Vibration Analysis of Buildings with Consideration to the Lateral Deformation of Floor Slabs

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Abstract

In the case of buildings with wide floor plans, the ground conditions are generally not uniform over the entire floor plans and in many cases the basements are planned to a different extent from the upper story plans. Thus, foundation motions of these buildings during earthquakes can not be equal over the entire building, and lateral deformation in the plane of slabs and torsional vibration will occur remarkably in the case where the buildings have long and narrow floor plans.

In the paper, building-soil interactions are analyzed by replacement with two-dimensionally distributed multi-mass systems in order to discuss above mentioned phenomena. Eigen-values and response values of building-soil systems are computed, where the basements are situated in the parts of the plans, and where the dislocation or the inclined layers of the grounds are assumed. From the results, it becomes clear that the dynamic behavior of buildings depends significantly on the lateral deformation of floor slabs and non-uniform conditions of the ground formation.

Introduction

The vibration problem of buildings is generally analyzed under the assumption that the floors show no deformation in their own plane. In the case of the buildings with long and narrow floor plans, however, the above assumption may not be applied. Such plans are generally found in school buildings, apartment houses, hospitals, railway stations, and so on. The author has ascertained that the floors deform due to ground motion particularly in low rise buildings through the microtremor tests of buildings and the observation of building behaviors during an earthquake. The author has previously discussed the relation between vibration and floor shapes of low rise buildings under condition where the columns are fixed at their bottoms in his published reports. In those papers, the vibration analysis was carried out for two-dimensionally distributed multi-mass system by considering the lateral deformation of floor slabs.

In the buildings with wide floor plans, the conditions between buildings and grounds are more complex: ground conditions around foundations are generally not uniform, the basements of buildings are often provided partially, and the time

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lag of earthquake waves may occur at the foundations. These conditions may also yield lateral deformation of floor slabs or torsional vibration in buildings.

In the present paper, the two-dimensionally distributed multi-mass system is taken in soils in the same conception as in buildings. The analysis on vibration interaction between two systems, building and soil, is carried out, and the effects of the above-mentioned underground conditions on the deformation of floor slabs can be discussed. Numerical computation studies are presented for various models of the building-soil system.

Building-Soil System

When the building with a rectangular floor plan has a basement in part of the plan as shown in Fig. 1 (a), the building-soil system can be idealized by replacement with a multi-mass system which is arranged in vertical and horizontal directions as shown in Fig. 1 (b). In other words, the masses of the building in every floor are replaced by discrete masses at each frame in the direction of short sides "Y" in Fig. 1 (a). The lateral stiffness of the horizontal members is calculated from bending-shear stiffness of slabs with beams. The equivalent stiffness of columns may be given by shear stiffness of frames and walls when the building is relatively low. The rectangular prism of soil is presumed for the extent of soil where the building-soil interaction will be active. The mass of the soil

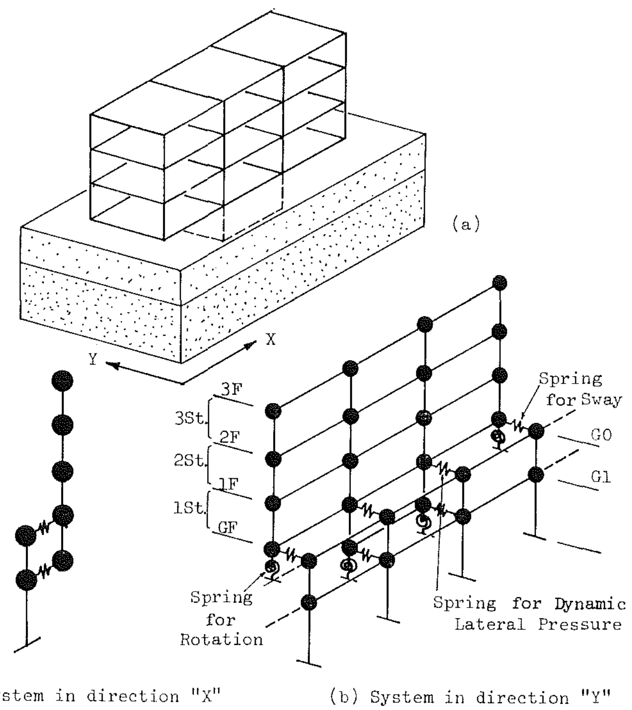

Fig. 1. The interaction model of the building and soil with the idealized multi-mass systems.

prism is lumped at discrete points distributed in the same conception as the mass system of the building. Stiffness of the linkages connecting adjacent pairs of masses is obtained from the shear stiffness of soil.

