On the Flexural Deflection of a Moderately Thick Plate

Part I. Equation of Deflection of a Thick Plate

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Résumé

Equation of deflection of a moderately thick plate is obtained by expanding components of displacement into power series in the coordinate perpendicular to the surface of the plate. Assuming that the thickness of plate is small, equations of deflection for a plate are given to the third order approximation. The present equations are compared with approximate equations hitherto obtained.

The method presented here enables us to calculate more precise equations of deflection with any desired accuracy.

§ 1. Notations and Fundamental Equations

Notations

x_i: rectangular coordinates, $(i=1, 2, 3)$

ξ_i: components of displacement, $(i=1, 2, 3)$

$\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial\xi_j}{\partial x_i}+\frac{\partial\xi_i}{\partial x_j}\right)$: components of strain, $(i, j=1, 2, 3)$

$\varepsilon_{kk}=\varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33}$,

$A_{ij}=\lambda\varepsilon_{kk}\delta_{ij}+2\mu\varepsilon_{ij}$: components of stress, $(i, j=1, 2, 3)$

with Lamé's constants λ and μ,

$(l, m)=l\lambda+m\mu$, $(l, m, n)=l\lambda^2+m\lambda\mu+n\mu^2$, etc.

h: thickness of plate,

$D=\frac{h^3(0, 1)(1, 1)}{3(1, 2)}$: flexural rigidity of plate, and

w_0: deflection of plate, *i. e.* vertical displacement of the middle plane of plate.

We shall take x_1- and x_2-axes on the middle plane of plate, x_3-axis being directed downwards. (cf. Fig. 1)

Love's treatment[1] of a moderately thick plate starts from the stress-analysis of a three-dimensional elastic body, and takes the trace of stress tensor to be equal to $\Theta_0+x_3\Theta_1$, where Θ_0 and Θ_1 are plane harmonic functions.

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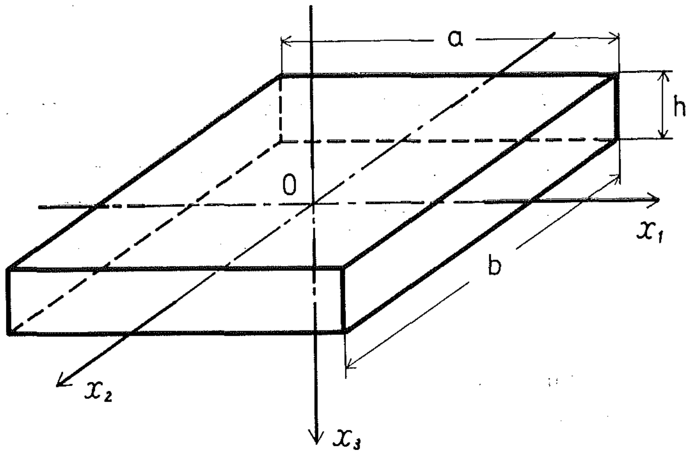

Fig. 1. Coordinate System.

Our present method is a natural extension of the method to treat the longitudinal vibration of a moderately thick bar proposed by one of the present authors, Takizawa[2], and is found to be similar to the method of initial functions by Vlasov[3].

Fundamental Equations

Equations of equilibrium of an elastic body read:

$$0=\frac{\partial A_{ij}}{\partial x_j}=(\lambda+\mu)\frac{\partial}{\partial x_i}\varepsilon_{kk}+\mu\Delta_3\xi_i\,, \qquad (i=1, 2, 3) \tag{1-1}$$

with

$$\Delta_3=\frac{\partial^2}{\partial x_k^2}=\Delta+\frac{\partial^2}{\partial x_3^2}\,.$$

The strained state (I) caused in a plate under the distributed external pressure p on its upper surface, can be decomposed into the two strained states, namely (II) and (III) in Fig. 2. The state (II) expresses the strained state under pressure $-p/2$ on the upper surface and $+p/2$ on the lower surface. While, the state (III) cor-

