Analysis of Chromatic Aberrations in Parametric Image-Upconversion Systems

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Abstract

This paper is concerned with chromatic and thickness aberrations in parametric image-upconversion systems in three cases as follows:

a) the chromatic aberrations of the upconverted image by a crystal are considered under conditions for eliminating thickness aberrations.

b) the effects of thickness of crystal on chromatic aberrations are generalized in the image-upconversion system given by Firester, hereafter referred to simply as F system.

c) the chromatic and thickness aberrations are investigated in the parametric image-upconversion system proposed by the authors, where the thick nonlinear crystal is placed in front of a chromatic lens which forms the upconverted image, hereafter referred to simply as MS system.

It is found that in the cases of a) and b) the finite thickness of the nonlinear crystal and chromatic aberrations due to dispersion of the crystal can not be eliminated at the same time, but the MS system can be free from the aberrations caused by dispersion and finite thickness of the crystal at the same time if the lens with properly chosen chromatic aberrations at upconverted wavelengths is used and the nonlinear crystal is properly located in front of the lens.

1. Introduction

Recently, the parametric image-upconversion system has stimulated great interest in real-time upconversion of infrared (IR) images to the visible region.[1]

A formula relating the location of the upconverted image to locations of the pump and the IR object is derived by Chiou[2] in the case where all rays are paraxial. He analyzed aberrations caused by the thickness of the nonlinear crystal and showed that these longitudinal-thickness aberrations vanish when both the pump wave and the object waves are parallel plane waves. Kumagami and Sakuraba[3] corrected image formation formulae of thick upconverter given by Chiou by a paraxial-ray tracing analysis[2] and exact calculations in

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the phase-match condition. It was shown in their analysis that the longitudinal-thickness aberration and the transverse image spread by the thickness do not exist in both cases where a pump source and an IR object are on the same transverse plane if the material is nondispersive and where a pump and an object are at the center of the dispersive material. The condition for eliminating the longitudinal thickness aberration and the transverse image spread by the thickness of crystal was given by Mishima and Sakuraba.(4) It was also pointed out by them(5) that phase matching configurations using extraordinary waves can introduce additional degradation to the image quality predicted by Chiou(2) and Kumagami and Sakuraba.(3)

Chromatic aberrations are caused by dispersion of the crystal material and image degradation caused by these aberrations becomes predominant for a large bandwidth upconverter, such as the one reported by Midwinter.(6) It is shown by Firester(7) that chromatic aberrations can be eliminated as a whole in the parametric image-upconversion systems where an object radiating two wavelengths is imaged by an objective chromatic lens and an infinitesimally thin crystal is located behind the lens and illuminated with a planar pump beam.

In this paper, chromatic aberrations of the upconverted image by a crystal are considered under conditions for eliminating thickness aberrations. The effects of thickness of the crystal on chromatic aberrations are also considered in the system given by Firester. Finally, a system, where a thick nonlinear crystal is placed in front of a chromatic lens, is proposed by the authors and the chromatic and thickness aberrations of this system itself are investigated theoretically.

To simplify analysis, the following assumptions are made:

(a) The two IR objects, the pump and the two sum-frequency waves are in phase-match condition throughout the interaction volume; the slight phase mismatch is neglected. The requirements for phase matched condition are

$$\omega_{ij} + \omega_p = \omega_{sj} \qquad (j = 1, 2), \tag{1}$$

$$k_{ij} + k_p = k_{sj} \qquad (j = 1, 2), \tag{2}$$

where the subscripts i, p, and s refer to IR, pump, and sum-frequency waves, respectively, ω is the angular frequency of the radiation, k is the corresponding wave vector and subscripts j=1 and 2 refer two IR objects and then they mean two sum-frequency waves.

(b) Because all rays are paraxial, all the angles are small enough to use the approximation $\tan\theta \fallingdotseq \theta$.

(c) Because crystal birefringence is negligibly small, the differences between directions of wave normals and those of rays are neglected.