Each of the masses of the building on foundation levels is connected to the adjacent masses of soil on the same levels by springs of rotation and sway. Each mass of the basement out of its foundation level is joined to the adjacent mass of soil with the spring by which dynamic lateral pressure on basement-walls is replaced.

As for a vibration system in the direction of "X", to simplify the analysis of the vibration, one may assume that each distance between the adjacent masses does not vary and that the displacement caused by rotation is extremely small, and thus, the one-dimensionally distributed multi-mass system without rotation spring as shown in Fig. 1 (c) may be used.

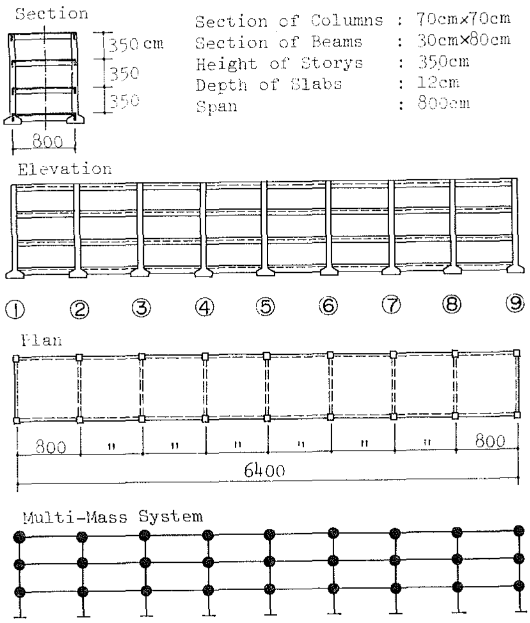

Fig. 2. Dimensions of the structure of prototype building.

Models for Computation

The prototype building for numerical computations is a three-story (height of story: 3.5 meters) reinforced concrete structure without a basement and formed into a rectangular floor plan with one bay (8 meters) by eight bays (64 meters in

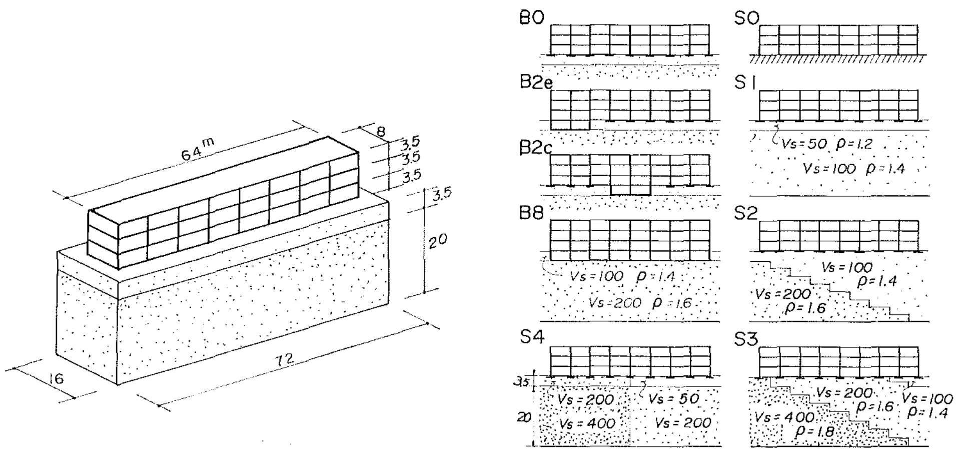

Fig. 3. The types of buildings and grounds for analysis.