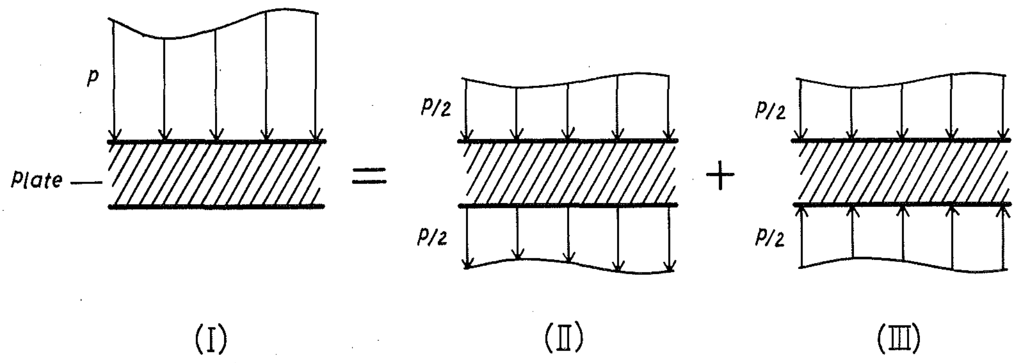

Fig. 2. Strained state (I), decomposed into (II) and (III).

responds to the strained state under pressure $-p/2$ on the upper and lower surfaces, respectively.

We shall treat here the problem in state (II). So, one can expand the components of displacement as follows:

$$\left.\begin{aligned} \xi_1 &= \sum_{k=0}^{\infty} u_{2k+1}\, x_3^{2k+1}, \\ \xi_2 &= \sum_{k=0}^{\infty} v_{2k+1}\, x_3^{2k+1}, \\ \xi_3 &= \sum_{k=0}^{\infty} w_{2k}\, x_3^{2k}, \end{aligned}\right\} \tag{1-2}$$

where, u_k, v_k, and w_k, are functions of x_1 and x_2.

By means of (1-2), the right-hand sides of eqs. (1-1) are also expressed in power series in x_3. Putting the coefficients of the same power in x_3 equal to zero in eqs. (1-1), we obtain the following equations*:

$$u_{2k+1} = \frac{(-1)^k}{(2k+1)!}\left[\Delta^k u_1 + \frac{k(1,1)}{(1,2)}\Delta^{k-1}\frac{\partial}{\partial x_1}\left\{\Xi_1 - \Delta w_0\right\}\right], \quad (k=0, 1, 2, \cdots) \tag{1-3}$$

$$v_{2k+1} = \frac{(-1)^k}{(2k+1)!}\left[\Delta^k v_1 + \frac{k(1,1)}{(1,2)}\Delta^{k-1}\frac{\partial}{\partial x_2}\left\{\Xi_1 - \Delta w_0\right\}\right], \quad (k=0, 1, 2, \cdots) \tag{1-4}$$

$$w_{2k} = \frac{(-1)^k}{(2k)!}\left[\Delta^k w_0 + \frac{k(1,1)}{(1,2)}\Delta^{k-1}\left\{\Xi_1 - \Delta w_0\right\}\right], \quad (k=0, 1, 2, \cdots) \tag{1-5}$$

with

$$\Xi_1 = \frac{\partial u_1}{\partial x_1} + \frac{\partial v_1}{\partial x_2}. \tag{1-6}$$

By means of (1-3)$\sim$(1-6), components of strain and stress can be expressed by u_1, v_1, and w_0.

§ 2. Boundary Conditions at the Upper and Lower Surfaces of the Plate

Boundary conditions at the surfaces of the plate shall be taken to be:

$$\pm\frac{p}{2} = A_{33} = \lambda\varepsilon_{kk} + 2\mu\varepsilon_{33}, \qquad \text{at} \quad x_3 = \pm\frac{h}{2}$$

$$0 = A_{31} = 2\mu\varepsilon_{31}, \qquad \text{at} \quad x_3 = \pm\frac{h}{2}$$

and

$$0 = A_{32} = 2\mu\varepsilon_{32}, \qquad \text{at} \quad x_3 = \pm\frac{h}{2}$$

i. e.