2. Image Upconversion by a Crystal

2.1 Image Formation by a Crystal

Fig. 1 shows the geometry of thick upconverter image-formation. The polychromatic IR object is located at (s, h), the point monochromatic pump-source is located at $(p, 0)$, and the sum-frequency images formed at (s'_1, h'_1) and (s'_2, h'_2). These waves are interacted at (t, y_t) in the nonlinear material.

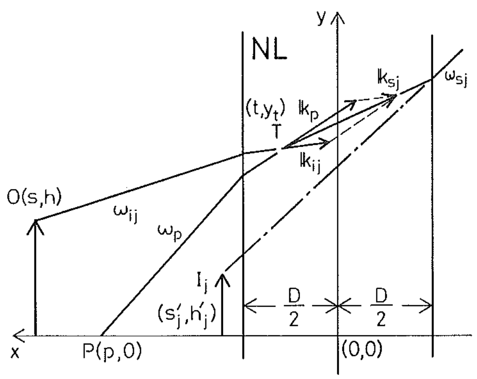

Fig. 1. Image formation for parametric upconversion by a single nonlinear crystal.

The upconverted image locations are given by

$$1/D_{sj} = \beta_j \,/\, D_{ij} + (1 - \beta_j) \,/\, D_p, \tag{3}$$

$$h'_j = h\beta_j D_{sj} \,/\, D_{ij}, \tag{4}$$

where

$$\left.\begin{aligned} \beta_j &= (n_{ij}\omega_{ij}) \,/\, (n_{sj}\omega_{sj}), \\ D_{ij} &= n_{ij}\,(s - 0.5D) + 0.5D - t, \\ D_p &= n_p\,(p - 0.5D) + 0.5D - t, \\ D_{sj} &= n_{sj}\,(s'_j + 0.5D) - 0.5D - t, \end{aligned}\right\} \tag{5}$$

and, ω_{ij}, ω_p and ω_{sj} are angular frequencies of the IR, pump and sum-frequency waves respectively, and n_{ij}, n_p and n_{sj} are indices of refraction for the IR, pump and sum-frequency waves respectively. The refractive indices n_{ij} and n_p are equal to unity if the object and the pump are inside the crystal.

Since both s'_j and h'_j are functions of t, the upconverted image suffers from thickness aberrations. Since both s'_j *and* h'_j depend on frequency of IR wave, the image suffers from chromatic aberrations, also.

2.2 Chromatic Aberrations in the Case where Thickness Aberrations are Eliminated

The condition for eliminating thickness aberrations is given by[4]

$$n_{i1}(s-0.5D)=n_{i2}\ (s-0.5D)=n_p(p-0.5D). \tag{6}$$

Under this condition given by Eq.(6), the upconverted image locations are given by

$$s'_j=(1/n_{sj})\left\{n_p\ (p-0.5D)+D\right\}-0.5D, \tag{7}$$

$$h'_j=h\beta_j. \tag{8}$$

Since s'_j and h'_j are independent of t, longitudinal aberrations and transverse image spread caused by finite thickness of the nonlinear material are eliminated.

The longitudinal chromatic-aberration, defined as the absolute value of difference between s'_1 and s'_2, is then

$$|\,s'_1-s'_2\,|=|\left\{n_p\ (p-0.5D)+D\right\}(n_{s2}-n_{s1})/(n_{s1}n_{s2})\,|. \tag{9}$$

It depends on pump location, material thickness and material properties.

The transverse image spread caused by chromatism, defined as absolute value of the difference between h'_1, and h'_2, is then

$$|\,h'_1-h'_2\,|=|\,h\,(\beta_1-\beta_2)\,|. \tag{10}$$

The aberration depends on frequencies of IR and refractive indices for IR, but it is independent of thickness of material and pump location.

In any event, it follows that, under the condition for eliminating thickness aberrations, the longitudinal chromatic aberration and transverse image spread by chromatism of the upconverted image by a crystal can not be eliminated at the same time.

3. Image Upconversion Systems Shown by Firester

Consider the image converter diagramed in Fig. 2 (F system). The IR radiation from an object 0 is collected by an objective lens L and the object is reimaged. A piece of nonlinear material NL is located behind the lens and illuminated with a pump beam of angular frequency ω_p. Firester has shown that if the leans has chromatic aberrations, then by properly locating the thin nonlinear material, the upconverted image can be made free of chromatic difference in longitudinal location but not chromatic difference in lateral position. In practice, the thickness of nonlinear crystal is neither infinitesimal nor infinite, so that the effect of nonlinear crystal thickness on image formation must be considered.

