A Study on Evaluation of the Thermal Radiation Effect

—Mean Radiant Temperature Weighted with the Absorption Factor—

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Abstract

As one of the elements influencing thermal sensation we have the surrounding wall temperature, and hitherto average radiant temperatures weighted with area ratio and with angle factor have been used in such investigations.

The present authors investigated this fact and at the same time by extending and developing Gebhart's absorption factor, a new concept of average radiant temperature which includes the emissivity of each wall in addition to angle factors between the human body and the surrounding walls was derived. The average radiant temperature takes into consideration the heat balance in addition to the geometric position relation to the human body in a room and the surrounding walls. The characteristics thereof are reported here.

1. Introduction

The main factors on the environmental side which exert an influence on thermal sensation are temperature, moisture and air current. Of these the environmental temperature is divided into air temperature and radiant temperature, and moreover the former plays a role in convective heat release which is exchanged between the human body and the ambient air. On the other hand the latter, namely radiant temperature is a factor which governs the heat loss by radiation from the body surface. In the course of its thermal give and take, the human body releases a net heat into all objects which are colder than the human body. And in reverse, the human body receives radiation heat from all objects with a higher temperature than the human body. As examples of these phenomena, we encounter numerous examples in daily life for instance the heated wall effect in summer in an office where the windows and walls are heated up by the sun, or for another instance cold radiation felt by persons in a frigid climate seated near the window. The radiation effect against the comfort of human beings has an important significance and recently numerous experimental research work related to this problem are being reported.

The radiation exchange can be calculated by the difference of raising to the

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4th power of the absolute temperature of the surfaces of objects, however for practical designing of air conditioning the surface temperature of each room wall by some method is used. In this averaging hitherto, the surface temperature of each wall is weighted by the surface area ratio or the angle factor between the human body and the wall, however in the present report a new concept of average radiant temperature is proposed. Here, the averaging is done by weighting with the absorption factor between the body and the surrounding walls. The drawback of these methods is that the average radiant temperature weighted with the surface area ratio can not be used for the evaluation of radiant temperature distribution of a human body in a room especially at various positions in the room. On the other hand, while the average radiant temperature with due consideration to the angle factor has eliminated the drawback of the area ratio average, in both cases the radiant temperature can be obtained by the simple relation between the person in the room and the dimension of the surrounding walls and position, in other words, the radiant temperature can only be obtained by a simple geometrical relation. In contrast the present authors' average radiant temperature weighted with the absorption factor not only gives the geometrical relation of the position of the person and the surrounding walls, but also gives the emissivity of all walls with due consideration to the heat balance. In the present paper, the induction of the radiant temperature averaged and weighted with the absorption factor and its characteristics together with a comparison with the radiant temperature obtained by averaging methods used hitherto were studied and the results will be reported.

2. Radiation heat transfer—method of expression using the absorption factor

In a closed space, regarding the radiation exchange between the various walls, Gebhart using the absorptoin factor expressed the results by using the so-called raising to the 4th power law.[1] This method is a calculating means which takes into consideration the direct radiation between walls and also the reciprocal radiation, the energy balance of the target wall following reciprocal radiation was also considered and the calculation of the heat balance equation and absorption factor is expressed by the two following equations.

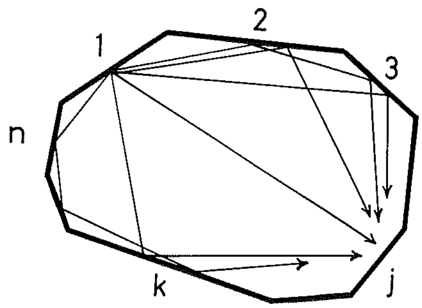

Fig. 1. Reciprocal thermal radiation.

$$Q_j = R_j S_j - B_{1j} R_1 S_1 - B_{2j} R_2 S_2 - \cdots - B_{jj} R_j S_j - \cdots - B_{nj} R_n S_n$$

$$= R_j S_j - \sum_{i=1}^{n} B_{ij} R_i S_i \qquad (1)$$

$$\Phi_{nj} \varepsilon_j + \Phi_{n1} \rho_1 B_{1j} + \cdots + (\Phi_{nn} \rho_n - 1) B_{nj} = 0 \qquad (2)$$

where

$R \equiv \varepsilon \cdot \sigma \cdot (T+273)^4$

Q_j : the net volume energy loss by radiation heat from the wall j [kcal/h]
R_i : the radiation heat flux at the wall i [kcal/m²h]
S_i : the area of the wall i [m²]
B_{ij} : the absorption factor (the wall S_i to the wall S_j) [−]
Φ_{ij} : the angle factor (the wall S_i to the wall S_j) [−]
T : the surface temperature [°C]
ε : emissivity [−]
ρ : reflectivity $(=1-\varepsilon)$ [−]
σ : Stefan-Boltzmann's constant $(=4.88\times10^{-8})$ [kcal/m²h°K⁴]

3. The Application of Absorption Factor to the Human Body and the Surrounding Walls

The absorption factor defined by Gebhart is mainly a means of expression of radiation heat exchange between walls, and moreover it is governed by a raising to the 4th power law.