* We understand that the zero-th power of the Laplacian operator is equal to unity, *i. e.* $\Delta^0 = 1$.

$$\pm\frac{p}{2}=\sum_{k=0}^{\infty}\left(\pm\frac{h}{2}\right)^{2k+1}\Big[(1,2)(2k+2)\,w_{2k+2}+(1,0)\,\Xi_{2k+1}\Big], \tag{2-1}$$

$$0=\sum_{k=0}^{\infty}\left(\pm\frac{h}{2}\right)^{2k}\left[\frac{\partial}{\partial x_1}w_{2k}+(2k+1)\,u_{2k+1}\right], \tag{2-2}$$

and

$$0=\sum_{k=0}^{\infty}\left(\pm\frac{h}{2}\right)^{2k}\left[\frac{\partial}{\partial x_2}w_{2k}+(2k+1)\,v_{2k+1}\right]. \tag{2-3}$$

By means of (1-3)∼(1-6), eqs. (2-1)∼(2-3) read:

$$\frac{4}{3}(1,1)\frac{p}{D}=\sum_{k=0}^{\infty}\frac{(-1)^k(2k+2)}{(2k+3)!}\left(\frac{h}{2}\right)^{2k}\Big[(2k+1,2k)\,\Delta^{k+2}w_0-(2k+3,2k+4)\,\Delta^{k+1}\Xi_1\Big], \tag{2-4}$$

$$\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}\left(\frac{h}{2}\right)^{2k}\left[\Delta^k u_1+\frac{2k(1,1)}{(1,2)}\Delta^{k-1}\frac{\partial}{\partial x_1}\Xi_1-\frac{(2k-1,2k-2)}{(1,2)}\Delta^k\frac{\partial}{\partial x_1}w_0\right]=0, \tag{2-5}$$

and

$$\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}\left(\frac{h}{2}\right)^{2k}\left[\Delta^k v_1+\frac{2k(1,1)}{(1,2)}\Delta^{k-1}\frac{\partial}{\partial x_2}\Xi_1-\frac{(2k-1,2k-2)}{(1,2)}\Delta^k\frac{\partial}{\partial x_2}w_0\right]=0, \tag{2-6}$$

with flexural rigidity D of plate of thickness h:

$$D=\frac{h^3(0,1)(1,1)}{3(1,2)}.$$

From (2-5) and (2-6), we have:

$$\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}\left(\frac{h}{2}\right)^{2k}\Big[(2k+1,2k+2)\,\Delta^k\Xi_1-(2k-1,2k-2)\,\Delta^{k+1}w_0\Big]=0, \tag{2-7}$$

and

$$\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}\left(\frac{h}{2}\right)^{2k}\Delta^k\Omega_1=0, \tag{2-8}$$

with

$$\Omega_1=\frac{\partial u_1}{\partial x_2}-\frac{\partial v_1}{\partial x_1}. \tag{2-9}$$

§ 3. Approximate Equations of Flexural Deflection of a Thick Plate

When the thickness h of plate is small compared with its lateral dimensions, we obtain u_1 and v_1, by means of (2-5)∼(2-7), as follows:

$$u_1=-\frac{\partial}{\partial x_1}w_0-\left(\frac{h}{2}\right)^2\frac{2(1,1)}{(1,2)}\Delta\frac{\partial}{\partial x_1}w_0-\left(\frac{h}{2}\right)^4\frac{2(1,1)(4,5)}{3(1,2)^2}\Delta\Delta\frac{\partial}{\partial x_1}w_0-\left(\frac{h}{2}\right)^6\frac{2(1,1)(27,68,43)}{15(1,2)^3}\Delta\Delta\Delta\frac{\partial}{\partial x_1}w_0-\cdots, \tag{3-1}$$

and

$$v_1 = -\frac{\partial}{\partial x_2} w_0 - \left(\frac{h}{2}\right)^2 \frac{2(1,1)}{(1,2)} \Delta \frac{\partial}{\partial x_2} w_0 - \left(\frac{h}{2}\right)^4 \frac{2(1,1)(4,5)}{3(1,2)^2} \Delta\Delta \frac{\partial}{\partial x_2} w_0 - \\ - \left(\frac{h}{2}\right)^6 \frac{2(1,1)(27,68,43)}{15(1,2)^3} \Delta\Delta\Delta \frac{\partial}{\partial x_2} w_0 - \cdots . \tag{3-2}$$

