Unsteady Separated Flow behind a Normal Plate Calculated by Discrete-Vortex Model

(Part 1. Velocity-Point Scheme)

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Abstract

A discrete-vortex model has been developed to predict the kinematic and dynamic characteristics of the separated flow past a normal plate immersed in a uniform approaching stream. This scheme utilizes the magnitude of velocities at the "velocity points" fixed in the vicinity of the separation points in order to determine the strength of the vortices newly introduced into the wake. Provided that the distance between the location of the velocity points and the separation points is appropriately selected, the drag force, the rate of shedding of vorticity from the separation points and the Strouhal number of the periodic vortex shedding are predicted with accuracy to an extent almost comparable to the previous calculation of the highest accuracy (Sarpkaya 1975). However, the time-averaged velocity and the root-mean-square values of the fluctuating velocities in the near wake are found to be rather poorly predicted by this model.

1. Introduction

Usteady separated flows past bluff bodies at high Reynolds numbers are of fundamental importance in fluids engineering. Calculation of such flows have mainly been performed by means of the discrete-vortex model. This model utilizes inviscid point vortices, which in some applications are endowed with an inner structure in order to avoid the intrinsic singularity of the point vortices, to represent the shear layers shed downstream from the separation points on the surface of the bluff bodies. The previous investigations in this category up to 1975 were summarized and critically reviewed by Clements and Maull [1]. Since then, the discrete-vortex model was applied to the calculation of the vortex shedding from an inclined flat plate in uniform shear flow by Kiya and Arie [2] and from a circular cylinder in steady and oscillating far flows by Stansby [3], and the calculation of the starting flow through a two-dimensional sharp-edged orifice by Evans and Bloor [4], among others.

It has been clarified from previous studies that the discrete-vortex model can predict with varying degrees of accuracy such important characteristics of the bluff bodies as fluctuating forces, vorticity-shedding rate at the separation points (which allows for estimation of the back pressure), and the Strouhal number of periodic

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vortex shedding. The Strouhal number is generally well predicted by this model whereas the time-averaged forces are, for instance, larger by several ten percent than experiments in the case of inclined flat plates calculated by Sarpkaya [5]. General appearance of predicted vortex patterns in the near wake is qualitatively similar to that observed experimentally.

Apart from a few exceptions, most investigators who used the discrete-vortex model for the calculation of separated flows over bluff bodies failed to show the the time-averaged velocity and fluctuating velocity components in the near wake of the bluff bodies. In view of the fact that accurate measurements of these quantities in recirculating flows are still very difficult, the value of the discrete-vortex model will be enhanced if such quantities can be accurately predicted. Clements [6] calculated the time-averaged velocity and amplitude of velocity fluctuations *outside* the wake at positions in the plane of a square-based body. The discrete-vortex model employed by him predicted the time-averaged velocity extremely well for a distance greater than about 0.6 base heights from the separation point, while the calculated amplitudes of velocity fluctuations were about half of the experimental results. The time-averaged velocity profiles in the recirculating region behind a downward-facing step were presented by Clements and Maull [1], the agreement between calculation and experiment (Tani et al. [7]) being fairly good. These studies, however, are not sufficiently detailed to illustrate the capability of the discrete-vortex model for predicting the properties of near wakes with recirculation. It is the main purpose of the present investigation to clarify the extent to which the discrete-vortex model can reproduce the time-averaged velocity and fluctuating quantities in the near wake of a flat plate set normal to uniform approaching stream. The normal flat plate has been selected here because this is one of the typical shapes of bluff bodies and because reliable measurements by Bradbury [8] are available for comparison purposes.

In Part I of this series of papers, a discrete-vortex scheme, which may be called as the velocity-point scheme, will be introduced. In this scheme, the strength of the vortices newly introduced into the wake, which will hereafter be designated as the nascent vortices, is determined by the use of the velocities at the points (called as the velocity points) fixed near the edges of the plate, the locations of the nascent vortices being chosen so as to satisfy the Kutta condition.

