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A Study of Pulsed Neutron Wave Propagation in Graphite by Using Electron Linear Accelerator

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Abstract

The present paper describes a study of the pulsed neutron wave propagation in graphite. The experiment is made on a rectangular assembly with a small cross section (30×30 cm^2), by using the 45 MeV electron linear accelerator at the Hokkaido University. The disperion law of the pseudo mode is obtained up to a fairly high frequency, about 6,000 rad/sec. The analysis is also made by the method of eigenfunction expansion based on a realistic scattering kernel.

1. Introduction

The method of neutron wave propagation experiment is one of the improtant techniques for studying the neutron thermalization, which provides more dynamical informations than exponential experiments. Several studies of the neutron thermalization in graphite have been made by the method of neutron wave propagation[1~3], and the dispersion law measured up to a fairly high frequency region. Williams indicated that the polycrystalline material such as graphite has a low critical frequency at which the discrete eigenvalue disappears. The mode which is experimentally observed beyond the critical frequency such as the discrete mode is called the pseudo mode. As the cross section of assembly decreases, the condition of the existence of the discrete mode becomes so severe that the dispersion law observed in the experiment corresponds to that of the pseudo mode.

DeJuren et al[4]. made an exponential experiment of small assemblies of graphite and observed that the decay of the neutron flux becomes nonexponential in the extremely small assemblies. One of the authors (T. M) analyzed the present behavior of the pseudo mode by the method of inelastic scattering expansion.

The present paper describes a study of pulsed neutron wave propagation in graphite. The experiment is made for a very small rectangular assembly of graphite by using the 45 MeV electron linear accelerator (LINAC) at the Hokkaido University. The pseudo mode observed in the neutron wave propagation experiment is analyzed by the method of eigenfunction expansion.

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2. Experiment

Figure 1 shows the experimental arrangement. The accelerated electron beam from the LINAC produces pulsed fast neutrons via the blemsstlahlung in a lead target, which are slowed down in the adjacent moderator of light water. The LINAC is operated under the condition of the 3 μsec pulse width and about 50 pps repitition rate. The lead blocks in the moderator help the neutrons to slow down by the inelastic scattering, as well as the shielding of the γ ray. The assembly of graphite, stacked by the blocks of nuclear reactor grade graphite, has a cross section $30 \times 30\ \text{cm}^2$ and a length 120 cm. Cadmium sheets cover the graphite

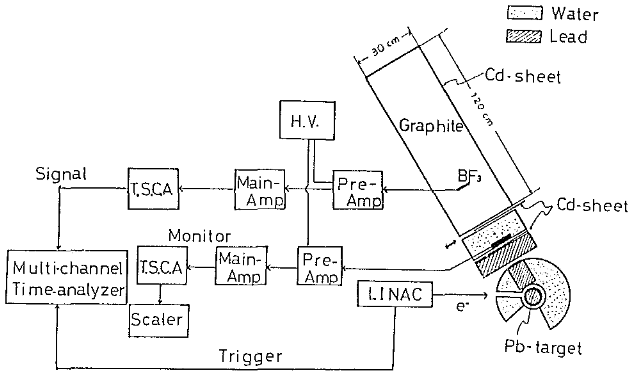

Fig. 1. Experimental arrangement.

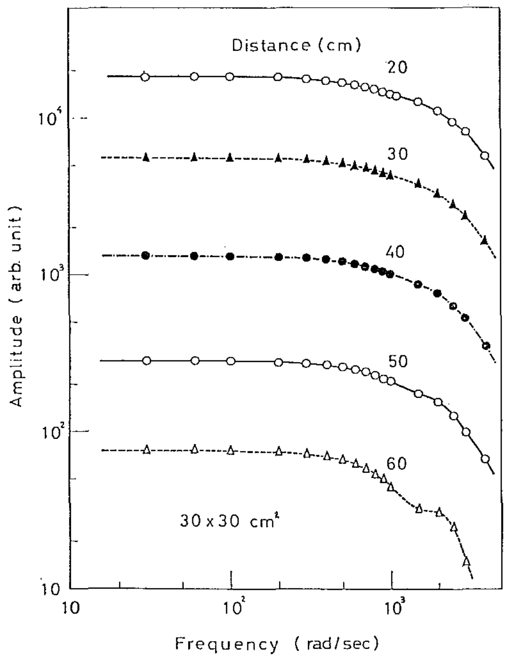

Fig. 2. Amplitude of neutron wave.

