Flow Pattern in the Vicinity of a Sphere or Disk Sinusoidally Vibrating in Liquid

Kunihiro KITANO Harumochi HIRANO Kazuo ENDOH

Abstract

Experimental studies where made on the inner and outer circulations in the vicinity of a sphere sinusoidally vibrating in viscous fluid.

When Reynolds number was smaller than a certain definite value affected by a/d, where a is the amplitude of vibration and d is the diameter of the sphere, it was observed that the outer circulation vanished and the inner circulation developed to the whole flow field. The critical condition obtained was expressed by:

$$0.06 \lesssim a/d \lesssim 1: \qquad (ad\omega/\nu)_c = 20$$

$$1 \lesssim a/d \lesssim 3.64: \qquad (a^2\omega/\nu)_c = 20$$

where ω is the angular velocity of vibration and ν is the kinematic viscosity of liquid.

In the region where the outer and inner circulations coexisted, the measured thickness of inner circulation η was expressed by:

$$0.2 \lesssim a/d \lesssim 1: \qquad \eta_d/d \propto (ad\omega/\nu)^{-1.55}$$

$$\eta_u/d \propto (d^2\omega/\nu)^{-0.62}$$

$$1 \lesssim a/d \lesssim 3.64: \qquad \eta_d/d \propto (ad\omega/\nu)^{-0.87}$$

$$\eta_u/d \propto (d^2\omega/\nu)^{-0.31}$$

where η_d was measured at the front of the sphere and η_u was measured at the rear of the sphere.

Introduction

When a solid body is oscillating in a fluid or a solid body is fixed in an oscillating fluid, steady streaming motions of the fluid, secondary flows, are generated under certain conditions.

Considerable work has been done in the investigation of these streaming motions both theoretically and experimentally. Carrière[5] reported a steady circulatory streaming motion in each quadrant around a cylinder, which was directed toward the cylinder along the axis of oscillation. Andrade[1] reported that the secondary flow was directed away from a cylinder along the axis of oscillation. Schlichting[10] used a cylinder oscillating in a fluid at rest and observed a similar type of streaming motion which was generated in a tank of water. Furthermore he carried out a theoretical analysis of this time independent flow. Andres and Ingard[2),3] investigated the streaming motion of the air around the cylinder in the sound field theoretically and experimentally and observed two types of secondary motion induced depending upon the Reynolds number. Holtsmark et al.[9] made detailed researches in this problem both experimentally and theoretically and found that the secondary streaming flow had two regions, namely the inner circulation and outer circulation. The inner circulation next to the cylinder is directed toward the cylinder along the axis of oscillation and outer circulation is directed away from the cylinder along the axis.

Afterwards, these studies have been continued by many investigators.[4,6,11~13] Most of them chose cylinders as the subject of the study. There are few reports regarding spheres or other shape of body. And there are very few reports about transition between the two types of the secondary flow, one is a streaming motion where inner circulation and outer circulation coexist, another is the flow where only inner circulation exists. It is the purpose of this paper to investigate experimentally the secondary flow behavior about spheres in wide range of a/d. In addition we observed the flow behavior around a vibrating disk as an example of other shapes.

1. Experimental Apparatus and Method of Observation

A sketch of the equipment used to impart a simple harmonic motion to a sphere is shown in Fig. 1. A slotted sliding bar is attached to a flywheel with a crank pin, and the location of the pin can be shifted along the radius of the wheel. Thus the amplitude of vibration, a, can be changed. By the combination of various speed and amplitude, it is possible to set up a large number of different

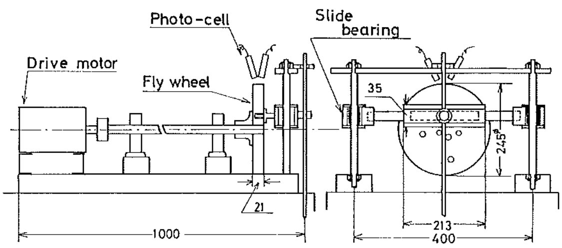

Fig. 1. Equipment for producing simple harmonic motion.

