot \Delta t'})}{(1 - e^{-\beta_m \cdot \Delta t})} - X_m' \right\} \times \frac{1}{E_m'} \tag{26}$$

Figure 9 shows a comparison of two cases A and B.

A : calculated by $\Delta t = 0.25$(h) from the bigining to the end (a dotted line)

B : calculated by $\Delta t = 1.0$ till the 13th day and change to $\Delta t = 0.25$ (h) thereafter (a broken line)

The results agree perfectly after the 14th day. It will be also seen that if there is a sudden change in heat supply and the time interval Δt is large then the calculated temperature fluctuates for a while. However, from the nature of this integration method the calculated temperature approaches an accurate value rapidly and even when it is fluctuating the average value or the integrated value of the results over the several intervals is always accurate. And of course when the variation of excitation is contenuous or when an interval Δt is short there is no trouble like that.

For the system in which thermal capacity is quite large, such as for the underground structure, or when a synthetic effect of the systems is in consideration, such as a heating system and a room in which time constants are very different for each other, the considerations of this sub-section are very important.

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(Papers above are written in Japanese)

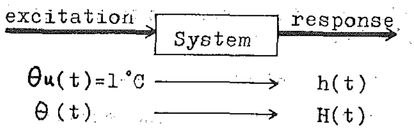

Fig. 1 Thermal system

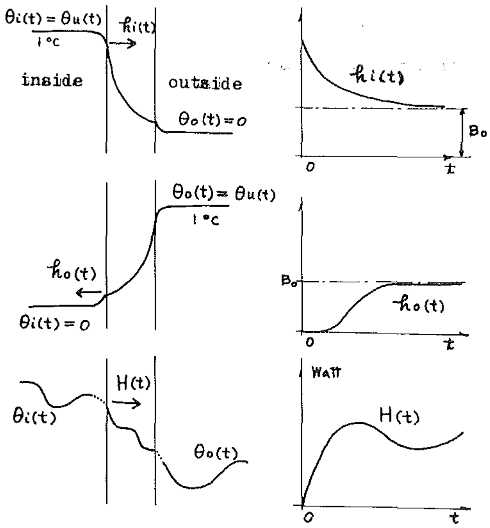

Fig 2. Indical response of heat flow

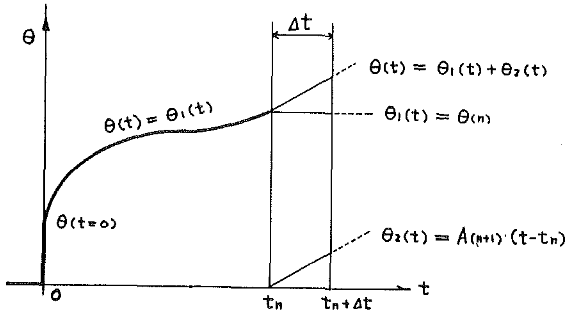

Fig. 3 Approximation of temperature

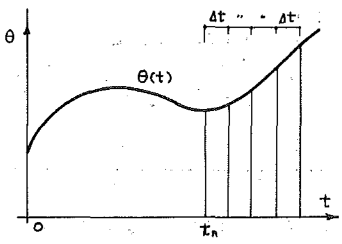

Fig. 4 Division of time

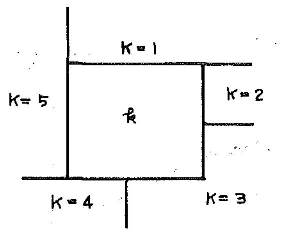

Fig. 5 Room k and adjoining room K

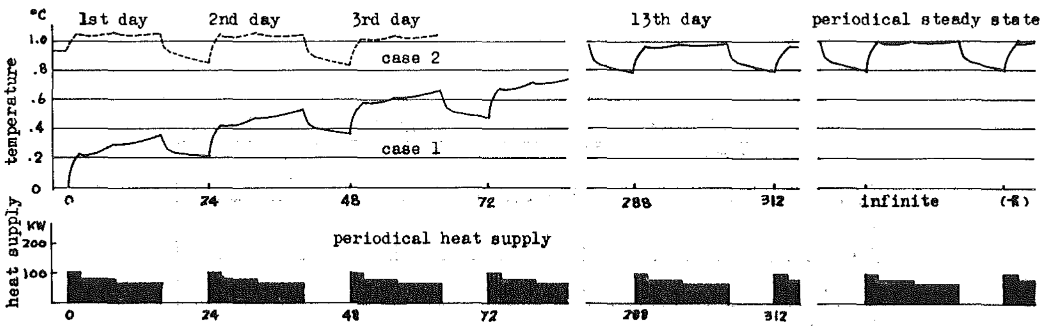

Fig. 6 Intermittent heat supply and calculated room air temperature

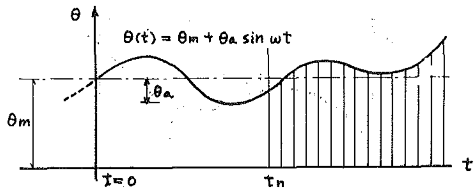

Fig. 7 Assuming of periodical steady state

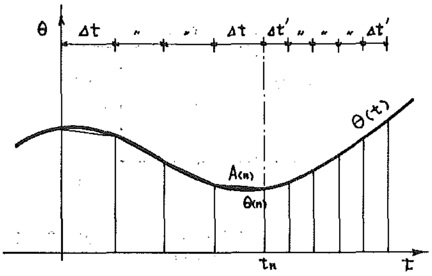

Fig. 8 Change of time interval

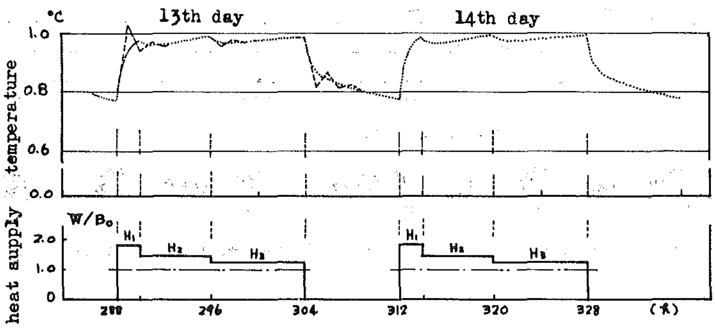

Fig. 9 Examples of calculation when Δt is changed