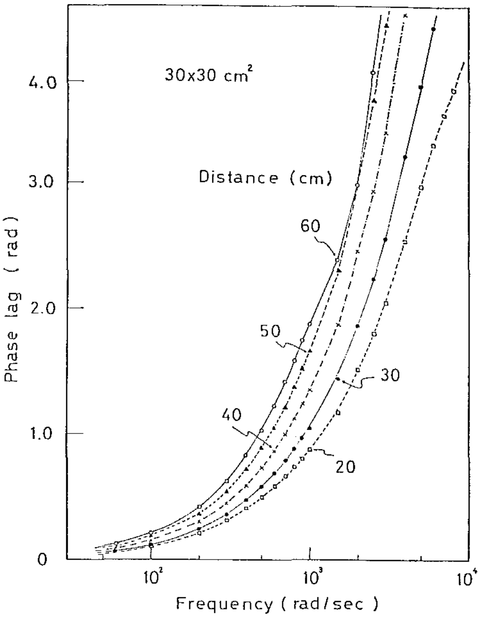

Fig. 3. Phase lag of neutron wave.

assembly in order to reduce the background counts of the room-returned neutrons.

The pulsed neutron wave propagation is measured along the axis of the assembly by a very small BF_3 counter (6 mmϕ × 15 mm), which reduces the integrating effect over the neutron wave distribution across the cross section. The signals of the counter are analyzed by the multi-channel time analyzer with the 10 μsec time width. A monitor (BF_3 counter: 1'ϕ × 240 mm) is located in the moderator to normalize the intense of the pulsed neutron source. The background count, which mainly comes from the epicadmium neutron in the moderator, is measured under the condition having a cadmium sheet in front of the assembly.

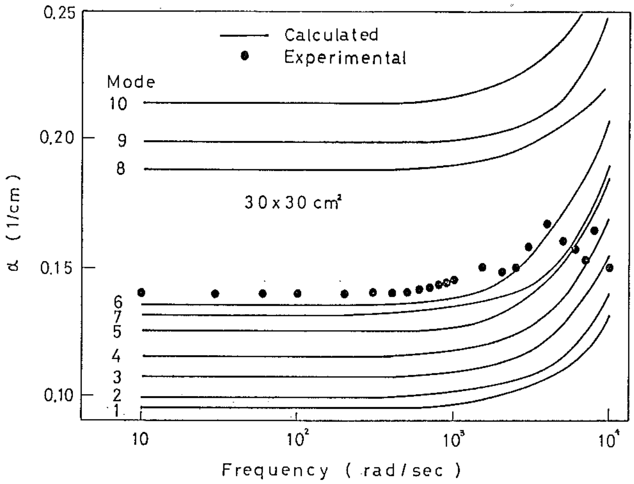

Fig. 4. Dispersion law of pseudo mode (α).

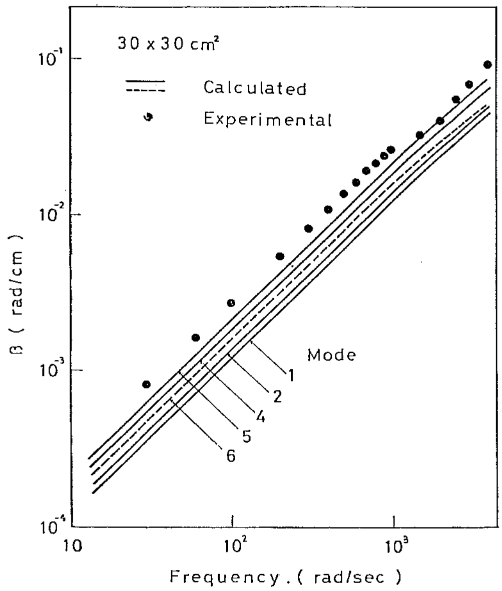

Fig. 5. Dispersion law of pseudo mode (β).