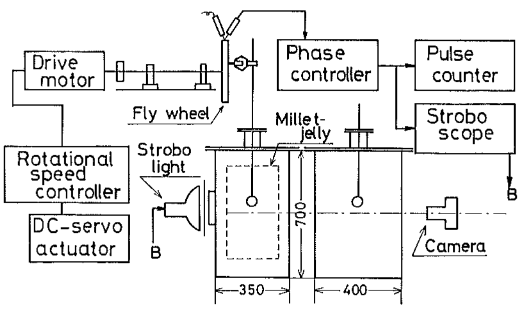

Fig. 2. Schema of experimental apparatus.

oscillating conditions. A small reflecting mirror was attached to the circumferential surface of the flywheel in any desired position. Light was reflected by the mirror and received by a photo-cell. Thus an electrical signal was taken for an arbitray phase of oscillation.

Figure 2 shows the diagram of the apparatus. The perspex container was filled with mixture of millet jelly and water. In order to visualize the fluid motion, aluminum dust method was used. Photographs of the flow were taken by a camera placed in front of the container. The sphere was stopped optically by using a stroboscope synchronized with the frequency of oscillation in an arbitrary position of oscillation. The stroboscope light source illuminated a vertical plane which included the axis of oscillation. The frequency of oscillation was measured by a pulse counter.

Spheres having three different diameters were employed, 1.90 cm, 3.81 cm and 7.13 cm. The sizes of disks were 2.9 cm (0.145 cm thick), 4.5 cm (0.255 cm and 1.13 cm thick), and 6.0 cm (0.1 cm and 0.3 cm thick) in diameter. Other variables included amplitude of vibration ranging from 0.42 to 6.91 cm, frequency ranging from 0.27 to 6.7 cycles/sec., and kinematic viscosity of fluid ranging from 0.857 to 15.7 cm^2/sec.

2. Experimental Results and Discussion

2.1 Transition between two types of flow

1) Sphere Figure 3 shows some photographs of the flow around a sphere for different conditions of oscillation. These photographs were taken in a stroboscopic light sychronized with the oscillation of the sphere. The sphere was stopped optically in the center position of oscillation when the sphere was moving in a

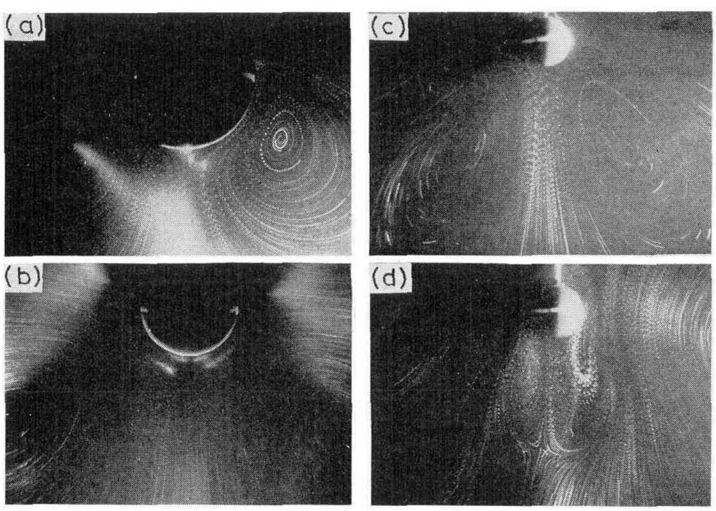

Fig. 3. Flow patterns around a vibrating sphere.

(a) $a/d=0.295$, $ad\omega/\nu=11.2$ (b) $a/d=0.271$, $ad\omega/\nu=77.1$
(c) $a/d=2.11$, $a^2\omega/\nu=13.0$ (d) $a/d=2.11$, $a^2\omega/\nu=28.9$

downward direction. For smaller values of Reynolds number the flow pattern qualitatively resembles to that observed by Carrière[5]. A single circulatory loop is present in each quadrant in the outside region of the sphere, and the flow is directed toward the sphere along the line of oscillation, as shown in Fig. 3 (a). When the Reynolds number is increased to a critical value, the inner circulation shrink down and a new circulation, outer circulation, in a reverse flow direction appears outside the initial one in each quadrant and appears as a "rest point", as shown in Fig. 3 (b). "Rest point" is a point where streamlines of inner and outer circulations gather. We observed a similar transition when a/d was larger than one, as shown in Fig. 3 (c), (d). This shrinkage of the inner circulation occurs rather markedly when a/d is smaller than one. When a/d is larger than one, however, this transition is not so sharp. Therefore we used the appearance of the rest point as the transition criterion.