The velocity-point scheme is then modified in Part II to permit temporal decay of the strength of vortices as a function of their age. It will be shown that an appropriate reduction in the strength of vortices yields the time-averaged forces, the vorticity-shedding rate at the separation points and the near-wake properties consistent with experiments. The results of the present calculation are discussed and summarized in § 4.

2. Velocity-Point Scheme

One of the most salient features of the discrete-vortex scheme is the determination of the strength and location of the nascent vortices. Since the velocity, shape and location of the separated shear layers oscillate in conjunction with the periodic vortex shedding from the plate, the strength and location of the nascent vortices should also vary as functions of time. Most investigators utilized the Kutta condition at the edges of the plate in order to relate the strength of nascent vortices with their locations. One of the most successful models was presented by Sarpkaya [5] who determined the strength of nascent vortices $\delta\Gamma$ in terms of the average (say U_m) of the velocities of the first four vortices in each shear layer by the use of the relation

$$\delta\Gamma=\frac{1}{2}U_m^2\,\delta t \tag{1}$$

where δt is the time interval of the introduction of the nascent vortices into the wake. Although Sarpkaya's scheme gives a reasonable agreement with experiment except that the time-averaged force acting on the plates are 20～30 percent larger than experiments, the authors are nevertheless of the opinion that equation (1) may not be justified at least conceptually because the average velocity U_m corresponds to the instantaneous convection velocity of the shear layers. The correct form of equation (1) should be

$$\delta\Gamma=\frac{1}{2}(V_1^2-V_2^2)\,\delta t\simeq\frac{1}{2}V_1^2\delta t \tag{2}$$

where V_1 and V_2 represent the velocities at the outer and inner edges of the shear layers.

A new discrete-vortex scheme will now be described which makes judicious use of equation (2) and still permits the oscillation of strength and location of the nascent vortices with respect to time. In this scheme, a point (velocity point) is introduced in the vicinity of each edge of the plate and the velocity U_s at this point will be used to determine the strength of the nascent vortices by the relation

$$\delta\Gamma=\frac{1}{2}U_s^2\delta t \tag{3}$$

The velocity point is selected to be located in front of the plate along a line which is perpendicular to the plane of the plate and drawn through the edge concerned. The distance between the velocity point and the edge of the plate is obviously one of the crucial parameters in the velocity-point scheme.

The position of the velocity points may be interpreted as the edge of the boundary layer at the separation points. Since, in accordance with the fluctuation of the separated shear layers, the thickness of the boundary layer at the separation points will oscillate with a certain period, it is desirable that the position of the velocity points should also be allowed to oscillate periodically as a function of time. Owing to the singularity at the edge of the plate (Ackerberg [9], [10]), however, the thickness of the boundary layer at the separation points cannot accurately be predicted by the conventional boundary-layer theories even if the assumption of quasi-steady flows is employed. This situation implies that the position of the velocity points must be determined by trial and error at least in the case of bluff bodies with salient edges.

Since the basic equations required in the discrete-vortex simulation of the separated flow past an inclined flat plate can be found elsewhere (see § 2 of Sarpkaya [5] and Kiya and Arie [11]), they will not be repeated here in the interest of space. However, it may be noted that the physical plane including the plate was transformed into a complex plane in which the plate became a circle. An image system of vortices, which replaced in a discete manner the vorticity of the separated shear layers, was constructed in the transformed plane so as to satisfy the condition of zero normal velocity on the surface of the circle. The velocity field which was the superposition of a uniform approaching flow and the flow induced by the vortex system was used to move the vortices in the physical plane. The drag force was calculated by means of the Blasius theorem extended to unsteady flows.

The definition sketch of the flow past a flat plate set approximately normal to a uniform approaching stream is shown in Fig. 1. The mid point of the plate of height $4a$ coincides with the origin of the Cartesian coordinate system, the x axis being taken in the direction parallel to the approaching flow and the y axis normal to the x axis. Thetime-averaged and fluctuating velocity components in the x and y directions are written as U, u' ; V, v' respectively. The values of various parameters relevant to the present calculation will be summarized in what follows together with the physical implication of the parameters.