The count of neutron wave $C(t, z)$ is obtained as follows:

$$C(t, z)=\frac{C_0(t, z)}{M_0}-\frac{C_{cd}(t, z)}{M_{cd}} \tag{1}$$

where M is the neutron count of the monitor, and the indices cd and 0 represent the measurements with and without the cadmium sheet, respectively. The characteristics of the neutron wave propagation is derived by the Fourier transformation of $C(t, z)$, where the distances are selected for 20, 30, 40, 50 and 60 cm.

Figures 2 and 3 show the experimental results for the amplitude and phase lag, respectively. Fitting the data at the far distances to a function $\exp(-\rho z)$, $(\rho=\alpha+j\beta)$ by the least square method, we obtained the dispersion law of the pseudo mode, shown in Figs. 4 and 5.

3. Analysis by the method of eigenfunction expansion

The experiment of the pulsed neutron wave propagation in Sec. 2 was analyzed by the method of eigenfunction expansion[5], using a realistic scattering kernel. The neutron flux propagating in the assembly is presented by the diffusion approximation as follows:

$$\begin{aligned}-D(E)\frac{dz^2}{d^2}\phi_{lm}(E, z, \omega)+\left\{\Sigma_a(E)+\Sigma_s(E)+D(E)\,B^2_{\perp lm}(E)+\frac{j\omega}{v}\right\}\phi_{lm}(E, z, \omega)\\ =\int_0^\infty dE'\,\Sigma(E'\to E)\,\phi_{lm}(E', z, \omega)+S(E)\,\delta(z)\end{aligned} \tag{2}$$

where

$D(E)$: diffusion coefficient
$\Sigma_a(E)$: absorption cross section
$B^2_{\perp lm}(E)$: energy dependent buckling of l and m-th transverse mode
v: velocity of neutron
ω: frequency of neutron wave
$\Sigma(E\to E')$: inelastic scattering cross section
$S(E)$: external source of neutron.

We assume the following neutron wave

$$\phi(E, z, \omega)=\phi(E, \omega)\,\mathrm{e}^{-\rho z} \tag{3}$$

Using the multi group picture for the variable of energy, and substituting Eq. (3) in Eq. (2), the following algebraic equation is obtained, i. e.

$$\sum_{j=l}^{N} A_{ij}\phi_l(E_j)=\rho_l^2\phi_l(E_i) \qquad \text{for} \quad i=1, 2, \cdots, N \tag{4}$$

where

$$A_{ij}=\left\{\frac{\Sigma_a(E_i)+\Sigma_s(E_i)}{D(E_i)}+\frac{j\omega}{v_i D(E_i)}+B^2_\perp(E_i)\right\}\delta_{ij}-\Sigma(E_j\to E_i)\frac{\Delta E_j}{D(E_i)} \tag{5}$$

The eigenvalue equation is derived from Eq. (4),

$$|\boldsymbol{A}-\rho^2 I|=0 \tag{6}$$

where $\boldsymbol{I}$ is a unit matrix. After calculating the eigenvalues and eignefunctions of Eq. (6), we can obtain the total wave as follows:

$$\phi(E, z, \omega)=\sum_{l=1}^{N} a_l \phi_l(E, \omega)\, \mathrm{e}^{-\rho z+j\omega t} \tag{7}$$

where the coefficient a_l is derermined by the boundary condition,

$$-D(E)\frac{\partial}{\partial z}\phi(E, z, \omega)\Big|_{z=0}=S(E)/2 \tag{8}$$

The calculation of the neutron wave was made on the following model: (1) energy mesh; the range of neutron energy 0.4 meV to 0.3 eV is divided into 30 groups with the equal lethargy intervals. The 1st to the 6th groups belong to the cold energy region, and the others to the thermal energy region. (2) inelastic scattering cross section $\Sigma(E\rightarrow E')$; the frequency distribution of phonons in graphite is averaged as follows:

$$g(\omega)=\frac{2}{3}g_{/\!/}(\omega)+\frac{1}{3}g_{\perp}(\omega) \tag{9}$$

where $g_{/\!/}(\omega)$ and $g_{\perp}(\omega)$ are the frequency distribution along the parallel and perpendicular direction to the planary structure of graphite, respectively. The calculated values by Young et al[6]. are used for the frequency distribution. The inelastic sacttering cross section is calculated by the UNCLE code[7] based on the incoherent approximation. (3) absorption; the $1/v$ dependence is assumed for the absorption cross section of neutrons for graphite, 3.4 mb at 0.0253 eV. (4) source; the external neutron source is assumed to have the Maxwellian distribution with the neutron temperature 298°K.

