Operational Principles of Synthetic Aperture Imaging Radars

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Abstract

The synthetic aperture imaging radar (SAR) enables us to monitor the surface of the Earth, day or night, even through cloud cover with the accuracy of an optical sensor. This is due to the use of microwave imaginary and coherent processing of the acquired data. The objective of this concise paper is to present a basic equation that governs the operation of the SAR sensor. The equation is shown to be an integral equation, the solution of which gives microwave reflectivity of the region of the Earth scanned by the radar. Two methods for solving the equation are proposed. One is simply a formalization of practical techniques well known in the SAR literature. The other is new and promising in the sense that it may reduce computational burden on the data processor.

Introduction

The spaceborne synthetic aperture imaging radar or SAR provides information about the surface of the Earth by transmitting pulses in the microwave range and measuring reflected energy. This is common to all radar systems and, in no sense, the main feature of the SAR. The real virtue of the SAR is the use of coherent data collected by the radar sensor which travels along the track of the spacecraft.

A process of recovering the reflectivity of the radar scanned area from the recorded echoes requires a tremendous amount of computation that forced the use of optical techniques in the early stage of the art. Because of the history, papers in this field are full of such words as coherent, focus, ..., etc. But the current trend in many fields is "going digital" and the SAR is no exception. Digital processing of SAR data is by far more flexible and reliable than optical processing. It is time we forgot about physical aspects of the SAR, at least temporarity, and concentrate our attention on its abstract, formal characteristics. This will free us from conventional techniques that are too closely linked with Fourier optics.

This paper presents an integral equation that connects targets and their radar echoes. A major problem in SAR signal processing is to find out computationally efficient ways for solving it. Two methods for solving the equation are presented.

No efforts will be made in this paper to provide a classical explanation of the principle of operation of the SAR. The reader is referred to references[1),2)].

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Description of the Problem

Referring to Fig. 1, we assume that a radar sensor travels along the track represented by a straight line LL′ that is fixed on the xz-plane and runs parallel to the x-axis. Let $(\xi, 0, z_0)$ be the current position of the radar and $S(t)$ be the transmitted signal from it at time t. The signal travels from the radar to a point target P on the xy-plane and is scattered back to the radar. The time required for the signal to make a round trip is given by

$$\tau = 2r/c \tag{1}$$

where c is the velocity of light. Hence the radar echo of the point, as received at the radar position ξ at time t is given by

$$R(\xi, t) = w(x, \rho) S(t-\tau) \tag{2}$$

where $w(x, \rho)$ is the reflectivity of the point (x, ρ) with ρ the distance from LL′ to the point $P(x, y)$. Observe that (x, ρ) uniquely specifies a point on the xy-plane. The distance r as used in (1) is given by

$$r = \sqrt{\rho^2 + (\xi - x)^2} \tag{3}$$

Assuming that targets are continuously distributed on the xy-plane, we integrate (2) to obtain the resultant radar echo:

$$R(\xi, t) = \iint_{(x, \rho)\in A} w(x, \rho) S(t-\tau)\, dx\, d\rho \tag{4}$$

where the region of integration A is the area illuminated by the radar antenna and τ is given by (1). Substituting (1) into (4) we obtain

$$R(\xi, t) = \iint_A w(x, \rho) \bar{S}(r_t - r)\, dx\, d\rho \tag{5}$$

where

$$\bar{S}(r) = S(2r/c) \tag{6}$$

and

$$r_t = ct/2 \tag{7}$$

To be more specific, the transmitted signal $S(t)$ is a sinusoidal function modulated by a baseband pulse:

$$S(t) = \exp(j\omega t) S_0(t) \tag{8}$$

where $S_0(t)$ represents the baseband pulse. Referring to (6) we see

$$\bar{S}(r) = S(2r/c) = \exp(j\omega 2r/c)\, S_0(2r/c) = \exp(j2kr) \bar{S}_0(r) \tag{9}$$