The present authors extended this method and applied it to the space between the human body and walls, and an attempt was made to linearize from raising the radiation to the 4th power and radiant temperature under a new concept and the radiative heat transfer coefficient was discussed.

Let us assume that in a closed space from an arbitrary wall j the net volume energy loss by radiation heat, may be obtained by the previously described equation (1) according to Gebhart.

Next let us assume a human body s at an arbitrary position in a closed space. The body has a minimal element. Equation (1) in which considerations are made for the reciprocal radiation exchange in the give and take of the radiation heat between various walls may be expressed by the following equation if applied between the human body and the surrounding walls. The meaning "element" will be dealt with in chapter 4.

$$Q_r = R_s A_s - \sum_{i=1}^{n} b_{is} R_i S_i \qquad (3)$$

where

Q_r : the net volume energy loss by radiation heat from the human body [kcal/h]
R_s : the radiaton heat flux at the human body surface [kcal/m²h]
A_s : the effective radiation area of the human body [m²]
S_i : the surface area of the wall i [m²]
b_{is} : the absorption factor between the wall i and the human body [−]

In the above equation since $R_s=\varepsilon_s\sigma(273+T_s)^4$, $R_i=\varepsilon_i\sigma(273+T_i)^4$ equation (3) may be rewritten as follows.

$$Q_r = \varepsilon_s \sigma (273 + T_s)^4 A_s - \sum_{i=1}^{n} b_{is} \varepsilon_i \sigma (273 + T_i)^4 S_i \quad (4)$$

In addition because special feature of the absorption factor, $\sum_{i=1}^{n} b_{si} = 1$ and moreover since $b_{si} \varepsilon_s A_s = b_{is} \varepsilon_i S_i$, equation (4) may be transformed to equation (5).

$$\begin{aligned} Q_r &= \sum_{i=1}^{n} b_{si} \varepsilon_s \sigma (273 + T_s)^4 A_s - \sum_{i=1}^{n} b_{is} \varepsilon_i \sigma (273 + T_i)^4 S_i \\ &= \sum_{i=1}^{n} b_{si} \varepsilon_s \sigma (273 + T_s)^4 A_s - \sum_{i=1}^{n} b_{si} \varepsilon_s \sigma (273 + T_i)^4 A_s \\ &= \sum_{i=1}^{n} b_{si} \varepsilon_s \sigma \left[(273 + T_s)^4 - (273 + T_i)^4 \right] A_s \quad (5) \end{aligned}$$

When equation (5) is subjected to factorization and rearranged, ultimately the following equation is obtained.

$$Q_r = \varepsilon_s \sigma k \left(\sum_{i=1}^{n} b_{si} T_s - \sum_{i=1}^{n} b_{si} T_i \right) A_s \quad (6)$$

where

Q_r : the radiation heat energy [kal/h]
b_{si} : the absorption factor between the human body and the wall i [—]
ε_s : the emissivity of the human body [—]
σ : Stefan-Boltzmann's constant $(=4.88 \times 10^{-8})$ [kcal/m^{2}h°K^4]
T_s : the surface temperature of the human body [°C]
T_i : the surface temperature of the wall i [°C]
A_s : the effective radiation area of the human body [m^2]

$$k \equiv \left[(T_s + 273)^2 + (T_i + 273)^2 \right] \cdot \left[(T_s + 273) + (T_i + 273) \right]$$

The temperature factor k in equation (6) is as shown in Fig. 2. Since the change is gradual, if the temperature range is limited, it becomes possible to regard it as a constant.

Further, in radiation heat exchange in an ordinary room, since long waves are mainly involved, objects generally seen in the room may be considered as

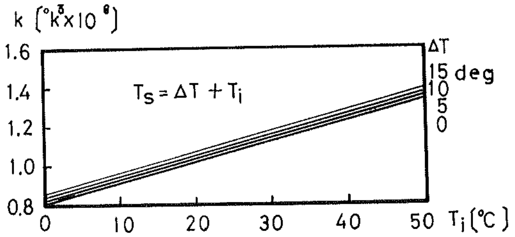

Fig. 2. Change of the temperature factor k.