From (3-1) and (3-2), we have:

$$\Omega_1 = \frac{\partial u_1}{\partial x_2} - \frac{\partial v_1}{\partial x_1} = 0 . \tag{3-3}$$

Eq. (3-3) shows that there is a function ϕ such that:

$$u_1 = \frac{\partial \phi}{\partial x_1}, \qquad v_1 = \frac{\partial \phi}{\partial x_2}, \tag{3-4}$$

with

$$\Xi_1 = \Delta \phi . \tag{3-5}$$

Introducing (3-4) into (3-1) and (3-2), we obtain:

$$\phi = -w_0 - \frac{2(1,1)}{(1,2)} \left(\frac{h}{2}\right)^2 \Delta w_0 - \frac{2(1,1)(4,5)}{3(1,2)^2} \left(\frac{h}{2}\right)^4 \Delta\Delta w_0 - \\ - \frac{2(1,1)(27,68,43)}{15(1,2)^3} \left(\frac{h}{2}\right)^6 \Delta\Delta\Delta w_0 - \cdots . \tag{3-6}$$

Substituting (3-5) and (3-6) into (2-4), we obtain:

$$\frac{p}{D} = \Delta\Delta w_0 + \frac{(13,16)}{10(1,2)} \left(\frac{h}{2}\right)^2 \Delta\Delta\Delta w_0 + \\ + \frac{(1479,3704,2332)}{840(1,2)^2} \left(\frac{h}{2}\right)^4 \Delta\Delta\Delta\Delta w_0 + \\ + \frac{(35969,135768,171420,72400)}{15120(1,2)^3} \left(\frac{h}{2}\right)^6 \Delta\Delta\Delta\Delta\Delta w_0 + \cdots . \tag{3-7}$$

Another method to obtain (3-7) is as follows. Applying operators:

$$\sum_{k=0}^{\infty} (-1)^k \frac{(2k+1, 2k+2)}{(2k)!} \left(\frac{h}{2}\right)^{2k} \Delta^k, \text{ and } \sum_{k=0}^{\infty} (-1)^k \frac{(2k+2)(2k+3, 2k+4)}{(2k+3)!} \left(\frac{h}{2}\right)^{2k} \Delta^{k+1},$$

respectively to (2-4) and (2-7), and eliminating Ξ_1, we have:

$$\sum_{k=0}^{\infty} (-1)^k \frac{(2k+1, 2k+2)}{(2k)!} \left(\frac{h}{2}\right)^{2k} \Delta^k p = \\ = 3(1,2) D \sum_{k=0}^{\infty} \sum_{i=0}^{k} (-1)^k \frac{(2i+2)(2i+1-k)}{(2i+3)!\,\{2(k-i)\}!} \left(\frac{h}{2}\right)^{2k} \Delta^{k+2} w_0 . \tag{3-8}$$

Assuming that the thickness of plate h is small, one can find that the inverse of the operator in the left-hand side of (3-8) takes the following form:

$$\vartheta = \frac{1}{(1,2)} \left\{ 1 + \frac{(3,4)}{2(1,2)} \left(\frac{h}{2}\right)^2 \Delta + \frac{(17,40,24)}{12(1,2)^2} \left(\frac{h}{2}\right)^4 \Delta\Delta + \frac{(1,1)^2(7,8)}{6(1,2)^3} \left(\frac{h}{2}\right)^6 \Delta\Delta\Delta + \cdots \right\}. \tag{3-9}$$

Applying this operator ϑ to both sides of (3-8), we also obtain (3-7).

Truncating the series in (3-7), one can obtain the approximate equations for deflection of a plate with any desired accuracy. We shall present here the equations to the third order approximation.