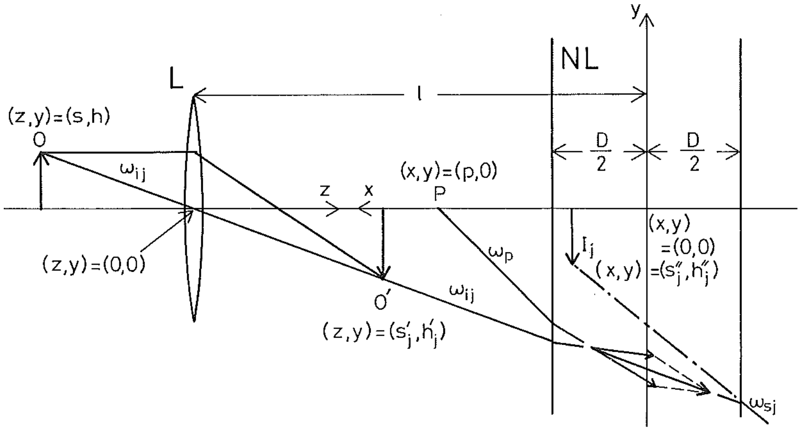

Fig. 2. Parametric image-upconversion systems for eliminating chromatic aberrations shown by Firester, where a finite thickness crystal is considered.

3.1 Image Formation by F System

The IR object 0 located at $(z, y)=(s, h)$ is reimaged at $0'_j$ $(z, y)=(s'_j, h'_j)$ by the thin chromatic lens L, then

$$1 / s'_j = 1 / s + 1 / f_j, \tag{11}$$

$$h'_j = h\, s'_j / s, \tag{12}$$

where f_j is the focal length of the lens for angular frequency ω_{ij}.

Equations expressing the image generated by thick nonlinear material are given by, in the new co-ordinate system xy,

$$1 / D_{sj} = \beta_j / D_{ij} + (1 - \beta_j) / \mathrm{D}_p, \tag{13}$$

$$h''_j = h'_j\, \beta_j\, D_{sj} / D_{ij}, \tag{14}$$

where

$$\left.\begin{aligned} D_{ij} &= n_{ij}\,(l - s'_j - 0.5D) + 0.5D - t, \\ D_p &= n_p\,(p - 0.5D) + 0.5D - t, \\ D_{sj} &= n_{sj}\,(s''_j + 0.5D) - 0.5D - t, \end{aligned}\right\} \tag{15}$$

and x=l-z and l is the distance between the lens L and the center of nonlinear crystal.

3.2 Chromatic Aberrations in the Case Thickness Aberrations are Eliminated

In the F system, the condition for eliminating thickness aberrations is given by

$$n_{i1}\ (l - s'_1 - 0.5D) = n_{i2}\ (l - s'_2 - 0.5D) = n_p\ (p - 0.5D). \tag{16}$$

If the objective lens is not corrected for chromatic aberrations and if the focal length of the lens are chosen by

$$1 / f_j = -1 / s + \{ l - 0.5D - n_p\ (p - 0.5D) / n_{ij} \}^{-1}, \tag{17}$$

then Eq.(16) is satisfied and the thickness aberrations are eliminated. The upconverted image locations are given by

$$s''_j = \{ n_p\ (p - 0.5D) + D \} / n_{sj} - 0.5D, \tag{18}$$

$$h''_j = (h\beta_j / s) \{ l - 0.5D - n_p\ (p - 0.5D) / n_{ij} \}. \tag{19}$$

The longitudinal chromatic aberration is given by

$$| s''_1 - s''_2 | = | \{ n_p\ (p - 0.5D) + D \} (n_{s2} - n_{s1})/(n_{s1} n_{s2}) |. \tag{20}$$

Thus by choosing $p{=}D(0.5{-}1/n_p)$, the longitudinal chromatic aberration vanishes.