gray bodies, and in addition the skin of human bodies and garment surface would have an emissivity close to that of a black body. Therefore, when the temperature range is assumed in a day to day living area, and if $\varepsilon_s \sigma k$ in equation (6) is collectively treated as a constant, the radiative heat transfer coefficient h_r may be expressed by the following equation.[2)]

$$h_r = \varepsilon_s \sigma k \tag{7}$$

Into equation (7) if the following values are substituted as concrete values in a day to day living area, as the radiative heat transfer coefficient h_r, $h_r \fallingdotseq 4.8$ kal/m²h °C can be obtained.[3)]

The values substituted ;

$$\varepsilon_s = 0.95 \quad [-]$$

$$\sigma = 4.88 \times 10^{-8} \quad [\mathrm{kcal/m^2 h ^\circ K^4}]$$

$$k = 1.05 \times 10^{8} \quad [^\circ \mathrm{K}^3]$$

Further, when emissivity ε_s or the temperature range is remarkably different from the above, as a matter of course the value h_r would follow that a value in accordance with the circumstances should be used.

In addition when equation (6) is expressed by the newly designated radiative heat transfer coefficient h_r, and a replacement by $\Sigma b_{si} T_i \equiv T_r$ is made, since $\Sigma b_{si} = 1$, the radiation heat transfer volume Q_r may be expressed by the following equation.

$$Q_r = h_r (T_s - T_r) A_s \tag{8}$$

where

T_r : the Environmental Radiant Temperature (the mean radiant temperature weighted by absorption factor)$\equiv \Sigma b_{si} T_i$ [°C]

Here in order to avoid confusion with other averages, the average radiant temperature weighted with the absorption factor in which the angle factor (cylinder element which will be described later) between the human body and each surface is included, will be referred to as the "environmental radiant temperature".[2,3,5,6)]

4. Discussion and evaluation of various types of average radiant temperatures

4-1. Radiant temperature weighted and averaged with area ratio

This is the average radiant temperature used hitherto generally, where the various wall temperatures are weighted with area ratio and is defined by the following equation.

$$\text{The mean radiant temperature weighted by the wall surface area ratio} = \frac{\Sigma S_i T_i}{\Sigma S_i} \tag{9}$$

where

S_i : the surface area of the wall i
T_i : the surface temperature of the wall i

However, as may be seen from equation (9) in radiant temperature weighted with area ratio, the difference arising from the location or position of the heat releasing body does not appear ; this constitutes a drawback in which evaluation of the distribution of radiant temperature effect can not be made.

4–2. Radiant temperature weighted and averaged with the angle factor

In order to remove the drawback of area ratio average radiant temperature which is not a function of position, taking the heat release object as a minute element, we may consider a means by weighting and averaging by a solid angle with the various walls. In order to differentiate radiant temperature designated by equation (9) and (11) which will be described later the radiant temperature averaged by the angle factor will be referred to as the "surrounding radiant temperature".

$$\text{The mean radiant temperature weighted by angle factors (the surrounding radiant temperature)} = \frac{\Sigma \phi_i T_i}{\Sigma \phi_i} \tag{10}$$

where

ϕ_i : the angle factor between the human body and the wall i ($\Sigma \phi_i = 1$)

This surrounding radiant temperature includes all radiant temperature weighted by the angle factor and is different from radiant temperatures weighted by area ratio.

When a human body is assumed to be the heat release body, it may be considered that by dealing with this as a cylinder element, this may be said to be a step forward as compared with assuming that it is a sphere element. Regarding this Ibamoto and Nishi derived the relation of angle factor against a cylinder element in the following manner.[4)]

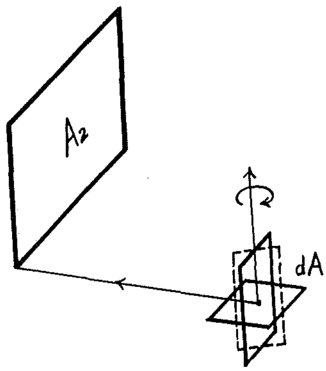

Fig. 3. Relation between flat surface element and flat surface.

As shown in Fig. 3, the angle factor relation between a minute flat surface element opposing a limited flat surface, has been analysed theoretically in the field of illumination technology and has been obtained by the surface integration method and the contour integration method. Now, if the minute surface element in Fig. 3 is rotated once around its own perpendicular axis, a minute cylinder element is formed. Utilizing this the angle factor relation of minute cylinder element may be lead forth as shown in Fig. 4 and 5.