A) Zero-th Order Approximation

Retaining the terms of $O(h^0)$ in (3-6) and in the right-hand side of eq. (3-7), we obtain:

$$\phi = -w_0, \tag{3-10}$$

$$p = D\Delta\Delta w_0. \tag{3-11}$$

Eq. (3-11) is the equation for deflection of a plate in the zero-th order approximation in our theory, and is nothing but the usual equation for a thin plate. Eqs. (3-4) with (3-10) read:

$$u_1 = -\frac{\partial}{\partial x_1} w_0, \quad \text{and} \quad v_1 = -\frac{\partial}{\partial x_2} w_0, \tag{3-12}$$

which are usually taken in the theory of thin plate.

B) First Order Approximation

When we retain terms of $O(h^2)$ in (3-6) and in the right-hand side of (3-7), we get:

$$\phi = -w_0 - \frac{2(1,1)}{(1,2)}\left(\frac{h}{2}\right)^2 \Delta w_0, \tag{3-13}$$

$$p = D\Delta\Delta\left\{w_0 + \frac{(13,16)}{10(1,2)}\left(\frac{h}{2}\right)^2 \Delta w_0\right\}. \tag{3-14}$$

Eq. (3-14) is the equation for deflection of a plate in our first order approximation. Equation given by Speare and Kemp[4] from Reissner's theory[5] has an elastic constant (12, 16) instead of (13, 16) in our expression (3-14).

C) Second Order Approximation

If we retain terms of $O(h^4)$ in (3-6) and in the right-hand side of eq. (3-7), we have:

$$\phi = -w_0 - \frac{2(1,1)}{(1,2)}\left(\frac{h}{2}\right)^2 \Delta w_0 - \frac{2(1,1)(4,5)}{3(1,2)^2}\left(\frac{h}{2}\right)^4 \Delta\Delta w_0, \tag{3-15}$$

for ϕ, and obtain equation:

$$p = D\Delta\Delta\left\{w_0 + \frac{(13,16)}{10(1,2)}\left(\frac{h}{2}\right)^2 \Delta w_0 + \frac{(1479, 3704, 2332)}{840(1,2)^2}\left(\frac{h}{2}\right)^4 \Delta\Delta w_0\right\}, \tag{3-16}$$

for deflection of a plate in our second order approximation.

D) Third Order Approximation

As for the third order approximation, we retain terms of $O(h^6)$ in (3-6) and in the right-hand side of eq. (3-7), and obtain:

$$\phi = -w_0 - \frac{2(1,1)}{(1,2)}\left(\frac{h}{2}\right)^2 \varDelta w_0 - \frac{2(1,1)(4,5)}{3(1,2)^2}\left(\frac{h}{2}\right)^4 \varDelta\varDelta w_0 - $$
$$- \frac{2(1,1)(27,68,43)}{15(1,2)^3}\left(\frac{h}{2}\right)^6 \varDelta\varDelta\varDelta w_0 , \tag{3-17}$$

$$p = D\varDelta\varDelta\left\{w_0 + \frac{(13,16)}{10(1,2)}\left(\frac{h}{2}\right)^2 \varDelta w_0 + \frac{(1479,3704,2332)}{840(1,2)^2}\left(\frac{h}{2}\right)^4 \varDelta\varDelta w_0 + \right.$$
$$\left. + \frac{(35969,135768,171420,72400)}{15120(1,2)^3}\left(\frac{h}{2}\right)^6 \varDelta\varDelta\varDelta w_0\right\} . \tag{3-18}$$

Eq. (3-18) is the equation for deflection of a plate in our third order approximation.

In a similar manner, we can continue the process of approximation, truncating the series in (3-6) and (3-7) at higher order terms in h, and can obtain the approximate equations for deflection of a plate with any desired accuracy.