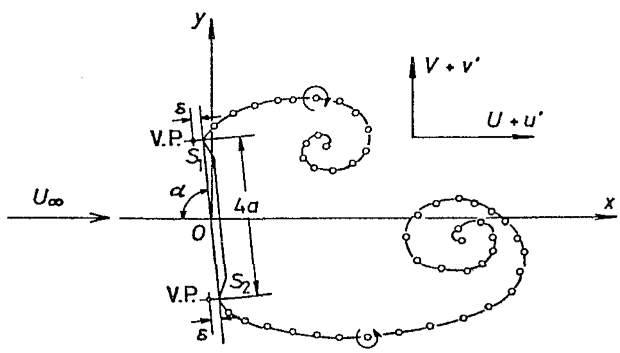

Fig. 1. Definition sketch and coordinate system. V. P. = velocity point.

(i) The computations were made for an inclined flat plate in the incidence of 85° to the approaching stream of velocity U_∞, which is an approximation to a normal plate. Any introduction of artificial asymmetry of flow to initiate the periodic vortex shedding from the normal plate can be avoided by this approximation. It seems that the difference between the characteristics of flows past the two plates is practically negligible.

(ii) The time step of the movement of vortices was taken to be $0.16(2a/U_\infty)$ which is twice the value employed by Kiya and Arie [11]. The use of this larger time step did not produce appreciable changes in the main features of the flow past the plate as compared with the case of the smaller time step. The time interval between the introduction of the new vortices into the wake was selected as $0.32\,(2a/U_\infty)$.

(iii) Since a considerable number of vortices exist in the flow field, it is probable that some vortices attain small separations and produce large velocities at each other's positions because of the singularity of the point vortices. This was avoided by the use of the cut-off vortex originally suggested by Chorin [12]. The cut-off radius σ was chosen as $0.05(2a)$ by taking into account the values of similar parameters employed by previous investigators (see § 3 of Kiya and Arie [11]).

(iv) It is probable that individual vortices approach too close to the rear face of the plate and this causes them to have unreasonably high velocities along the plate owing to the presence of the image vortices within the circle in the transformed plane. These vortices were removed form the flow field whenever they came nearer to the rear face of the plate than the distance of $0.05(2a)$.

(v) In order to keep the computation time within reasonable bounds, the vortices in a given vortex cluster were combined into an equivalent single vortex whose strength was the sum of the individual strengths and whose position was the center of gravity of the cluster. This procedure was employed when a cluster passed beyond a downstream distance of about $x/(2a)=6$. In some calculations, however, this procedure was not employed.

(vi) The velocity points are assumed to be situated in the vicinity of each edge of the plate respectively at the same distance from the edge concerned. This assumption seems to be reasonable in view of the fact that the plate employed in the present calculation is practically equivalent to a normal plate. The nascent vortices are located in the plane of the plate. Computations were made for the values of the parameters described above in (i)–(v) by systematically changing the distance δ between the velocity points and the edges of the plate in the range $\delta/(2a)=0.01\sim0.15$.

The results of computations will now be presented. Figure 2 shows the distribution of vortices in the wake of the plate calculated for $\delta/(2a)=0.10$. The vortex clusters are reasonably well shaped and generally consistent with experimentally observed vortex patterns. The vortex clusters for $\delta/(2a)=0.01$ were found to be rather irregular not only in their forms but in their relative positions while