Further, the above element is not a mere dot, and it does not have a dimen-

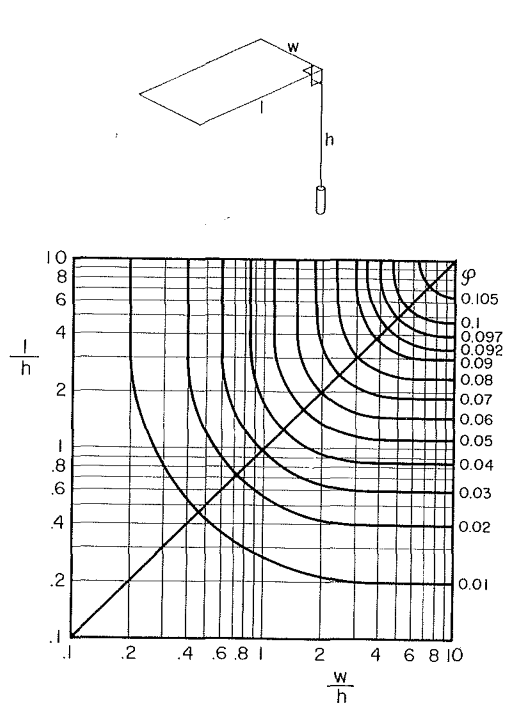

Fig. 4. Angle factor between cylinder element and horizontal rectangle[4].

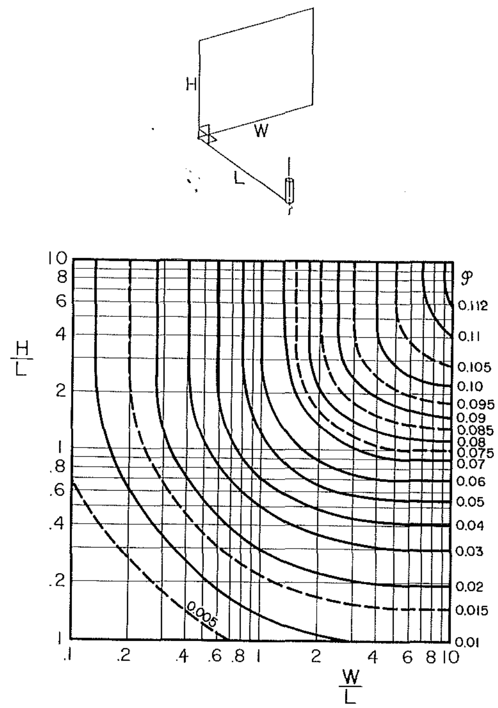

Fig. 5. Angle factor between cylinder element and vertical rectangle[4].

sion, but the direction of the surface alone may be considered. In other words, it follows the same thinking as in illumination technology when the minute recipient reflecting surface and the light source surface are in parallel or at right angles where the illuminance calculations are different.

When the human body is assumed to be a cylinder element, the surrounding radiant temperature calculated from Fig. 4, 5 and equation (10) is a definite advance over the radiant temperature weighted and averaged by a sphere element.

4–3. Radiant temperature weighted and averaged by the absorption factor

The angle factor of element and wall can only be decided by the geometrical position.

Now, the new radiant temperature equation (8) previously lead forth by the present authors, is expressed in a similar form to equation (9) and (10), we get the next equation. Further as previously described to avoid confusion equation (11) will be referred to in this paper as "the environmental radiant temperature".

$$\text{The mean radiant temperature weighted by the absorption factors (the environmental radiant temperature)} = \frac{\Sigma b_i T_i}{\Sigma b_i} \tag{11}$$

where

b_i : the absorption factor between the human body and the wall $i(\Sigma b_i=1)$

The characteristic feature of the enviromental radiant temperature which encloses the angle factor against cylinder element, is that considerations are not only made for the geometric position but also for the emissivity of each wall surface in the absorption factor b_i.

4-4. Special features of the environmental radiant temperature

The special features of the environmental radiant temperature described above are investigated using basic examples, in addition a comparative investigation is conducted on the actual number values which arise from various radiant temperatures brought about by the difference in averaging.[3,7)]

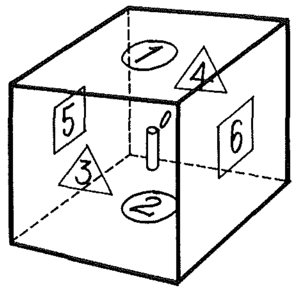

Fig. 6. Cubic room and situation of element.

Example 1. In the cubic room shown in Fig. 6 when the spherical or the cylinder element is No. 0, calculate the absorption factors $b_{01} \sim b_{06}$ between these elements and each wall.