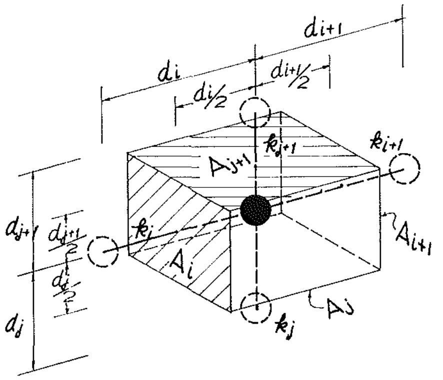

Fig. 4. Sectional area 'A' and distance 'd' to calculate the spring constant of soil.

total length), as shown in Fig. 2. The types of basements under the prototype building are set up into three variant models as shown in Fig. 3 : two cases where two bays of basement are at an end or center of the plan, and the other case where the basement is on the whole plan. The underground condition combined with the above mentioned building is assumed to consist of two soil layers. The top loyer is of $Vs=100$ m/sec and $\rho=1.4$, the lower layer is of $Vs=200$ m/sec and $\rho=1.6$. This condition is named the prototype underground condition in this paper. The spring constant of soil, k, in the multi-mass system is given according to the well-known equation $k=A\cdot\rho\cdot Vs^2/d$, where A is equal to the sectional area as shown in Fig. 4, ρ is the density of soil, Vs is the velocity of S wave, d is the distance between adjacent masses.

In order to discuss the effect of irregular underground conditions on the vibration of the building-soil system, characteristics of the ground with dislocation or with inclined layers as shown in Fig. 3 is investigated in relation to the prototype building.

It is assumed in the paper that the damping coefficient of the building is equal to the damping coefficient of soil ; the vertical displacement of masses is not raised ; the prism of soil which has an area of 16 m by 72 m limited by a band of 4 m in width around the building and with a depth of 23.5 m (the top layer is 3.5 m deep, the lower layer is 20 m deep) is taken as the reasonable extent of the building-soil interaction.

The procedure of numerical computation for analysis is carried out as follows.

(1) Eigenvalues of these models are calculated.

(2) Participation facters for modes are calculated to uniform input force for "X" or "Y".

(3) Dynamic response is computed with modal analysis by using the eigenvalues, the participation factors and the constant response shear-force coefficient "$q=1$" of one mass system regardless of natural periods of the interaction system.

In the above procedure the coefficient "$q=1$" is adopted as a response spectrum of a white noise instead of use of individual recorded earthquake waves.

Results of Numerical Computation

Natural periods and maximum values of "βu" from the first order to seventh order of all models are shown in Table 1 where "βu" is the product of participation facter "β" and eigenvector "u".

Several normal modes of each model are shown in Fig. 5. The modes on the levels of GF, 1F, 2F and 3F (cf. Fig. 1) in the buildings are projected on a horizontal plane and the modes along the vertical axis of a frame are shown as an example. The low order modes of ground surface and underground is

TABLE 1. Natural period "T (sec.)" and "βu_{max}"

Model		1 st	2 nd	3 rd	4 th	5 th	6 th
BO	T	0.6452	0.6072	0.4322	0.1948	0.1861	0.1722
	βu_{max}	0.9257	0.	1.4537	−0.7918	2.4225	0.
B2e	T	0.7108	0.6673	0.4272	0.2031	0.1955	0.1853
	βu_{max}	0.6659	0.1989	1.4407	1.0238	−0.7810	2.2511
B2c	T	0.7091	0.6261	0.4271	0.1997	0.1842	0.1721
	βu_{max}	0.7655	0.	1.4409	0.8932	0.1990	0.
B8	T	0.6743	0.6754	0.4249	0.2559	0.2302	0.2163
	βu_{max}	0.6555	0.	1.4240	−0.0660	0.7593	0.
S1	T	0.7362	0.6917	0.4811	0.3303	0.3171	0.2795
	βu_{max}	0.9769	0.	2.2534	1.1639	−1.3887	0.
S2	T	0.6475	0.6082	0.4360	0.2709	0.2371	0.1956
	βu_{max}	1.0942	−0.0809	1.7453	1.5276	0.9952	−1.0366
S3	T	0.6187	0.5635	0.4236	0.1925	0.1818	0.1683
	βu_{max}	1.2712	0.5654	1.3162	−0.3537	1.2481	−0.3473
S4	T	0.7051	0.5961	0.4250	0.1963	0.1954	0.1737
	βu_{max}	1.0325	0.9292	1.3119	−1.1070	1.6911	−0.0616