The transverse image spread caused by chromatism is given by

$$| h''_1 - h''_2 | = | (h / s) \{ (l - 0.5D) (\beta_1 - \beta_2) - n_p\ (p - 0.5D) (\beta_1 / n_{i1} - \beta_2 / n_{i2}) \} |. \tag{21}$$

Thus by choosing $p{=}0.5D$ and $l{=}0.5D$, transverse image spread vanishes.

In any event, in the case thickness aberrations are eliminated, longitudinal chromatic aberration and transverse image spread caused by chromatism can not be eliminated at the same time.

3.3 Plane-Wave Pump-Beam Case

Plane-wave pump-beam incidence, normal to the crystal, is considered next. In this case, the pump distance p becomes infinite and the image is located at

$$s''_j = [\{ n_{ij}\ (l - s'_j - 0.5D) + 0.5D - t \} / \beta_j + 0.5D + t] / n_{sj} - 0.5D, \tag{22}$$

$$h''_j = h'_j. \tag{23}$$

Since s''_j depends on t and frequency of IR wave, the upconverted images suffer from longitudinal thickness aberration and longitudinal chromatic aberration.

Since h''_j depends on subscript j, the upconverted image suffers from transverse image spread caused by chromatism. But since h''_j is independent of t, transverse image spread caused by thickness of the crystal becomes zero. The point object is imaged into two straight lines I_1 and I_2 which are parallel to the x axis. The upconverted images are shown in Fig. 3.

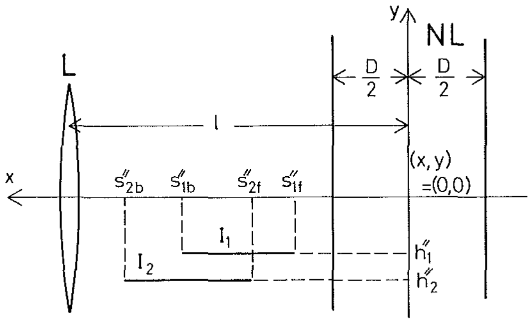

Fig. 3. Image formation in the Firester system where $p \to \infty$ and $D \neq 0$.

The transverse image spread caused by chromatism, defined as the distance between the straight lines I_1 and I_2, are given by

$$| h_1'' - h_2'' | = | hs (f_2 - f_1) / \{ (s + f_1)(s + f_2) \} |. \tag{24}$$

By choosing s=0 or $f_1 = f_2$, the quantity $|h_1'' - h_2''|$ becomes zero.

The sum-frequency-wave focal points, generated at the front and back surfaces of the material, are obtained by equating t to +0.5D and −0.5D, respectively:

$$s_{jf}'' = \{ n_{ij} (l - s_j' - 0.5D) / \beta_j + D \} / n_{sj} - 0.5D \quad \text{for } t = 0.5D, \tag{25}$$

$$s_{jb}'' = \{ n_{ij} (l - s_j' - 0.5D) + D \} / (\beta_j n_{sj}) - 0.5D \quad \text{for } t = -0.5D. \tag{26}$$

The longitudinal thickness aberration, defined as the difference between these two focal points, is then

$$| s_{jf}'' - s_{jb}'' | = | (D / n_{sj})(1 - 1 / \beta_j) |. \tag{27}$$

The longitudinal-thickness aberration for plane-wave pump-beam depends only on the material properties, but not upon the IR position. The aberration becomes zero only when D=0, since β_j is less than unity.

3.4 Chromatic Aberrations in the Case of Plane-Wave Pump-Beam and Infinitesimally Thin Material

Although the longitudinal chromatic aberration in this case was considered by Firester, the transverse image spread caused by chromatism has not been considered as yet.