Solution 1. Gebhart's method is extended, and the elements will be considered as number zero wall. In other words various elements do not hinder the radiation between the walls themselves, and it will be considered that only the value of the absorption factor $b_{0i}(\equiv b_i)$ exsists. To the radiant energy balance of each wall (including the element), when equation (2) is applied and rearranged the following equation is obtained.

$$\left.\begin{array}{l}
-b_j+\phi_{10}\rho_1 B_{1j}+\phi_{02}\rho_2 B_{2j}+\phi_{03}\rho_3 B_{3j}+\phi_{04}\rho_4 B_{4j}+\phi_{05}\rho_5 B_{5j}+\phi_{06}\rho_6 B_{6j}=-\phi_{0j}\varepsilon_j \\
\phi_{10}\rho_0 b_j \quad -B_{1j}+\Phi_{12}\rho_2 B_{2j}+\Phi_{13}\rho_3 B_{3j}+\Phi_{14}\rho_4 B_{4j}+\Phi_{15}\rho_5 B_{5j}+\Phi_{16}\rho_6 B_{6j}=-\Phi_{1j}\varepsilon_j \\
\phi_{20}\rho_0 b_j+\Phi_{21}\rho_1 B_{1j} \quad -B_{2j}+\Phi_{23}\rho_3 B_{3j}+\Phi_{24}\rho_4 B_{4j}+\Phi_{25}\rho_5 B_{5j}+\Phi_{26}\rho_6 B_{6j}=-\Phi_{2j}\varepsilon_j \\
\phi_{30}\rho_0 b_j+\Phi_{31}\rho_1 B_{1j}+\Phi_{32}\rho_2 B_{2j} \quad -B_{3j}+\Phi_{34}\rho_4 B_{4j}+\Phi_{35}\rho_5 B_{5j}+\Phi_{36}\rho_6 B_{6j}=-\Phi_{3j}\varepsilon_j \\
\phi_{40}\rho_0 b_j+\Phi_{41}\rho_1 B_{1j}+\Phi_{42}\rho_2 B_{2j}+\Phi_{43}\rho_3 B_{3j} \quad -B_{4j}+\Phi_{45}\rho_5 B_{5j}+\Phi_{46}\rho_6 B_{6j}=-\Phi_{4j}\varepsilon_j \\
\phi_{50}\rho_0 b_j+\Phi_{51}\rho_1 B_{1j}+\Phi_{52}\rho_2 B_{2j}+\Phi_{53}\rho_3 B_{3j}+\Phi_{54}\rho_4 B_{4j} \quad -B_{5j}+\Phi_{56}\rho_6 B_{6j}=-\Phi_{5j}\varepsilon_j \\
\phi_{60}\rho_0 b_j+\Phi_{61}\rho_1 B_{1j}+\Phi_{62}\rho_2 B_{2j}+\Phi_{63}\rho_3 B_{3j}+\Phi_{64}\rho_4 B_{4j}+\Phi_{65}\rho_5 B_{5j} \quad -B_{6j}=-\Phi_{6j}\varepsilon_j
\end{array}\right\} \quad (12)$$

where

$b_j(=b_{0j})$: the absorption factor (human body to wall)
B : the absorption factor (wall to wall)
ϕ : the angle factor (human body to wall)
Φ : the shape factor (wall to wall)
ε : emissivity
$\rho(=1-\varepsilon)$: reflectivity

By solving simultaneous equation (12), the absorption factors between the human body and the walls $b_1 \sim b_6$ can be obtained.

Here, the human body is taken as an element, and we have $\phi_{10}=\phi_{20}=\cdots=\phi_{60}=0$ when the multiplication opponent ρ_0 disappears from the simultaneous equation.

Further, especially in the case of a cube, since all $\Phi=0.2$, the calculated results are aggregated and the following important meaning was found.

I. When a sphere element (a cube is also possible) is at the center of the cubic room space and when equal ϕ values ($\phi=1/6$) exsist,

I-1. When all other values are equal ρ with the exception of ρ_* solving the simultaneous equation gives the following equation.

$$b_* = \frac{1}{6} \times \frac{(5+\rho)\,\varepsilon_*}{(5-\rho)-(3+\rho_*)\,\rho}$$

I-2. When all are equal ρ, we have $b_j=1/6$, which results in no relation to the surrounding ρ value.

II. When a non sphere element is at the center of the room space or when a sphere element which departs from the center of the room space exsists and unequal ϕ values exsist,

II-1. When all are of equal ρ we get the following equation.

$$b_j = \frac{5\varepsilon}{5+\rho}\,\phi_j + \frac{\rho}{5+\rho}$$

Only when the surrounding wall is a complete black body ($\rho=0$), $b_j=\phi_j$.

II-2. When ρs are unequal, a strong influence, which makes formulating almost impossible, is exerted upon b_* value by ρ_*. Therefore, in general cubes it becomes still more complicated, however if a computer is used it is easy to obtain real number values.

Example 2. In a room such as in Fig. 7 when a human element (cylinder) is placed at 1 meter above the floor center, the absorption factors $b_1 \sim b_6$ between the human body and each wall surface and the environmental radiant temperature are calculated. Here, the emissivity of each wall is $\varepsilon_{1\sim5}=0.9$, $\varepsilon_6=0\sim1$ and the wall surface temperature in the room is $T_{1\sim5}=20°\mathrm{C}$, $T_6=5°\mathrm{C}$.