The observed critical Reynolds numbers of transition are plotted in Fig. 4. In Fig. 4, in addition to the data obtained by the author, results are included which are calculated from the data of Andres and Ingard[2] and Tatsuno[12]. The experimental values of the present work (○) agree well with the other author's data which were obtained for a cylinder. A line passing through these points divides the diagram into two parts. The region on the larger Reynolds number side of the line correspond to the coexistence of the outer and inner circulations. The region on the smaller Reynolds number side of the line correspond to the existence of only inner circulation. From Fig. 4 it is seen that critical Reynolds number, $(d^2\omega/\nu)_c$, is a function of the ratio of amplitude to sphere's diameter a/d. And the correlation between $(d^2\omega/\nu)_c$ and a/d changes before and after $a/d=1$. The data can be correlated by the following equations.

$$(d^2\omega/\nu)_c = 20(a/d)^{-1} \qquad (0.06 \lesssim a/d \lesssim 1) \tag{1}$$

and

$$(d^2\omega/\nu)_c = 20(a/d)^{-2} \qquad (1 \lesssim a/d \lesssim 3.64) \tag{2}$$

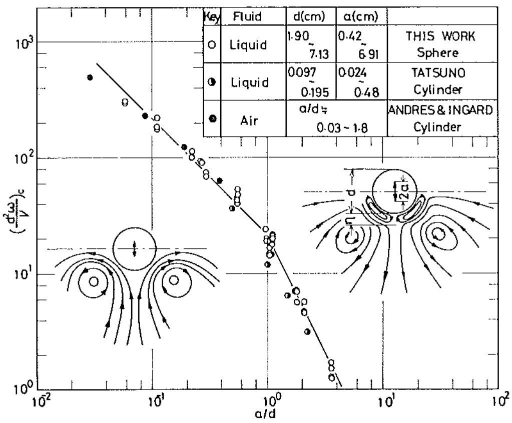

Fig. 4. Relationship between a/d and $d^2\omega/\nu$ at the transition point for sphere.

The above equations are transformed into more simple form. From equation (1)

$$(ad\omega/\nu)_c = 20 \qquad (0.06 \lesssim a/d \lesssim 1) \qquad (3)$$

and from equation (2)

$$(a^2\omega/\nu)_c = 20 \qquad (1 \lesssim a/d \lesssim 3.64) \qquad (4)$$

Namely, transition can be expressed by one critical Reynolds number in each range of a/d.

2) **Disk** A similar experiment was carried out for a disk. Figure 5 shows photographs of flow around a disk for different conditions of oscillation. The same transition were observed as that observed for a sphere. The flow corresponding to the smaller Reynolds number is shown in Fig. 5 (a), and the flow corresponding to the larger Reynolds number is shown in Fig. 5 (b). The observed critical Reynolds number of the transition is plotted in Fig. 6. From Fig. 6, it can be considered that there is no effect of the disk thickness on transition as far as the present experiment is concerned. The proportional equation of the best fit of the data can be written as follows.

$$(d^2\omega/\nu)_c \propto (a/d)^{-1.7} \qquad (0.07 \lesssim a/d \lesssim 1.38) \qquad (5)$$

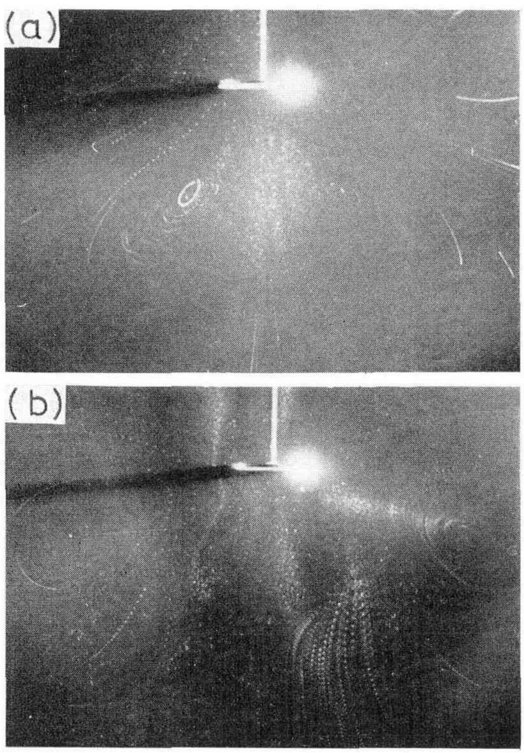

Fig. 5. Flow patterns around a vibrating disk.
$a/d = 1.38$ (a) $d^2\omega/\nu = 6.51$ (b) $d^2\omega/\nu = 16.1$