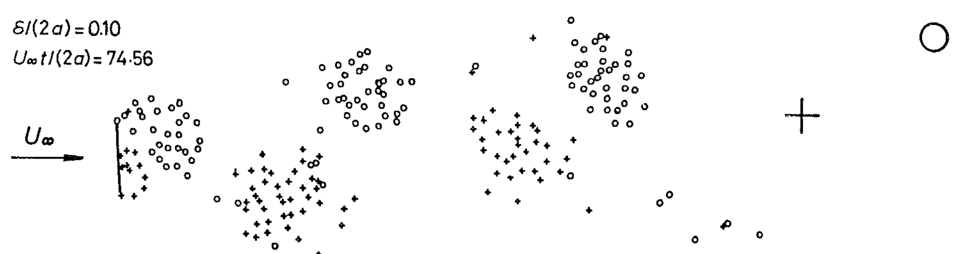

Fig. 2. Vartex pattern in the wake of flat plate of 85° incidence. ○ : clockwise vortices, +: counterclockwise vortices. Larger circle and cross imply single point vortices which have replaced corresponding vortex clusters.

those for $\delta/(2a)=0.15$ were generally satisfactory. The number of vortices included in a vortex cluster increased with a decrease in $\delta/(2a)$ and hence smaller values of $\delta/(2a)$ were accompained by larger vortex clusters. The position of the nascent vortices was observed to become more and more remote from the edges of the plate as the distance δ decreased. This may be interpreted by the fact that, as δ decreases, the velocities at the velocity points, i. e. the strength of the nascent vortices, become larger so that they must be situated at larger distances from the edges of the plate in order to satisfy the Kutta condition.

Figure 3 shows the temporal variations of the drag coefficient $C_D(=\text{Drag}/[(1/2)\rho U_\infty^2(4a)])$ and the rate of shedding of vorticity into the separated shear layers $\partial\Gamma/\partial t$ calculated for $\delta/(2a)=0.1$. Amplitudes of the wave forms of C_D and $\partial\Gamma/\partial t$

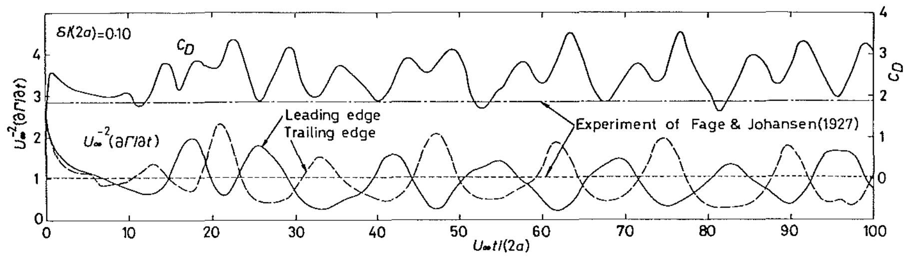

Fig. 3. Temporal variations of drag coefficient C_D and rate of shedding of vorticity from separation points $U_\infty^{-2}(\partial\Gamma/\partial t)$. $\alpha=85°$.

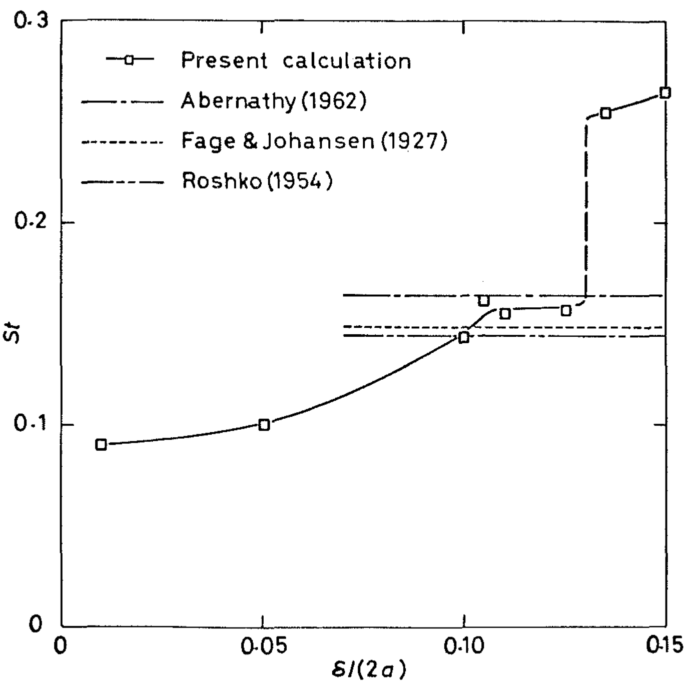

Fig. 4. Variation of Strouhal number St with respect to $\delta/(2a)$. Note that the abscissa has no meaning for experimental data.