Solution 2. When the human element is considered as the number zero wall, since the model room is a 6 faced body, equation (12) and a heat balance of a similar form can be lead forth. When a solution by substituting the given real

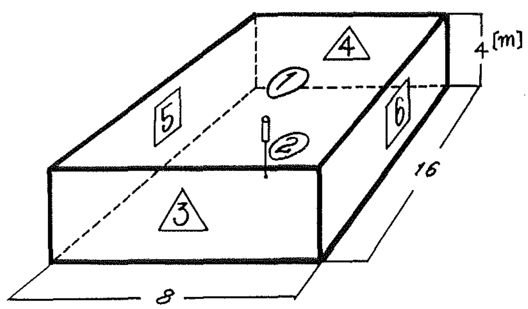

Fig. 7. Sketch of model room.

number values is made, the results of Fig. 8, 9 and Table 1 can be obtained. Further, the position of the elements are shifted in the same room under the same conditions in example 2 the change of absorption factors in Fig. 10 may be expressed as Fig. 11~15, and in addition the distribution of the environmental

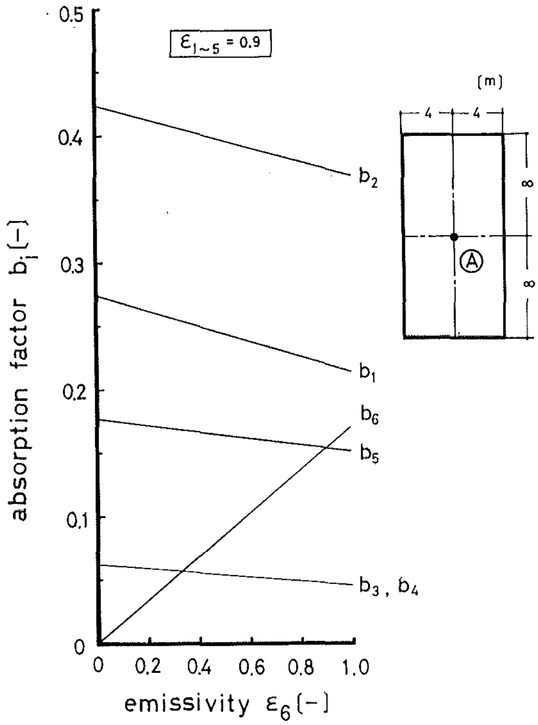

Fig. 8. Absorption factors $b_{1\sim6}$ at position Ⓐ.

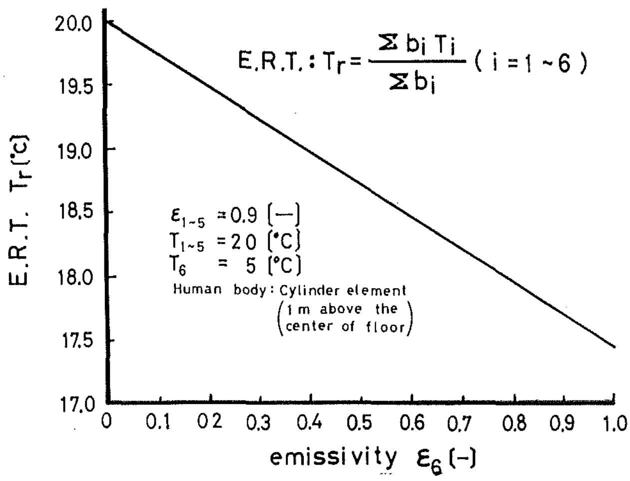

Fig. 9. Change of E.R.T.(Environmental Radiant Temperature)

TABLE 1 Comparison of various kinds of mean radiant temperatures

Kind of mean radiant temp.	Definition	Example $T_{1\sim5}=20°C$ $T_6=5°C$ $\varepsilon_{1\sim6}=0.9$
Arithmetic mean	$\frac{\sum_{i=1}^{6} T_i}{6}$	17.50°C
Wall area mean	$\frac{\Sigma S_i T_i}{\Sigma S_i}$	17.86°C
Angle factor (sphere element)	$\frac{\Sigma \mathring{\phi}_i T_i}{\Sigma \mathring{\phi}_i}$	18.10°C
Angle factor (cylinder element)	$\frac{\Sigma \phi_i T_i}{\Sigma \phi_i}$	17.68°C
Emissivity & angle factor	$\frac{\Sigma \varepsilon_i \phi_i T_i}{\Sigma \phi_i}$	15.85°C
Emissivity & angle factor	$\frac{\Sigma \varepsilon_i \phi_i T_i}{\Sigma \varepsilon_i \phi_i}$	17.67°C
Absorption factor	$\frac{\Sigma b_i T_i}{\Sigma b_i}$	17.70°C

Human element: 1 meter above the center of floor.
T_i: wall surface temp., S_i: wall area, ε_i: emissivity, ϕ_i: angle factor, b_i: absorption factor.