where

$$k = \omega / c \tag{10}$$

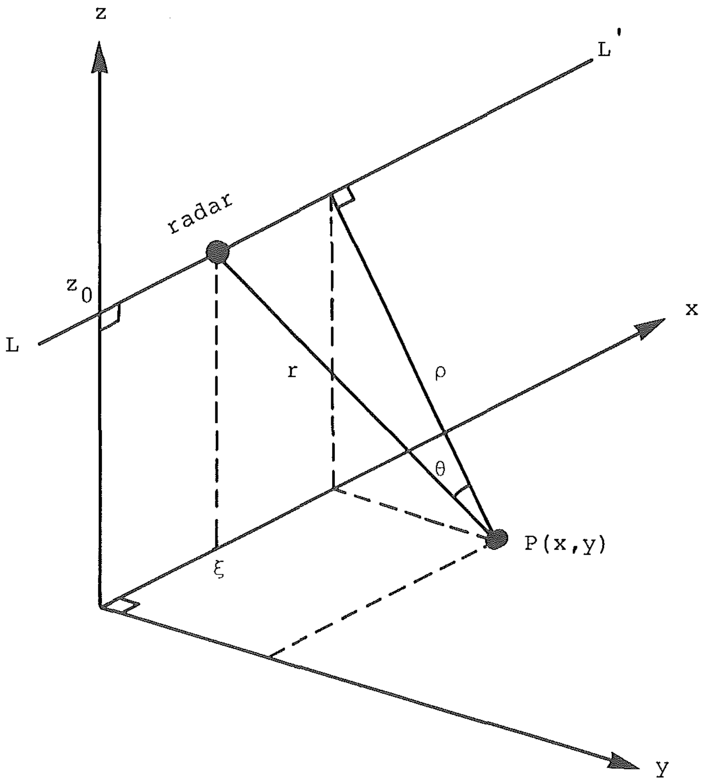

Fig. 1 Co-ordinate system of the SAR

$$\bar{S}_0(r)=S_0(2r/c) \tag{11}$$

With (9), the integral (5) is written as

$$\begin{aligned} R(\xi,\ t) &= \iint_A w(x,\ \rho)\exp(j2k(r_t-r))\bar{S}_0(r_t-r)\,dx\,d\rho \\ &= \exp(j\omega t)\,\bar{R}(\xi,\ r_t) \end{aligned} \tag{12}$$

where (7) and (10) have been used and $\bar{R}(\xi,\ r_t)$ is given by

$$\bar{R}(\xi,\ r_t)=\iint_A w(x,\ \rho)\exp(-j2kr)\bar{S}_0(r_t-r)\,dx\,d\rho \tag{13}$$

with r and r_t as defined by (3) and (7), respectively. $\bar{R}(\xi,\ r_t)$ represents the received signal at ξ after synchronous detection of $R(\xi,\ t)$. The integral equation (13) gives the basic functional relationship between the reflectivity $w(x,\ \rho)$ and the received signal $\bar{R}(\xi,\ r_t)$. A major signal processing task associated with the synthetic aperture imaging radar is to solve the integral equation to find $w(x,\ \rho)$ when $\bar{R}(\xi,\ r_t)$ is given.

Few attempts have been made in the literature to define the SAR problem by a single equation. Now that we have established the fundamental equation governing the problem, we can concentrate our attention on solving it.

Solution of the Integral Equation

Conventional Approach

Assuming that

$$(\xi-x)^2 \ll \rho^2$$

r, given in (3), can be replaced by approximations

$$r \cong r_1 \triangleq \rho+(\xi-x)^2/2\rho \tag{14}$$

or, more roughly

$$r \cong r_2 \triangleq \rho \tag{15}$$

We further assume that the baseband pulse $\bar{S}_0(r)$ is of the form

$$\bar{S}_0(r)=\exp(j\beta r^2) \tag{16}$$

With (14), (15) and (16) substituted into (13), we have

$$\bar{R}(\xi, r_t)=\int_{\rho_1}^{\rho_2} \exp(j\beta(r_t-\rho)^2)\exp(-j2k\rho)\int_{\xi-\frac{l}{2}}^{\xi+\frac{l}{2}} w(x,\rho)\exp(-jk(\xi-x)^2/\rho)\,dx\,d\rho \tag{17}$$

where we have assumed that A, the area illuminated the radar, is defined by a rectangular region

$$\xi-l/2 \geq x \leq \xi+l/2, \qquad l>0 \tag{18}$$

$$\rho_1 \leq \rho \leq \rho_2, \qquad 0<\rho_1<\rho_2 \tag{19}$$

Introducing a zero-one function

$$P_\Delta(x)=\begin{cases} 1, & |x| \leq \Delta/2 \\ 0, & \text{elsewhere} \end{cases} \tag{20}$$

we can rewrite (17) as

$$\bar{R}(\xi, r_t)=\int_{-\infty}^{\infty}\exp(j\beta(r_t-\rho)^2)p_{\rho_2-\rho_1}(\rho-\frac{\rho_1+\rho_2}{2})$$

$$\exp(-j2k\rho)\int_{-\infty}^{\infty} w(x,\rho)p_l(\xi-x)\exp(-jk(\xi-x)^2/\rho)\,dx\,d\rho \tag{21}$$

Eq. (21) is seen to consist of repeated convolutions, with respect to x and ρ, respectively. Hence we can solve (21) for $w(x, \rho)$ by appropriate deconvolution procedures. Deconvolutions with respect to ρ and x correspond to operations known in the literature as range compression and azimuth compression, respectively.