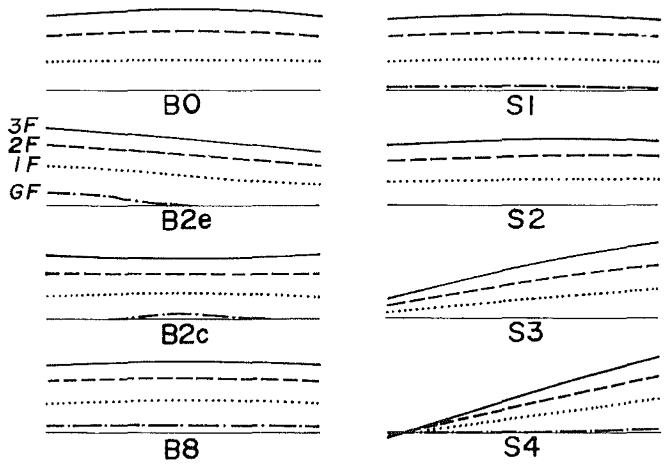

Fig. 5(a). First normal modes of all models.

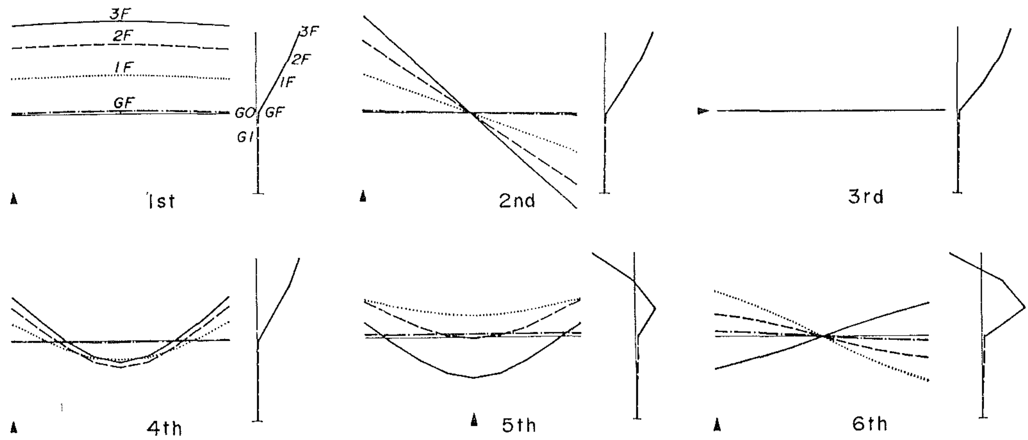

Fig. (5) b. Normal modes of "*BO*".

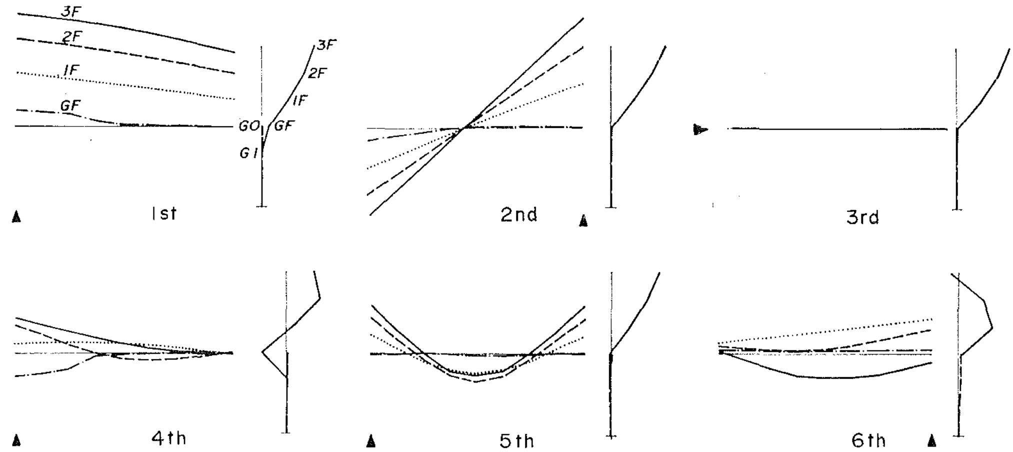

Fig. (5) c. Normal modes of "*B2e*".