The calculation by the method of eigenfunction expansion were made for two assemblies with different cross sections: 100×100 cm^2 and 30×30 cm^2. In the case of the cross section 100×100 cm^2, at first, Fig. 6 shows the frequency dependence of the eigenvalue α, and Fig. 7 the eigenfunctions of several modes calculated at the frequency 3,000 rad/sec, respectively. From the consideration of the calculated eigenfunctions, it is reasonable to classify the modes into the cold and thermal modes: 1st to 6th are the cold modes, and 7th to 30th the thermal ones. In the case of the cross section 100×100 cm^2, both the modes are well seperated in the low frequency region. The 7th mode which corresponds to the fundamental one has the lowest value of α in the low frequency region, and hence can survive at a distance far from source. As the 7th mode has the spectrum of amplitude extremely close to the Maxwellian distribution, the observed total wave might be dominated by the 7th mode in the experiment of the external source with the near Maxwellian spectrum.

On the other hand, Fig. 8 shows some eigenfunctions calculated for the assembly with the cross section 30×30 cm^2. The eigenvalue α is compared with the experiment, shown in Fig. 4. The eigenvalues α of the cold modes 1st to 6th are smaller than the that of the 7th mode. Therefore the 7th mode, which has a large amplitude, decays more rapidly than those cold modes. This is shown in the spectrum of the amplitude of the total wave which is derived by the integration of the $1/v$ detector efficiency (Fig. 9). The amplitude of the total wave is compared with the experiment, shown in Fig. 10. The results of the experiment and the calculations are in good agreement both for the frequencies 30 and 3,000 rad/sec.

From the calculations for the cases $100\times100\ \mathrm{cm}^2$ and $30\times30\ \mathrm{cm}^2$, we can understand the pseudo mode of the neutron wave experiment by the method of

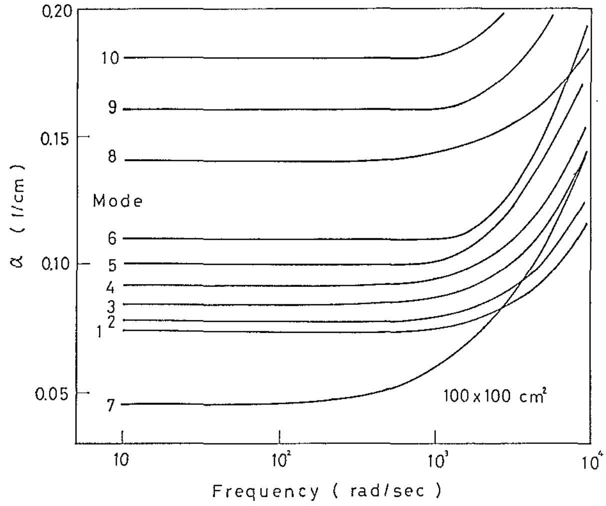

Fig. 6. Eigenvalue α for assembly $100\times100\ \mathrm{cm}^2$.

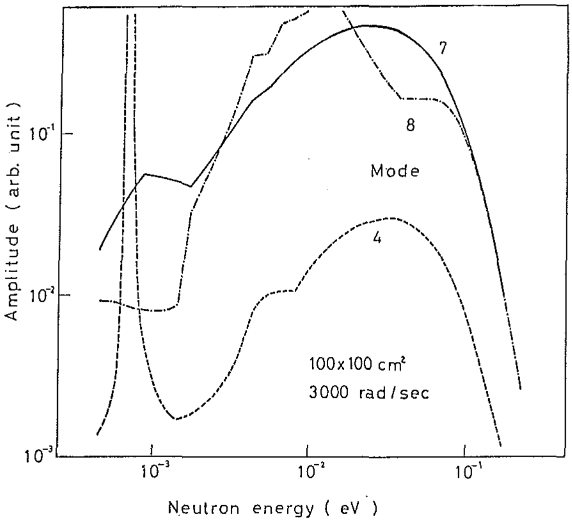

Fig. 7. Eigenfunction for assembly $100\times100\ \mathrm{cm}^2$.

eigenfunction expansion as follows: the pseudo mode is presented by the 7th mode which has the lowest eigenvalue α between the thermal modes and the amplitude spectrum which is very close to the Maxwellian distribution.