The two integrals in (21) are essentially Fresnel transforms. The inverse Fresnel transform can be most naturally implemented by an optical system. Digital processing techniques for the SAR, found in the literature, all simulate optical processing. But under digital environments other techniques should be possible. What follows is a new approach along this way of thought.

New Method

The Fresnel transform approach is a direct consequence of the approximations (14) and (15). We take an alternate approach. Set

$$\rho = r\cos\theta \tag{22}$$

$$x = \xi + r\sin\theta \tag{23}$$

Then (13) will take the form

$$\bar{R}(\xi, r_t) = \iint_{(r,\theta)\in B} w(\xi + r\sin\theta,\ r\cos\theta)\exp(-j2kr)\bar{S}_0(r_t - r)\,r\,d\theta\,dr \tag{24}$$

where we model the radar illumination B by

$$r_1 \leq r \leq r_2 \tag{25}$$

$$-\delta/2 \leq \theta \leq \delta/2 \tag{26}$$

Then (24) can be written as

$$\begin{aligned}\bar{R}(\xi, r_t) &= \int_{r_1}^{r_2} \bar{S}_0(r_t - r)\exp(-j2kr)\,r\int_{-\delta/2}^{\delta/2} w(\xi + r\sin\theta,\ r\cos\theta)\,d\theta\,dr \\ &= \int_{r_1}^{r_2} \bar{S}_0(r_t - r)\exp(-j2kr)g(\xi, r)\,dr \end{aligned}\tag{27}$$

where

$$g(\xi, r) = r\int_{-\delta/2}^{\delta/2} w(\xi + r\sin\theta,\ r\cos\theta)\,d\theta \tag{28}$$

It is easily seen that $g(\xi, r)$ can be found by deconvolution. A more difficult problem remains to be solved, i.e., we must solve (28) for $w(\cdot, \cdot)$. While an exact solution to it has not been found yet, an approximate solution can be found by noting

$$\sin\theta \cong \theta \tag{29}$$

$$\cos\theta \cong 1 \tag{30}$$

which are valid if $\delta/2$ is extremely small:

$$\delta/2 \ll 1 \tag{31}$$

With (29) and (30), eq. (28) can be written

$$g(\xi, r) \cong r\int_{-\delta/2}^{\delta/2} w(\xi + r\theta,\ r)\,d\theta = \int_{-\infty}^{\infty} p_\delta(\theta/r)\,w(\xi - \theta,\ r)\,d\theta \tag{32}$$

Deconvolution of w from (32) can be performed by simple inverse filtering or recursive techniques[3].

While the approximation (30) makes it possible to solve (28) by deconvolution, resulting errors may not be ignored; errors due to the use of (30) may impair resolution. Moderate improvement can be achieved if (30) is replaced by

$$\cos\theta \cong \overline{\cos\theta} \tag{33}$$

where $\overline{\cos\theta}$ is the average of $\cos\theta$ over $-\delta/2 \leq \theta \leq \delta/2$.

Concluding Remarks

A fundamental integral equation that governs the operation of the SAR has been presented. Range compression and azimuth compression have been given explicit mathematical definitions ; they are two successive operations on the received signal in search of a solution to the equation.

Two different techniques for solving the integral equation have been developed. The Fresnel approach is commonly used in SAR data processing since it is based on well-establised Fourier optics. An alternative approach to the problem has been proposed in this paper. Comprehensive comparison of the two methods and search for more accurate solutions of (28) are in progress and will be reported when complete.

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References

1) C. Elachi, T. Bicknell, R. L. Jordan, and C. Wu, Proc. IEEE, 70, 1174 (1982).
2) N. Fugono, Journal of the Remote Sensing Soc. of Japan, 1, 49, (1981).
3) T. Shimono and H. Kitajima, Trans. IECE of Japan, 63-D, 233 (1980).