generally increased as δ decreased. High frequency irregular fluctuations superposed upon fundamental periodic wave forms of $\partial\Gamma/\partial t$ were observed for $\delta/(2a)=0.01$ and were found to correspond to equally irregular oscillations of the locations of the nascent vortices.

The Strouhal number of the vortex shedding from the plate St can be estimated from the wave forms of $\partial\Gamma/\partial t$. The results are plotted in Fig. 4 as a function of $\delta/(2a)$. Although the Strouhal number generally increases with increased value of $\delta/(2a)$, it is observed that St attains practically a constant value between 0.143 and 0.162 in the range $0.1\leqq\delta/(2a)\leqq 0.125$, which agrees extremely well with experimentally observed values (Fage and Johansen [13], Roshko [14] and Abernathy [16]). The Strouhal number suddenly increases at $\delta/(2a)=0.13$ to the value of about 0.255 and changes only slightly for larger values of $\delta/(2a)$. Accordingly, in order to make a good prediction of the Strouhal number, the value of $\delta/(2a)$ should be in the range $0.1\leqq\delta/(2a)\leqq 0.125$.

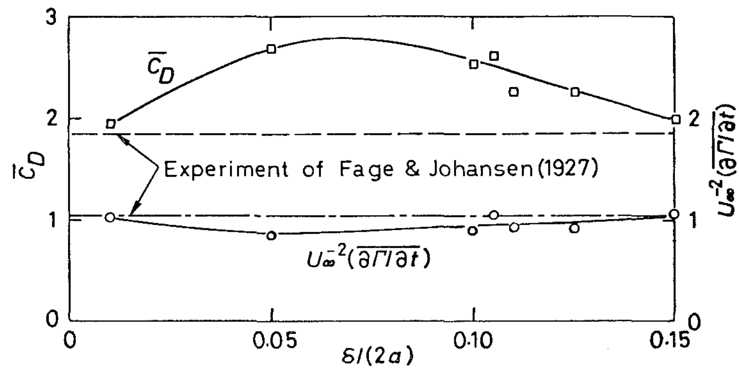

Fig. 5. Variation of time-averaged drag coefficient $\bar{C}_D$ and rate of shedding of vorticity from separation points $U_\infty^{-2}(\overline{\partial\Gamma/\partial t})$ with respect to $\delta/(2a)$. Note that the abscissa has no meaning for experimental data.

The time-averaged values of C_D and $\partial\Gamma/\partial t$ over a few cycles of the periodic vortex shedding, which will be denoted by $\bar{C}_D$ and $\overline{\partial\Gamma/\partial t}$ respectively, are showm in Fig. 5 plotted against the nondimensional parameter $\delta/(2a)$. The value of $\overline{\partial\Gamma/\partial t}$ is important in view of the fact that the back-pressure coefficient C_{pb} of the plate can be estimated from

$$C_{pb}=1-2U_\infty^{-2}(\overline{\partial\Gamma/\partial t}) \tag{4}$$

In the above-mentioned range of $\delta/(2a)$, the calculated values of $\bar{C}_D$ are larger by $20\sim 40$ percent than the experimental value of Fage and Johansen [13]. This accuracy is almost comparable to the result of Sarpkaya [5]. The values of $\overline{\partial\Gamma/\partial t}$ are observed to be practically independent of the parameter $\delta/(2a)$. An arithmetic average of four values of $\overline{\partial\Gamma/\partial t}$ obtained in the range $0.1\leqq\delta/(2a)\leqq 0.125$ is about 0.9, which is 9 precent smaller than the value 1.04 estimated on the basis of a measured back-pressure coefficient of a normal plate corrected for the wind-tunnel blockage effect (Maskell [15]). On the other hand, however, if the back-pressure coefficient is to be obtained from the calculated value of $\overline{\partial\Gamma/\partial t}$, one must expect a larger error of about 26 percent.