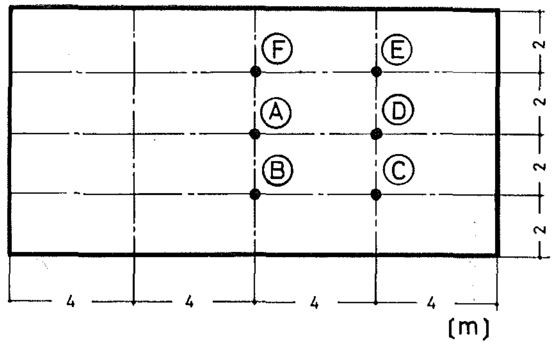

Fig. 10. Position of element in the model room.

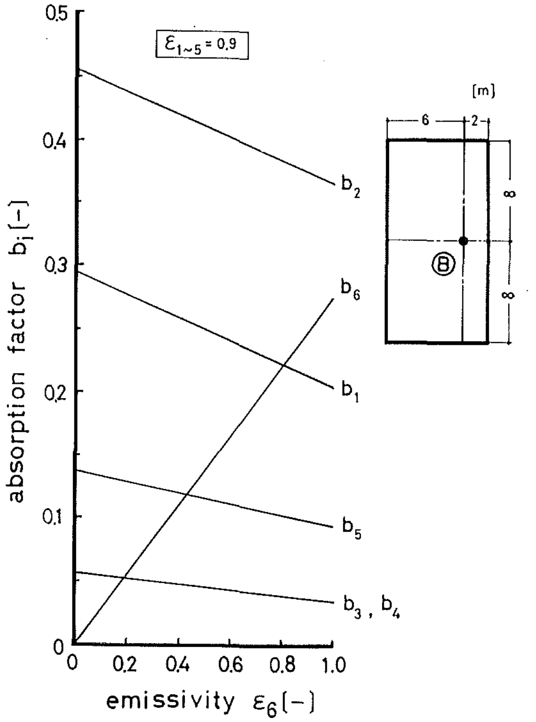

Fig. 11. Absorption factors $b_{1\sim6}$ at position Ⓑ.

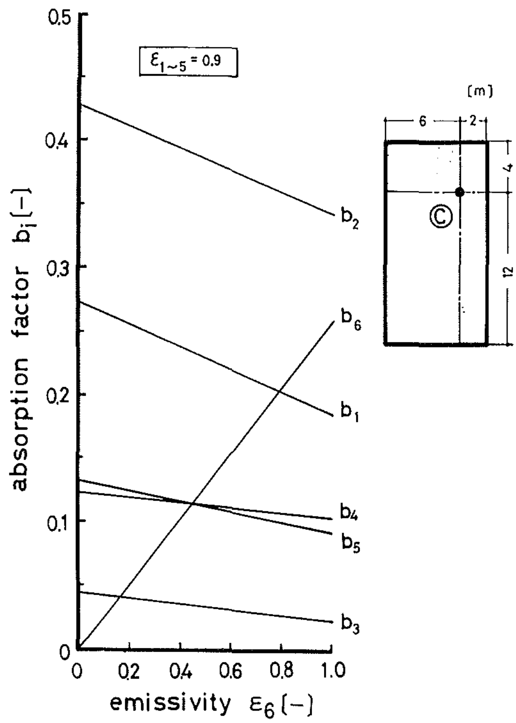

Fig. 12. Absorption factors $b_{1\sim6}$ at position Ⓒ.

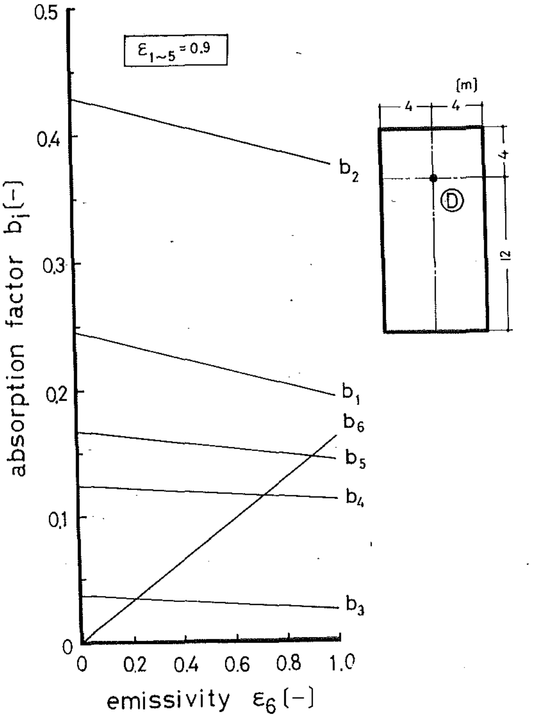

Fig. 13. Absorption factors $b_{1\sim6}$ at position Ⓓ.