§ 4. Comments for Boundary Conditions at the Vertical Plane

As for the boundary conditions at the vertical plane of the plate, we shall consider as follows:

a) For the case of **fixed boundary**, boundary conditions should be:

$$\xi_1 = \xi_2 = \xi_3 = 0 . \qquad \text{at} \quad x_1 = \text{const.} \quad \text{and} \quad x_2 = \text{const.}$$

b) For a **simply supported plate**, these conditions may read:

$$\xi_3 = 0 , \quad \text{and} \quad A_{11} = A_{12} = 0 , \qquad \text{at} \quad x_1 = \text{const.}$$

and

$$\xi_3 = 0 , \quad \text{and} \quad A_{22} = A_{21} = 0 , \qquad \text{at} \quad x_2 = \text{const.}$$

or, in the alternative forms:

$$\xi_2 = \xi_3 = 0 , \quad \text{and} \quad A_{11} = 0 , \qquad \text{at} \quad x_1 = \text{const.}$$

and

$$\xi_1 = \xi_3 = 0 , \quad \text{and} \quad A_{22} = 0 . \qquad \text{at} \quad x_2 = \text{const.}$$

c) For **free boundary** at the vertical plane, we should take:

$$A_{11} = A_{12} = A_{13} = 0 , \qquad \text{at} \quad x_1 = \text{const.}$$

and

$$A_{22} = A_{21} = A_{23} = 0 . \qquad \text{at} \quad x_2 = \text{const.}$$

These boundary conditions contain terms of integral powers of h and are expanded into the power series in x_3 in the expressions of displacements and stress, and we should take appropriate terms of higher order of h and x_3 for solving approximate equations of the plate, namely eqs. (3-11), (3-14), (3-16), (3-18), etc.

§ 5. Discussion

i) *Approximate equations obtained from eq. (3-8)*

From eq. (3-8), we obtained approximate equations in the forms of eqs. (3-11), (3-14), (3-16), etc. It is also possible to obtain approximate equations from eq. (3-8) in a somewhat different manner.

Retaining terms of $O(h^2)$ in the left-hand side of (3-8), we obtain:

$$\left\{1-\frac{(3,4)}{2(1,2)}\left(\frac{h}{2}\right)^2\Delta\right\}p = D\Delta\Delta w_0\,, \tag{5-1}$$

which is slightly different from an equation given by Timoshenko and Krieger[6], and by Salerno and Goldberg[7] from Reissner's theory[5]. Our equation contains $h^2/4$ in the left-hand side in (5-1), while these authors[6),7)] took $h^2/5$ instead of $h^2/4$.

Neglecting terms of $O(h^2)$ in eq. (3-8), we have:

$$p = D\Delta\Delta w_0\,, \tag{5-2}$$

which is nothing but the usual equation for deflection of a thin plate and coincides with our zero-th order approximation (3-11). While, if we retain terms of $O(h^2)$ in both sides of eq. (3-8), we obtain:

$$\left\{1-\frac{(3,4)}{2(1,2)}\left(\frac{h}{2}\right)^2\Delta\right\}p = D\left\{1-\frac{1}{5}\left(\frac{h}{2}\right)^2\Delta\right\}\Delta\Delta w_0\,. \tag{5-3}$$

Applying the inverse of the operator in the right-hand side of (5-3) to both sides of eq. (5-3), and retaining terms of $O(h^2)$, one can find that eq. (5-3) is reduced to the following equation:

$$\left\{1-\frac{(13,16)}{10(1,2)}\left(\frac{h}{2}\right)^2\Delta\right\}p = D\Delta\Delta w_0\,, \tag{5-4}$$

which differs also from the equation derived by the authors cited above[6),7)], who took an elastic constant (12, 16) instead of (13, 16) in (5-4). We can find that equations having differential operators applied to p, such as in (5-1) and (5-4), are reduced to eq. (5-2), when pressure p is distributed *uniformly* over the surface of the plate. Here we showed eqs. (5-1) and (5-4) merely for the sake of comparison with the equations given by the authors cited above.

ii) *On the problem of a plate in state (III)*

It is also necessary to treat the problem in state (III), if we wish to have the detailed stress in state (I). We can obtain the fundamental equations for state (III) by means of a similar procedure as was given in § 1 and § 2. In this case, components of displacement shall be taken to be:

$$\left.\begin{aligned}\xi_1 &= \sum_{k=0}^{\infty} u_{2k}\, x_3^{2k}\,, \\ \xi_2 &= \sum_{k=0}^{\infty} v_{2k}\, x_3^{2k}\,, \\ \xi_3 &= \sum_{k=0}^{\infty} w_{2k+1}\, x_3^{2k+1}\,.\end{aligned}\right\} \tag{5-5}$$