The condition for eliminating longitudinal chromatic aberration is obtained by equating D to zero and s_1'' to s_2'' :

$$n_{i1}\ (l - s_1')\ /\ (\beta_1 n_{s1}) = n_{i2}\ (l - s_2')\ /\ (\beta_2 n_{s2}). \tag{28}$$

Then it follows that

$$\Delta_\lambda\ (l - s_1') + (\lambda_p + \lambda_{i1})\ (s_1' - s_2') + \Delta_\lambda\ (s_1' - s_2') = 0, \tag{29}$$

where Δ_λ is the difference between two IR wavelengths. Thus by choosing

$$l - s_1' = \{(\lambda_p + \lambda_{i1})\ /\ \Delta_\lambda\}\ (s_2' - s_1'), \tag{30}$$

the longitudinal location of the upconverted image is independent of wavelength to the first ofder. This is the condition introduced by Firester(7). The exact condition for eliminating the longitudinal chromatic aberration is given by

$$l = \{s_2'\ (\lambda_p + \lambda_{i2}) - s_1'\ (\lambda_p + \lambda_{i1})\}\ /\ \Delta_\lambda. \tag{31}$$

Under the condition, transverse image spread caused by chromatism is considered next. For this case, the transverse image spread is given by Eq.(24). It is independent of l, but depends on focal length of the lens and the IR object location. Evidently there will be zero of the transverse image spread caused by chromatism when $f_1 = f_2$ or $s=0$.

The conditions for eliminating thickness and chromatic aberrations in the F system are summarized in Table 1. In any event, it follows that in the F system the aberrations caused by the finite thickness of the nonlinear crystal and the chromatic aberrations due to the dispersion of the crystal can not be eliminated at the same time.

Table 1 Conditions for eliminating thickness and chromatic aberrations in the Firester system.

Case		Condition	
		Longitudinal condition	Lateral condition
Thick nonlinear material	Thickness aberation is eliminated	$p = D\ (0.5 - \frac{1}{n_p})$	$p = 0.5D$ and $l = 0.5D$
	Plane-wave pump-beam		$f_1 = f_2$ or $s = 0$
Thin nonlinear material and plane-wave pump-beam		$l = \dfrac{s_2'\ (\lambda_p + \lambda_{i2}) - s_1'\ (\lambda_p + \lambda_{i1})}{\Delta_\lambda}$	$f_1 = f_2$ or $s = 0$

4. Image Upconversion Systems for Eliminating Chromatic and Thickness Aberrations

The chromatic and thickness aberrations are next investigated in the parametric image-upconversion system as shown in Fig. 4 (MS system), where the thick nonlinear material NL is placed in front of a chromatic lens L. In the MS system, an IR object radiating two angular frequencies ω_{i1} and ω_{i2} is imaged with a different location because of thickness and chromatism of nonlinear material. These differences can be corrected by the lens having an appropriate chromatic aberration.

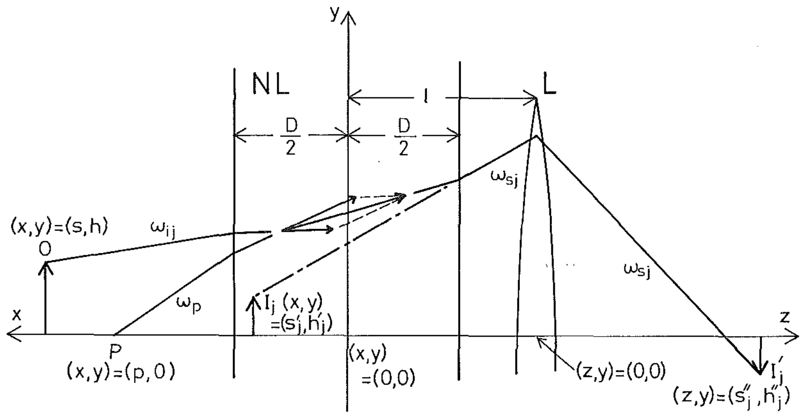

Fig. 4. Parametric image-upconversion systems for eliminating chromatic and thickness aberrations at the same time by proper location of the nonlinear crystal in front of the lens of properly chosen chromatic aberrations.

4.1 Image Formation in the MS System

The IR object 0 located at $(x, y)=(s, h)$ is upconverted at $(x, y)=(s_j', h_j')$ by a thick nonlinear material NL with a point pump source P located at $(x, y)=(p, 0)$, then Eqs.(3),(4) and (5) are obtained. Under the condition for eliminating thickness aberrations given by Eq.(6), the upconverted image locations $(x, y)=(s_j', h_j')$ are given by Eqs.(7) and (8).