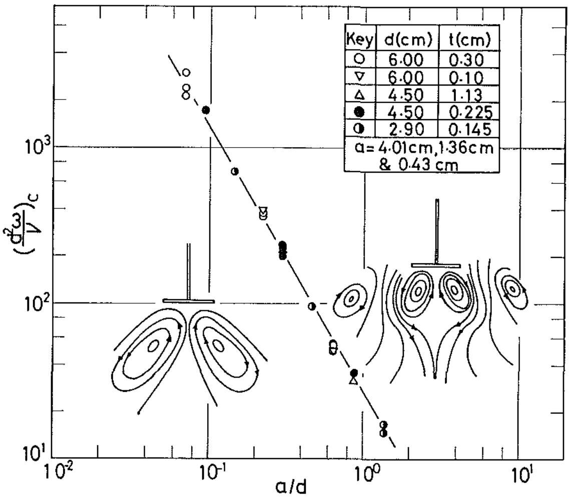

Fig. 6. Relationship between a/d and $d^2\omega/\nu$ at the transition point for disk.

2.2 Correlation between circulation thickness and Reynolds number

In the large Reynolds number region where the outer and inner circulations coexist, the inner circulation thickness was measured for a sphere. In order to eliminate the effect of the supporting rod, the inner circulation thickness was measured at the lower part at a sphere. As shown in Fig. 7, the inner circulation thickness was denoted by η_d when the sphere was stopped optically in the center position of a downward movement of oscillation, and similarly η_u indicates the inner circulation thickness when the sphere was moving in the reverse direction at it's center position. In a previous paper[7,8], we reported that under the condition in which a/d is sufficiently small, η_d/d and η_u/d are numerically equal and can be expressed as a function of $d^2\omega/\nu$.

The thickness of inner circulation η_d vs. amplitude of vibrating sphere is shown in Fig. 8. As shown in the figure, η_d/d is affected by a/d at a/d above 0.2. The data obtained by this experiment at $a/d>0.2$ are plotted in Fig. 9 and Fig. 10.

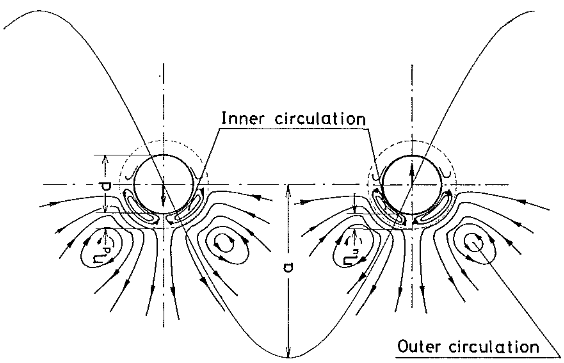

Fig. 7. Thickness of inner circulations at the front and rear of sphere.

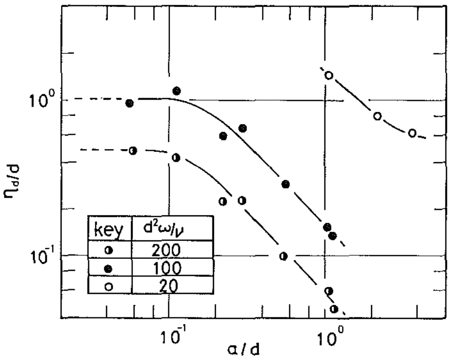

Fig. 8. Relation of the thickness of inner circulation η_d and amplitude of vibrating sphere.

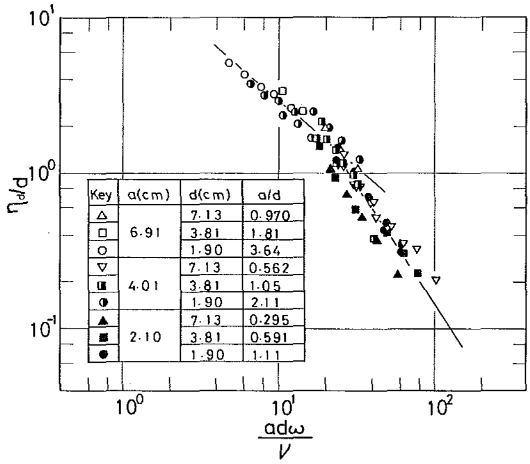

Fig. 9. Experimental correlation between the thickness of inner circulation η_d, which was measured at the front of sphere, and Reynolds number.