The convection velocity U_v of the vortex clusters in the wake is shown in Fig. 6 plotted against the distance downstream of the plate, the parameter $\delta/(2a)$ being selected as 0.10. The convection velocity has been defined as the velocity induced at the center of gravity of the vortex clusters concerned. It is found that the convection velocity increases slowly in the downstream direction. Fage and Johansen [13] measured the velocity of the rolled-up vortices in the wake of a normal flat plate in the region $x/(2a)=4.0\sim24.4$ to show that the velocity was equal to $0.766\,U_\infty$. In the present calculation, however, the convection velocity of this value is realized approximately in the region $25\leqq x/(2a)\leqq35$ upstream of which the nondimensional convection velocity U_v/U_∞ increases from 0.4 to 0.75. It is not possible to compare the present results with other calculations because the previous users of the discrete-vortex model failed to show, within the authors' knowledge, the convection velocity of the vortex clusters. The smaller convection velocity in the vicinity of the plate may be explained by the larger strength of the vortex clusters in comparison with the strength of rolled-up vortices in actual wakes. This conjecture is supported by the fact that the cancellation of the vorticity, which may mostly be represented in the discrete-vortex model by a number of elemental vortices entering a vortex cloud of opposite sign, is much smaller than that in the actual wakes in which $34\sim70$ percent of the original vorticity shed from the separation points are cancelled in the formation region of the rolled-up vortices.

In fact, it was observed in the present calculation that the convection velocity decreases with increased average number of elemental vortices in vortex clusters, which is approximately proportional to their strength. In the region more downstream, the vertical distance between two negihbouring vortex clusters of opposite sign increases so that the longitudinal velocity component in the upstream direction induced at the center of each vortex cluster by all other vortex clusters will decrease, the result being an increase in its convection velocity in the downstream direction.

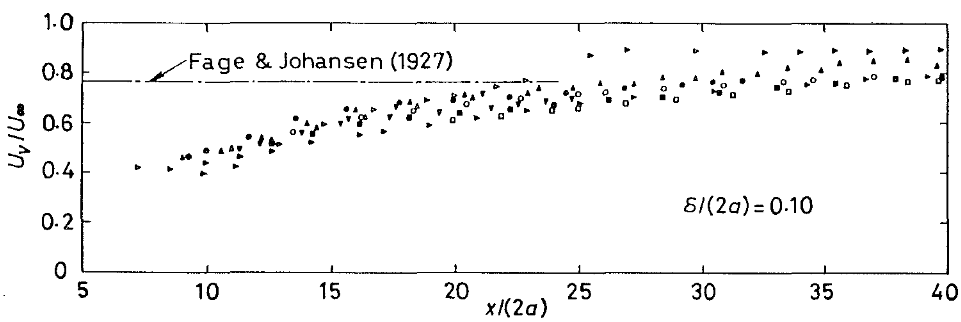

Fig. 6. Variation of convection velocity U_v of rolled-up vortices in the downstream direction. Open and solid symbols are for clockwise and counterclockwise vortices.