The upconverted images I_1 and I_2 are reimaged by a thin chromatic lens L located at the back of the material NL. Then the image locations $(z, y)=(s_j'', h_j'')$ are given by

$$1/s_j'' = -1 \,/\, (l + s_j') + 1 \,/\, f_j, \tag{32}$$

$$h_j'' = -h_j's_j'' \,/\, (l + s_j'), \tag{33}$$

where $z=-l-x$, l is the distance between the material NL and the lens L, and f_j

is the focal length of the lens L for angular frequency ω_{sj}.

4.2 Chromatic Aberrations in the Case Thickness Aberrations are Eliminated

In this case, the longitudinal chromatic aberration is given by,

$$|s_1'' - s_2''| = |\{1/f_1 - 1/(l+s_1')\}^{-1} - \{1/f_2 - 1/(l+s_2')\}^{-1}|. \quad (34)$$

The quantity $|s_1''-s_2''|$ can be calculated by Eqs.(34) and (7). The image can be made free of the longitudinal chromatic aberration by properly locating the lens L:

$$l = -(s_1' + s_2')/2 \pm \sqrt{(s_1' - s_2')/(1/f_2 - 1/f_1) + \{(s_1' - s_2')/2\}^2}. \quad (35)$$

It is noted that the value of the radical of Eq.(35) is the plus sign because the quantity of $(1/f_2 - 1/f_1)$ can be chosen for the plus or minus sign. It is also noted that the upconversion system can be realized as the value of the right hand side of Eq.(35) is larger than $0.5D$.

The transverse image spread caused by chromatism is shown by

$$|h_1'' - h_2''| = |h_1'f_1/(l+s_1'-f_1) - h_2'f_2/(l+s_2'-f_2)|. \quad (36)$$

The quantity $|h_1''-h_2''|$ can be calculated by Eqs.(36), (7) and (8). The image can be made free of the transverse image spread by chromatism by properly locating the lens L:

$$l = \{h_2'f_2(s_1'-f_1) - h_1'f_1(s_2'-f_2)\}/(h_1'f_1 - h_2'f_2). \quad (37)$$

If the right hand side of Eq.(37) is greater than $0.5D$, the transverse image spread caused by chromatism can be eliminated.

4.3 Condition for Eliminating Thickness and Chromatic Aberrations Simultaneously

To eliminate the thickness aberrations, the relationship between the locations of IR object 0 and pump source P should be given by Eq.(6). Under the condition for eliminating thickness aberrations the chromatic aberrations are considered next. The condition for eliminating the longitudinal chromatic aberration and the transverse image spread caused by chromatism simultaneously, is obtained by equating the right hand sides of Eqs.(34) and (36) to zero:

$$l = (s_2'h_1' - s_1'h_2')/(h_2' - h_1') \quad (38)$$

and

$$1/f_1 - 1/f_2 = (h_1' - h_2')^2/\{h_1'h_2'(s_2' - s_1')\}. \quad (39)$$

Combining Eqs.(7), (8), (38) and (39), it follows that

$$l = \frac{D}{2} + \frac{\beta_1 / n_{s2} - \beta_2 / n_{s1}}{\beta_2 - \beta_1} \times \{ n_p (p - 0.5D) + D \} \tag{40}$$

and

$$\frac{1}{f_1} - \frac{1}{f_2} = \frac{(\beta_1 - \beta_2)^2}{\beta_1 \beta_2} \times \frac{n_{s1} n_{s2}}{n_{s1} - n_{s2}} \times \frac{1}{n_p (p - 0.5D) + D} \tag{41}$$

It should be noted that the arrangement of optical elements can be realized in the case where the value of l is larger than $0.5D$.