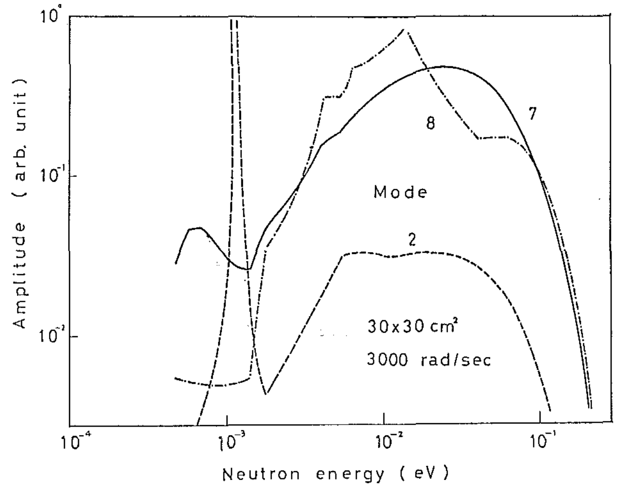

Fig. 8. Eigenfunction for assembly 30×30 cm^2.

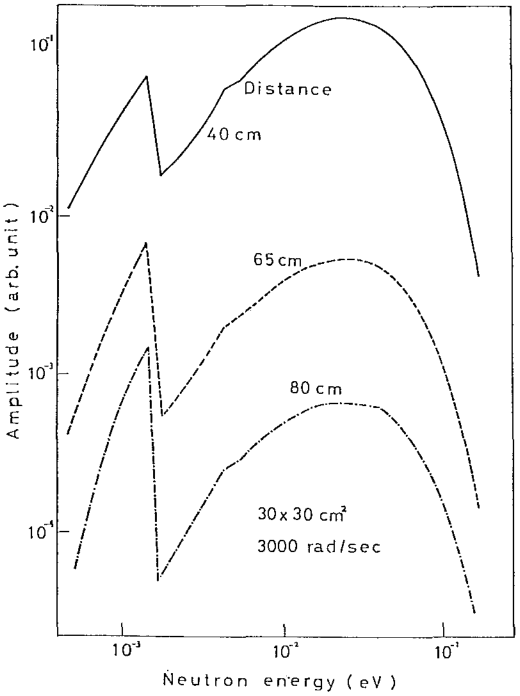

Fig. 9. Spectrum of amplitude of total wave.

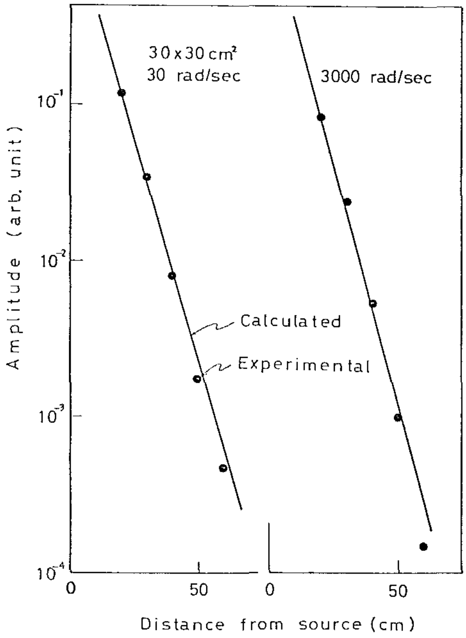

Fig. 10. Decay of amplitude of total wave.

4. Conclusion

In the present paper, the pseudo mode of the neutron wave was studied on an assembly of graphite with the very small cross section. The intense neutron source provided by the 45 MeV LINAC compensated the strong decay of the neutron wave in the small assembly and enabled the measurements of the pseudo mode. The experimental dispersion law of the pseudo mode is in good agreement with the calculated one by the method of eigenfunction expansion based on the realistic scattering kernel. Therefore we have obtained a clear understanding about the pseudo mode: namely it corresponds to the mode (the 7th mode in the used model of the calculation) which has the lowest eigenvalue α between the thermal modes and the spectrum of amplitude very close to the Maxwellian distribution.

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