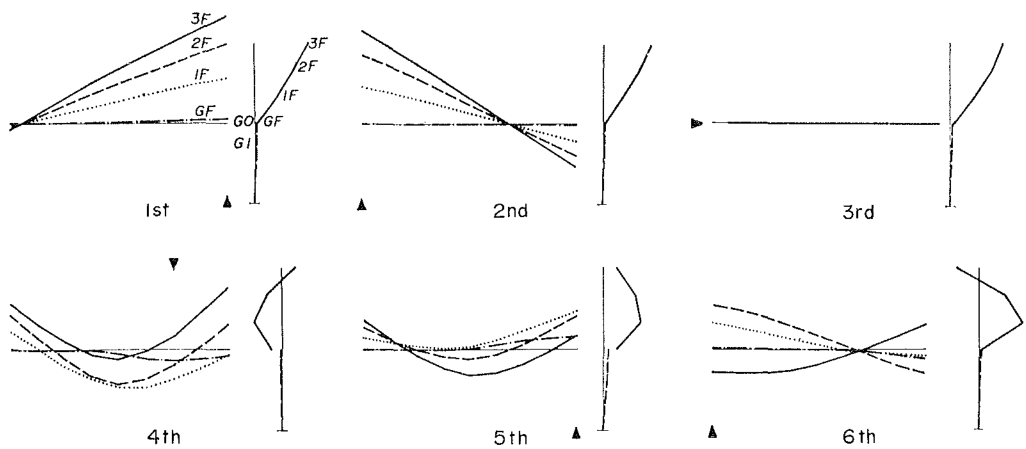

Fig. (5) d. Normal modes of "*S4*".