It is possible to calculate the condition eliminating thickness and chromatic aberration at the same time by Eqs.(6), (38) and (39) or (6), (40) and (41). Under the condition of Eqs.(6), (40) and (41), the image which is freed of thickness and chromatic aberrations is located at

$$s_1'' = s_2'' = [\, 1/f_1 - (h_1' - h_2')/\{ h_1' (s_1' - s_2') \} \,]^{-1}, \tag{42}$$

$$h_1'' = h_2'' = -s_1'' (h_1' - h_2') / (s_1' - s_2'). \tag{43}$$

Combining Eqs.(42), (8) and (39), s_1'' and s_2'' become

$$s_1'' = s_2'' = (\beta_1 - \beta_2) / \{ (\beta_1 / f_1) - (\beta_2 / f_2) \} . \tag{44}$$

And combining Eqs.(43), (44), (8) and (39), it follows that

$$h_1'' = h_2'' = h\beta_1\beta_2 \{ (1/f_1) - (1/f_2) \} / \{ (\beta_1/f_1) - (\beta_2/f_2) \} . \tag{45}$$

The location which is made free of thickness and chromatic aberrations depend on β_1, β_2, f_1, f_2 and h.

4.4 Chromatic Aberrations in the Case of Infinitesimally Thin Material

In this case, locations of upconverted image I_j are obtained by equating D to zero and t to zero:

$$1/(n_{sj} s_j') = \beta_j / (n_{ij} s) + (1 - \beta_j) / (n_p p), \tag{46}$$

$$h_j' = h\beta_j n_{sj} s_j' / (n_{ij} s). \tag{47}$$

Since image formation by the lens L is the same as that in the case of section 4.1, the longitudinal chromatic aberration is given by Eq.(34), and the transverse image spread caused by chromatism is shown by Eq.(36).

The aberration free condition is given by Eqs.(38) and (39) or by Eqs.(40) and (41).

5. Conclusions

Chromatic and thickness aberrations of parametric image-upconversion systems were investigated in this paper. The nonlinear interaction between each object wave at two different wavelengths and a monochromatic pump wave was considered. The preliminary result of this paper has been reported in a communication.[8]

Chromatic aberrations could be summarized as follows;

1) Chromatic aberrations of the upconverted image by a crystal are investigated under the condition for eliminating thickness aberrations. In this case the chromatic aberrations can not be eliminated.

2) The aberrations caused by the finite thickness of the nonlinear crystal and the chromatic aberrations due to the dispersion of the crystal can not be eliminated at the same time in the upconversion system shown by Firester.

3) The upconversion system where the thick nonlinear crystal is placed in front of a chromatic lens is proposed by authors (MS system).

4) The MS system can be free from the aberrations caused by dispersion and finite thickness of the crystal at the same time if the lens with properly chosen chromatic aberrations at upconverted wavelengths is used and the nonlinear crystal is properly located in front of the lens.

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References

1) Sakuraba, I.: "The magnification for optical parametric image conversion in thin nonlinear materials", Bulltein of the Faculty of Engineering, Hokkaido University, No.61, pp.27–31, March 1971.

2) Chiou, W.: "Geometric optics theory of parametric image upconversion", J. Appl. Phys., vol.42, pp.1985–1993, April 1971.

3) Kumagami, T. and Sakuraba, I.: "Image formation formulae in parametric image upconversion", J. IECE. Japan, vol.56-C, pp.129–130, February 1973 (in Japanese).

4) Mishima, T. and Sakuraba, I.: "Thickness aberrations in parametric image upconver-

sion", J. IECE. Japan, vol.56-C, pp.508–509, August 1973 (in Japanese).

5) Mishima, T. and Sakuraba, I.: "Phase-match condition and image formation in parametric image upconversion", J. IECE. Japan, vol.57-C, pp.229–230, July 1974 (in Japanese).

6) Midwinter, J. E.: "Infrared up-conversion in lithium-niobate with large bandwidth and solid acceptance angle", Appl. Phys. Lett., vol.14, pp.29–32, January 1, 1969.

7) Firester, A. H.: "Image upconversion: part III", J. Appl. Phys., vol.41, pp.703–709, February 1970.

8) Mishima, T. and Sakuraba, I.: "Chromatic aberrations in parametric image-upconversion systems", J. IECE. Japan, vol.57-C, pp.312–318, September 1974 (in Japanese).