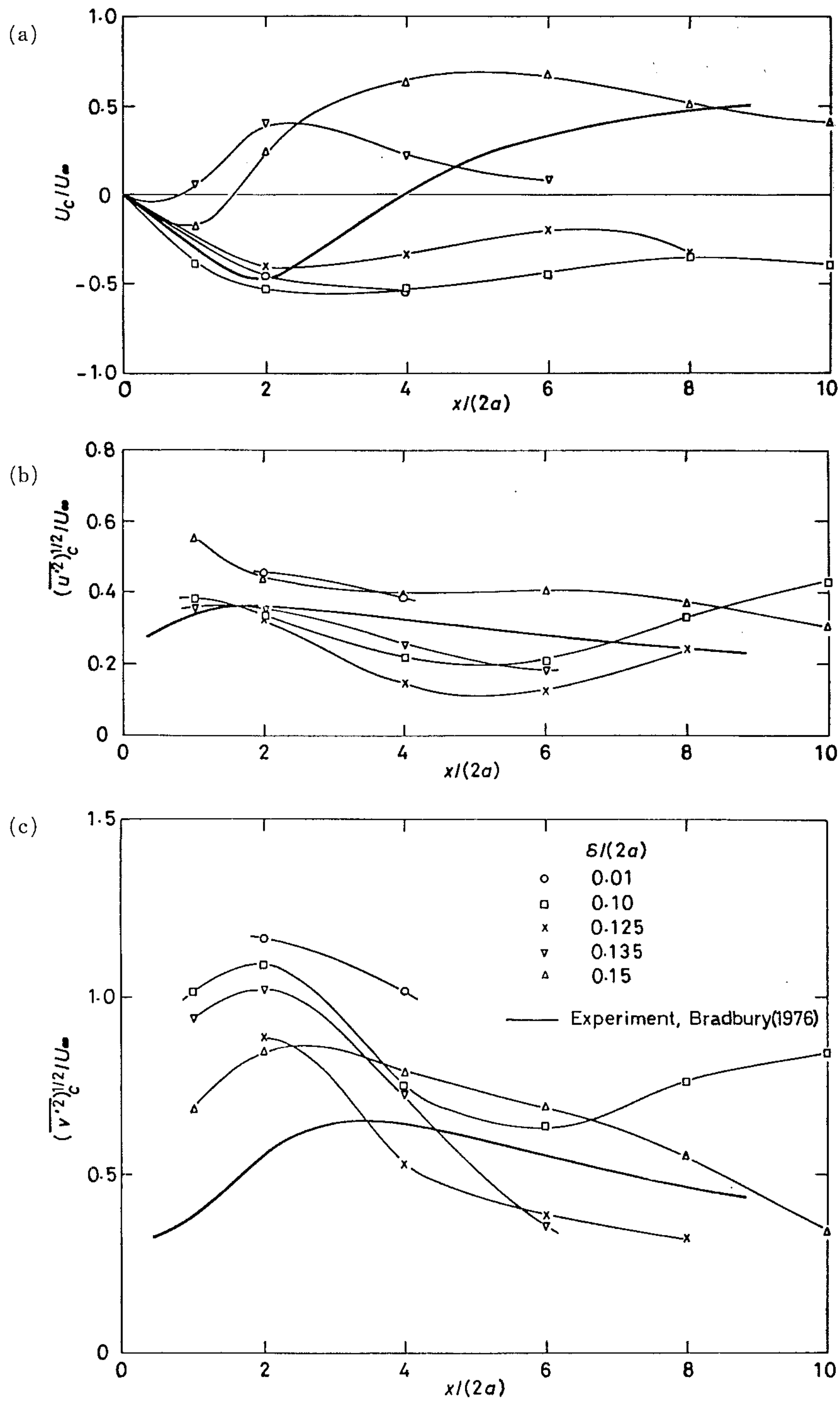

Fig. 7. Variations of time-averaged longitudinal velocity and root-mean-square values of fluctuating velocity components along the center of wake. (a) U_c/U_∞, (b) $(\overline{u'^2})_c^{\frac{1}{2}}/U_\infty$, (c) $(\overline{v'^2})_c^{\frac{1}{2}}/U_\infty$.