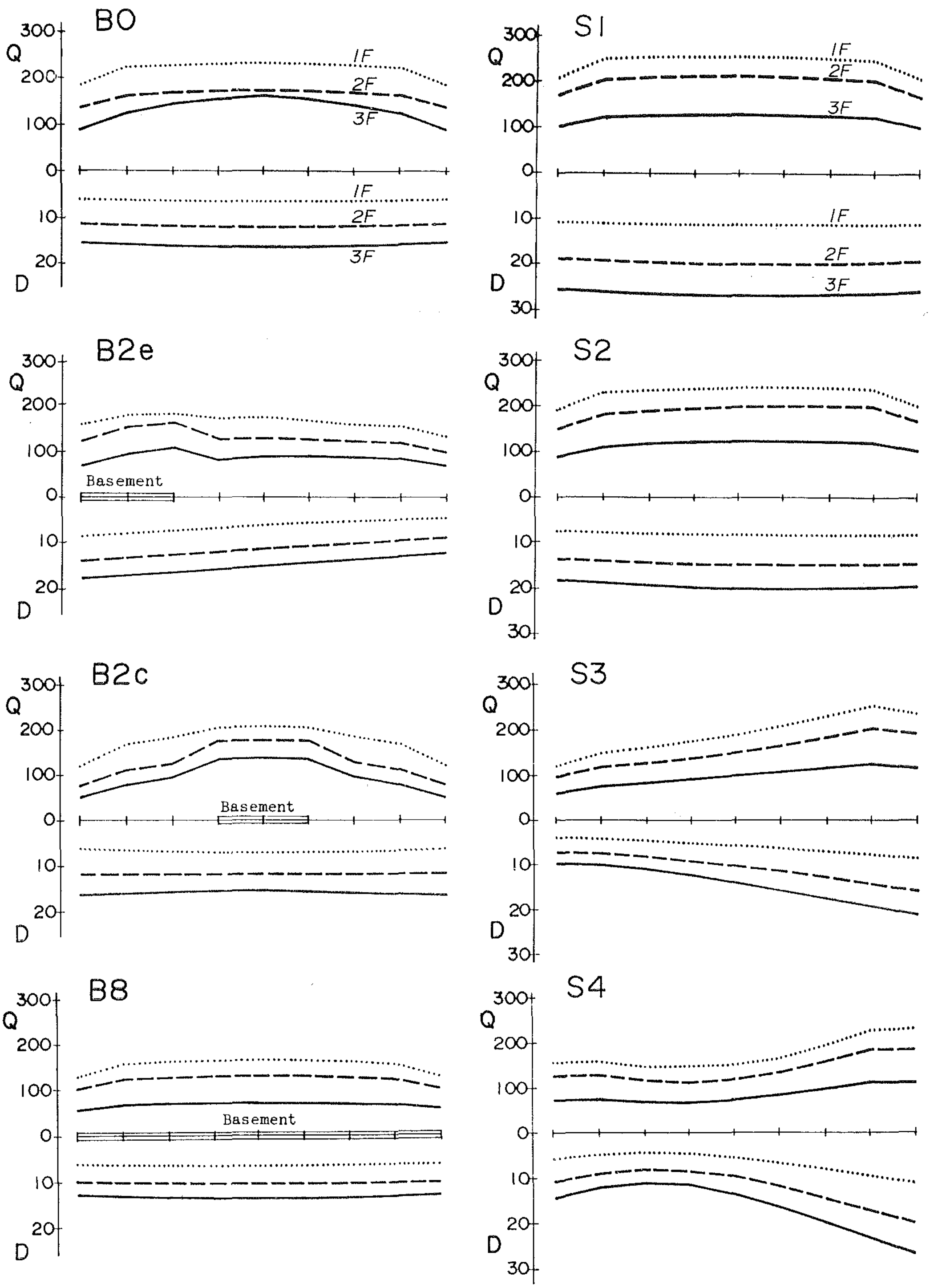

Fig. 6. Maximum response shear force of column, Q(ton), and maximum response displacement of mass point, D(cm).

exceedingly small.

Response values of the maximum displacements of masses and the maximum shear forces of columns are shown in Fig. 6 on all models.

The behaviors in the directions, "X" and "Y", are independent of each other in the case of buildings analyzed in the paper.

Discussion

(1) Comparison of the results of interaction system with those of each independent system—building or soil

Comparing the natural periods of the building-soil system "BO" with those of the prototype building "SO" with fixed foundations, both the first and second periods of the former are longer by about 30% than those of the latter, mainly by rotation in "BO". On the other hand, when the natural periods of "BO" and "SO" are compared with those of the prototype underground "S", the 4th to 7th periods of "SO" are accidentally close to the first period of "S" in spite of the independence of each other, so that the 4th to 7th periods of "BO" are drawn closer toward the first period of "S" as shown in Fig. 7.

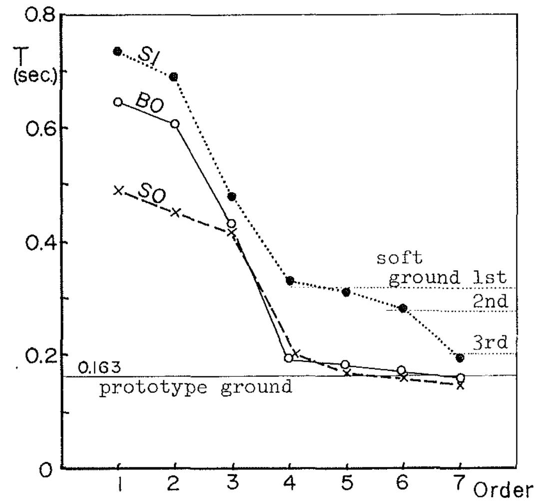

Fig. 7. Natural periods 'T' of each order in the model "SO", "$S1$" and "BO".

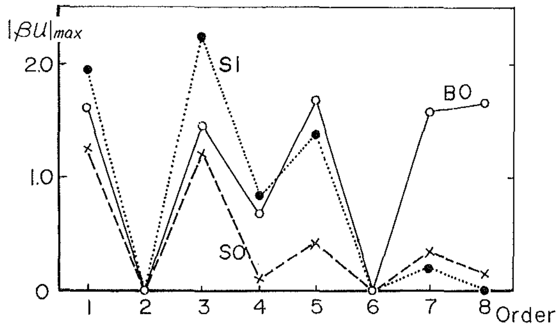

Fig. 8. '$|\beta u|_{\max}$' of each order in the model "SO", "$S1$" and "BO", where 'β' is participation factor and 'u' is eigen-vector.

In the case of the building-soil "$S1$" with the ground consisting of soft soil, the 4th to 7th periods of building-soil system also are remarkably close to the 1st to 3rd periods of the soft ground alone, while the 4th to 7th periods of "SO" are shorter than the 1st to 3rd periods of the soft ground.

Fig. 8 shows the maximum value of "βu" in each order on the model "BO", "SO" and "$S1$". The 3rd mode is the transverse motion in the direction "X". The 2nd and 6th modes are torsional motions, and "$\beta u_{\max}$" of these two modes are zero for uniformed input waves. The other modes except the above mentioned three modes are transverse motions in the direction "Y". In comparison between these modes "BO" or "$S1$" and "SO" in each order, it is confirmed that the 4th and 5th "βu_{max}" of "$S1$" and the 5th to 8th "βu_{max}" of "BO" are larger than the "βu_{max}" of "SO" and furthermore are close to the first period of the ground. It is recognized in the above discussion that a high order mode of a building-soil system should not be disregarded in the analysis when the period is close to the first period of the ground.