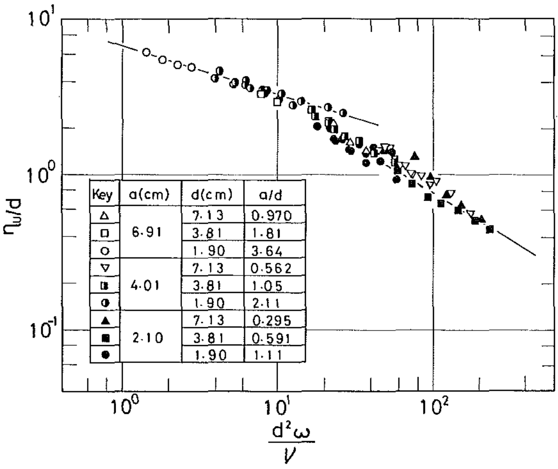

Fig. 10. Experimental correlation between the thickness of inner circulation η_u, which was measured at the rear of sphere, and Reynolds number.

From there figures, η_d/d was considered to be a function of $ad\omega/\nu$, whereas η_u/d a function of $d^2\omega/\nu$. But the slope of these lines change before and after $a/d \fallingdotseq 1$. They may be expressed by the following equations.

$0.2 \lesssim a/d \lesssim 1$

$$\eta_d/d \propto (ad\omega/\nu)^{-1.55} \tag{6}$$

$$\eta_u/d \propto (d^2\omega/\nu)^{-0.62} \tag{7}$$

and

$1 \lesssim a/d \lesssim 3.64$

$$\eta_d/d \propto (ad\omega/\nu)^{-0.87} \tag{8}$$

$$\eta_u/d \propto (d^2\omega/\nu)^{-0.31} \tag{9}$$

The results are summarized in Fig. 11. In this figure, the shadowed portion corresponds to the existence of only inner circulation, and outside of this portion corresponds to the coexistence of the outer and inner circulations. The slope of a straight line through the origin is equal to a/d. In the region where a/d is smaller than one, transition is determined by $ad\omega/\nu$, whereas it can be expressed by $a^2\omega/\nu$ in the region where a/d is larger than one.

The dependency of the inner circulation thickness upon the oscillating condition is also presented in the figure.

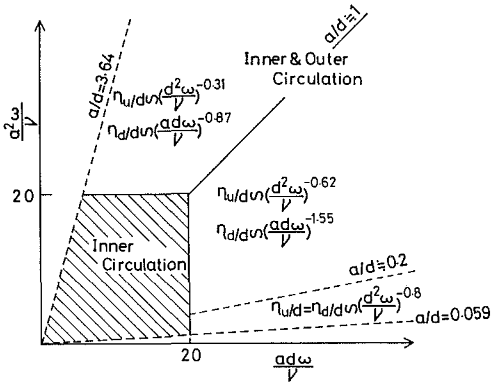

Fig. 11. Chart showing parameters that define flow pattern around a vibrating sphere.

3. Conclusion

Experimental studies were made on the steady circulatory streaming motion induced in the vicinity of a sphere sinusoidally vibrating in viscous fluid in the range of $0.059 < a/d < 3.64$. In the region of $0.2 \lesssim a/d$, η_d/d was considered to be a function of $ad\omega/\nu$ and η_u/d was considered to be a function of $d^2\omega/\nu$. The exponents of the dimensionless variables of the correlations change before and after $a/d \fallingdotseq 1$.

When the Reynolds number is smaller than a critical value, it was observed that the outer circulation vanished and only the inner circulation occupied the whole flow field. The critical Reynolds number of the transition obtained was expressed by

$$(0.06 \lesssim a/d \lesssim 1) \qquad (ad\omega/\nu)_c = 20$$

$$(1 \lesssim a/d \lesssim 3.64) \qquad (a^2\omega/\nu)_c = 20$$

Acknowledgement

The authors are greatly indebted to Dr. H. Imai for his valuable advice and criticism during the preparation of this paper.

Nomenclature

a = amplitude of sinusoidally vibrating sphere or disk	[cm]
d = diameter of sphere or disk	[cm]
t = thickness of disk	[cm]
η = thickness of inner circulation	[cm]
η_d = measured thickness of inner circulation at the front of sphere	[cm]
η_u = measured thickness of inner circulation at the rear of sphere	[cm]
ν = kinematic viscosity	[cm²/sec]
ω = angular velocity of vibration	[radian/sec]

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