Figure 7 shows the nondimensional plots of the time-averaged longitudinal velocity component U_c/U_∞ and the root-mean-square values of the longitudinal and vertical velocity fluctuations, $(\overline{u'^2})_c^{\frac{1}{2}}/U_\infty$ and $(\overline{v'^2})_c^{\frac{1}{2}}/U_\infty$, along the center-line (x-axis) of the wake. The experimental results of Bradbury [8] are also included in Fig. 7 for the purpose of comparison. The values of U_c/U_∞ obtained for $\delta/(2a)=0.01$ and 0.1 are much smaller than the measured ones and continue to be negative in the region $x/(2a)\geqq 4.0$ where the experiment shows no recirculation. It seems clear that larger extent of the recirculation region is brought about by larger strength of the vortex clusters in comparison with the actual strength of the rolled-up vortices.

On the other hand, the $U_c/U_\infty \sim x/(2a)$ curve calculated for larger values of $\delta/(2a)=0.15$ is well above the experimental curve with a small recirculation region in the range $0\leqq x/(2a)\leqq 1.4$. As mentioned previously, the strength of the vortex clusters in this case was found to be much weaker than those for other values of $\delta/(2a)$. At first sight it may seem that, if an appropriate value of $\delta/(2a)$ is selected, the calculated $U_c/U_\infty \sim x/(2a)$ curve will coincide with the experimental one. However, this was not the case. The computations were repeated for another two values of $\delta/(2a)$, i. e. 0.125 and 0.135, the results being again included in Fig. 7. The peculiar behaviour of the $U_c/U_\infty \sim x/(2a)$ curve for $\delta/(2a)=0.135$ demonstrates the impossibility of selecting an appropriate value of $\delta/(2a)$ such as to yield good agreement between calculation and experiment with regard to the time-averaged velocity distribution. The authors are of the opinion that the introduction of some artificial means to simulate the cancellation of vorticity in the formation region of rolled-up vortices is inevitable in order to obtain the $U_c/U_\infty \sim x/(2a)$ curve consistent with experiment.

As will be seen in Fig. 7(b), the calculated values of $(\overline{u'^2})_c^{\frac{1}{2}}/U_\infty$ are distributed in the vicinity of the experimental curve, no systematic change of this quantity with respect to $\delta/(2a)$ being observed. The general level of $(\overline{v'^2})_c^{\frac{1}{2}}/U_\infty$ is higher than the experiment especially in the region $0\leqq x/(2a)\leqq 3$ for most values of $\delta/(2a)$, where the calculated values of $(\overline{v'^2})_c^{\frac{1}{2}}/U_\infty$ are about twice the experimental results on the average. Thus the agreement between calculation and experiment is not satisfactory.

3. Concluding Remarks

In the present paper, a discrete-vortex model designated as the velocity-point scheme has been developed to predict the characteristics of the separated flow past a normal plate immersed in a uniform approaching stream. This scheme utilizes the magnitude of velocity at the "velocity point" fixed in the vicinity of the edges of the plate in order to determine the strength of the vortices newly introduced into the wake. An appropriate distance δ between the velocity point and the edges of the plate was found to exist in the range $\delta/(2a)=0.1\sim 0.125$. If a value of δ in this range is employed, the Strouhal number agrees extremely well with experiment while the time-averaged values of the drag coefficient and rate of shedding of vorticity from the separation points are approximately 9 percent smaller and 20∼40 percent larger than experimental results respectively. These errors are among the smallest of the previous calculations of the separated flow past inclined flat plates including the one set approximately normal to the approaching stream. The time-averaged velocity and fluctuating quantities in the wake, however, are quite unsatisfactorily predicted by the velocity-point scheme.

One of the main shortcomings of the discrete-vortex model in general is an insufficient cancellation of vorticity in the formation region. The cancellation of vorticity is represented in the discrete-vortex model mainly by a certain number of point vortices entering a given vortex cluster of opposite sign. The point is that only an insufficient mixing actually occurs. Accordingly, in order to realize an appropriate amount of vorticity cancellation, an artificial reduction in the strength of vortices must be incorporated into the scheme. Such an approach will be made in Part II of this series of papers.

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