These results would depend not only on such inherent characteristics as the building-soil interaction, but also perhaps on the translation method of ground into an equivalent system, such as the width and depth of soil or the boundary conditions applied to the idealized model. Thus, it may be necessary to discuss further on the modeling of soil system.

(2) Vibration characteristics of the buildings with various basements

The response shear force of columns, Q, of the buildings with the basements is much different from the building "BO" without basement, particularly in higher stories. As is evident from Fig. 9 it can be seen that the shear force of columns in the top stories of all models, the shear force of "$B8$" with the same basement plan to the ground floor plan decreases to about two-thirds at the end of the columns, and to about one-third at the center column as compared with those of "BO".

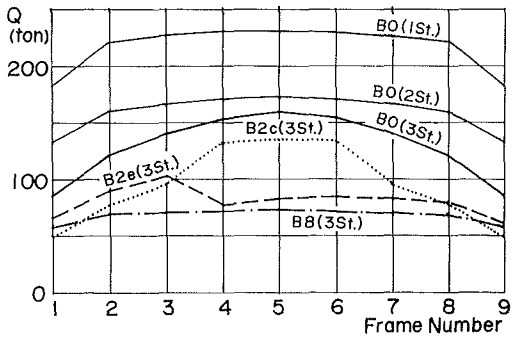

Fig. 9. Effect of the location of basements on maximum response shear force of columns, Q.

In the case of "$B2e$" with the two bays of basement at the end of the building, torsional vibration appears and the shear force of columns above the inner end of the basement is maximum in every story. Since the shear force distribution of columns in the building "$B2c$" with the two bays of basement at the center is extremely not uniform, the shear force of the center columns is about twice as much as that of the end in the first story (1St.) and is about three times in the third story (3St.). In other words, the shear force of the columns of the buildings with a partial basement is more than that of "$B8$" and less than that of "BO". The distribution of the shear force in the buildings with a partial basement is more uneven than both of "BO" and "$B8$", particularly the shear force of the columns above the basement is larger than that of other columns in the same story.

(3) Vibration characteristics of the buildings on dislocation or inclined layers

Both the natural modes and response values of the building "$S2$" on inclined and soft layers are not so much different from "BO" on the prototype ground condition, and torsional vibration appears slightly in spite of the nonsymmetrical ground conditions. In contrast, all modes of "$S3$" on inclined and hard layers are torsional, therefore the response value is nonsymmetrical and the maximum shear force of columns is larger on the soft soil side than "*BO*" with symmetrical ground condition as shown in Fig. 10. Because the stiffness distribution of every building is uniform in the vertical direction in section (3), the distribution shapes of response values are similar on every floor against each other in each building.

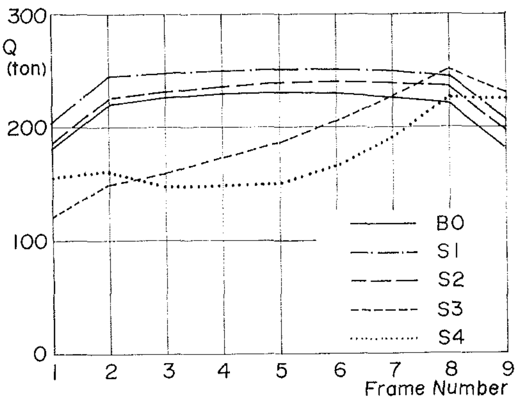

Fig. 10. Effect of the condition of grounds on maximum response shear force of columns in the first story, Q.

In the case of the building "*S4*" on dislocated ground, the distribution of response values is convex with minimum on the hard soil side, and the response values above the dislocation are relatively small.

Conclusions

So far as the analysis in the present paper is concerned, the characteristics of the building-soil system with a long and narrow floor plan may be summarized as follows.

1) The high order mode of the building-soil system generally neglected in modal analysis should not be disregarded when the period of the system is close to the prominent period of the ground.

2) The existence of basements generally effects the decrease of the shear force in spite of its shape. In buildings with partial basements, the shear force in the columns does not be distributed uniformly and takes a maximum value in the columns above the basement on each floor.

3) The vibration modes of the buildings on dislocation or inclined layers are torsional, and the response values are distributed unevenly on each floor.

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