Introducing (5-5) into equations of equilibrium (1-1), we have:

$$u_{2k} = \frac{(-1)^k}{(2k)\,!}\left[\Delta^k u_0 + \frac{k(1,1)}{(0,1)}\Delta^{k-1}\frac{\partial}{\partial x_1}\left\{\Xi_0 + w_1\right\}\right], \quad (k=0,1,2,\cdots)$$

$$v_{2k} = \frac{(-1)^k}{(2k)\,!}\left[\Delta^k v_0 + \frac{k(1,1)}{(0,1)}\Delta^{k-1}\frac{\partial}{\partial x_2}\left\{\Xi_0 + w_1\right\}\right], \quad (k=0,1,2,\cdots)$$

and

$$w_{2k+1} = \frac{(-1)^k}{(2k+1)\,!}\left[\Delta^k w_1 - \frac{k(1,1)}{(0,1)}\Delta^{k}\left\{\Xi_0 + w_1\right\}\right], \quad (k=0,1,2,\cdots) \tag{5-6}$$

with

$$\Xi_0 = \frac{\partial u_0}{\partial x_1} + \frac{\partial v_0}{\partial x_2}\,.$$

Boundary conditions at the upper and lower surfaces of the plate shall read:

$$-\frac{p}{2} = A_{33} = \lambda\varepsilon_{kk} + 2\mu\varepsilon_{33}\,, \qquad \text{at} \quad x_3 = \pm\frac{h}{2}$$

$$0 = A_{31} = 2\mu\varepsilon_{31}\,, \qquad \text{at} \quad x_3 = \pm\frac{h}{2} \tag{5-7}$$

and

$$0 = A_{32} = 2\mu\varepsilon_{32}\,. \qquad \text{at} \quad x_3 = \pm\frac{h}{2}$$

Introducing (5-5) into (5-7) with (5-6), we obtain the following equations:

$$\frac{p}{2} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)\,!}\left(\frac{h}{2}\right)^{2k}\left[(2k-1,\,2k-2)\,\Delta^k w_1 + (2k-1,\,2k)\,\Delta^k \Xi_0\right], \tag{5-8}$$

$$\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)\,!}\left(\frac{h}{2}\right)^{2k}\left[\Delta^{k+1}u_0 + \frac{(2k+1,\,2k)}{(0,1)}\Delta^k\frac{\partial}{\partial x_1}w_1 + \right.$$
$$\left. + \frac{(2k+1)\,(1,1)}{(0,1)}\Delta^k\frac{\partial}{\partial x_1}\Xi_0\right] = 0\,, \tag{5-9}$$

and

$$\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)\,!}\left(\frac{h}{2}\right)^{2k}\left[\Delta^{k+1}v_0 + \frac{(2k+1,\,2k)}{(0,1)}\Delta^k\frac{\partial}{\partial x_2}w_1 + \right.$$
$$\left. + \frac{(2k+1)\,(1,1)}{(0,1)}\Delta^k\frac{\partial}{\partial x_2}\Xi_0\right] = 0\,. \tag{5-10}$$

From eqs. (5-9) and (5-10), we have:

$$\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)\,!}\left(\frac{h}{2}\right)^{2k}\left[(2k+1,\,2k)\,\Delta^{k+1}w_1+(2k+1,\,2k+2)\,\Delta^{k+1}\Xi_0\right]=0, \quad (5\text{-}11)$$

and

$$\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)\,!}\left(\frac{h}{2}\right)^{2k}\Delta^{k+1}\Omega_0=0\,, \quad (5\text{-}12)$$

with

$$\Omega_0=\frac{\partial u_0}{\partial x_2}-\frac{\partial v_0}{\partial x_1}\,. \quad (5\text{-}13)$$

Eqs. (5-8)~(5-10) (or eqs. (5-8), (5-11), and (5-12)) can be used to obtain approximate equations for a moderately thick plate in state (III).

In Part II of our paper, we shall give solutions of the problem for a moderately thick plate under distributed pressure over the surface of the plate, when the plate is simply supported at its edge.

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