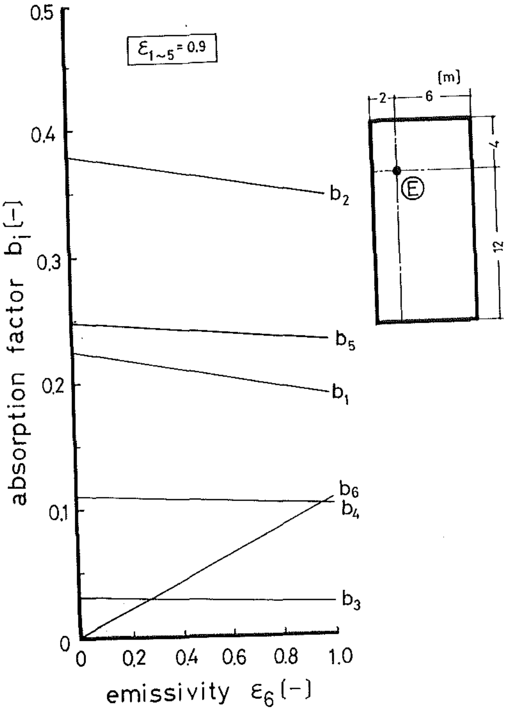

Fig. 14. Absorption factors $b_{1\sim6}$ at position Ⓔ.

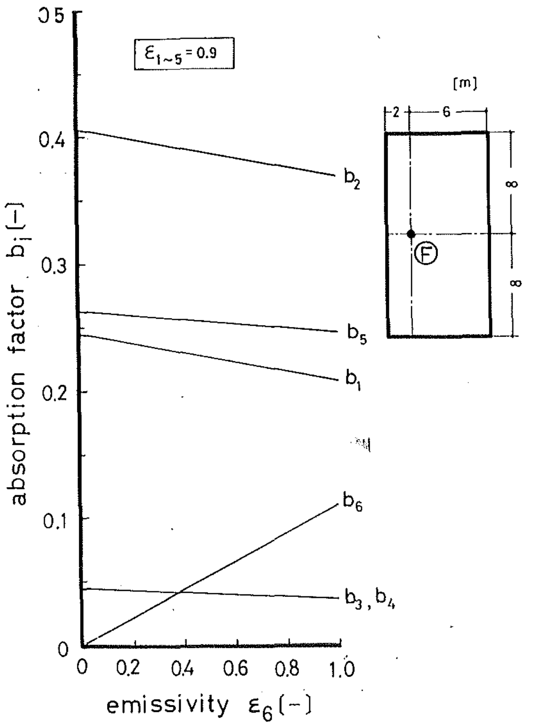

Fig. 15. Absorption factors $b_{1\sim6}$ at position Ⓕ.

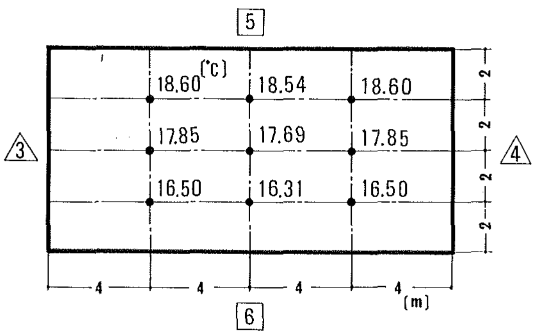

Fig. 16. Distribution of E.R.T.

radiant temperature is given in Fig. 16.

References

1) B. Gebhart: A new method for calculating radiant exchanges, ASHRAE Trans. Vol. 65, 1959, p. 321.
2) K. Ibamoto and T. Mochida: Warmth diagram for thermal sensation analysis and its application in environmental design, The international symposium on the environment in buildings, held in England, 1972.
3) K. Ibamoto and T. Mochida: A new estimating method of radiation at thermal environment, SHASE autumnal meeting, 1974, p. 35.
4) K. Ibamoto and Y. Nishi: Thermal sensation analysis and its application to air-conditioning, SHASE Trans. Vol. 42, No. 3, 1968, p. 1.
5) K. Ibamoto and T. Mochida: A thermal environment design method using warmth diagram, SHASE autumnal meeting, 1970, p. 33.
6) K. Ibamoto and T. Mochida: A rational heating design for a man-comfort-thermal-system, The international symposium on the environment in buildings, held in England, 1972.
7) T. Mochida and Y. Atoda: The radiation effect from the cold walls to the human body, SHASE winter meeting, 1975, p. 5.