Theory and Analysis of Incomplete Composite Plates

Koichi SATO
(Received August 28, 1991)

Abstract

This paper offers a set of partial differential equations designed to enable static analysis of incomplete composite plates with regard to the thickness of a steel plate. The paper also analyzes the relationship of lateral deflections among complete composite plates, incomplete composite plates, and individual plates which do not interact with regard to simply supported rectangular composite plates. $\beta=0$ signifies complete composite plates. $0<\beta<1$ signifies incomplete composite plates. $\beta=1$ signifies individual plates which do not interact. The paper also submits a diagram that represents the essential features of the lateral deflection characteristics of simply supported rectangular incomplete composite plates that will be found suitable for most design purposes. Lateral deflections can be derived by the use of the diagram without too much difficulty. The present method can also be applied to the finding of lateral deflections of continuous incomplete composite plates and other types of load.

1. INTRODUCTION

In recent years bridge-slab technology has increasingly come to use hybrid structures composed of such different materials as concrete slabs and steel plates. A concrete composite steeldeck plate (called here a "composite plate") consists of a concrete slab reinforced on its underside by a relatively thin flat steel plate (Fig. 1). Although a composite plate offers the combined advantages of both concrete and steel, we cannot usually rely on a natural bond between such materials. Headed stud connectors, which are welded to the steel plate and cast in the concrete, are therefore used to make the whole act as a composite plate (Fig. 2). Headed stud connectors, however, are deformed by a horizontal shear. Complete interaction is therefore impossible. If the parts of plates composed of two materials are not interconnected, each material acts separately, while an actual composite plate is intermediate between a complete composite plate and individual plates which do not interact. The same is true of an actual composite girder; i. e., a composite plate acts basically in a manner similar to a composite girder. A theory of incomplete composite plates seems to be given on p. 564 of Ref.1. In that paper, however, it appears that the thickness of the steel plate is small compared with that of concrete slab, and is of a negligible order of magnitude. But, if the steel plates are thick, we cannot neglect a consideration of their thickness. The

Department of Civil Engineering, Faculty of Engineering, Hokkaido University

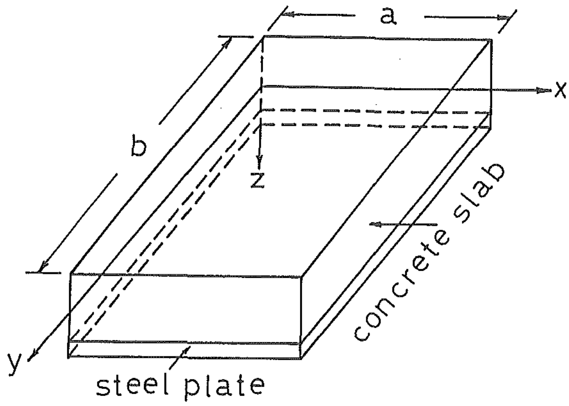

Fig. 1 Concrete composite steeldeck plate

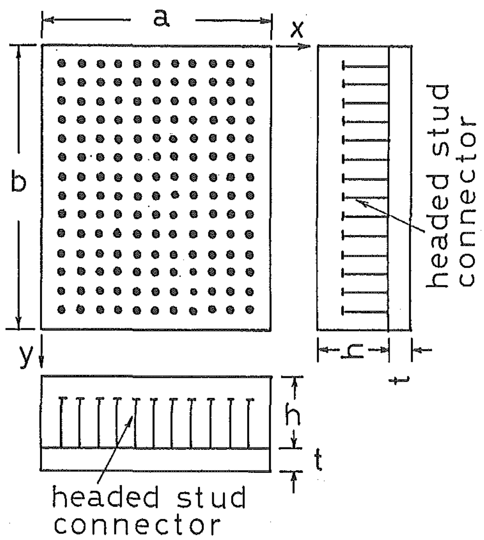

Fig. 2 Composite plate

theory offered in the present paper can be applied to the formulation of a system of equations which govern the elastic bending of incomplete composite plates consisting of two layers of isotropic materials. The main objectives for this paper are as follows: (1) To offer a set of partial differential equations designed to enable static analysis of incomplete composite plates with regard to the thickness of a steel plate; (2) To show that our theory of incomplete composite plates includes the theory of the incomplete composite plates given on p. 564 of Ref. 1; (3) To analyze the relationship of lateral deflections among complete composite plates, incomplete composite plates, and individual plates; and (4) To submit a diagram that represents the essential features of the lateral deflection characteristics of incomplete composite plates that will be found suitable for most design purposes.

2. GENERAL CONCEPTS AND ASSUMPTIONS

The plate theory of composite plates used in this paper is based on the small deflection theory, generally attributed to Kirchhoff and Love.
All loads considered in this paper are static. If the concrete part of the section is transformed into an equivalent steel plate (Fig. 3), the sectional properties of composite plates can be estimated from

$$A_c=h,\ \ A_s=t,\ \ A_v=A_s+\frac{A_c}{\bar{n}},$$
$$s=\frac{h+t}{2},\ \ s_c=\frac{A_s}{A_v}s,\ \ s_s=\frac{A_c}{\bar{n}A_v}s,$$
$$I_c=\frac{h^3}{12},\ \ I_s=\frac{t^3}{12},\ \ I_v=I_s+\frac{I_c}{\bar{n}}+A_v s_c s_s,$$
$$D_v=E_s' I_v,$$
$$\bar{n}=\frac{E_s'}{E_c'},\ \ E_c'=\frac{E_c}{1-\nu_c^2},\ \ E_s'=\frac{E_s}{1-\nu_s^2} \qquad \cdots\cdots(1)$$

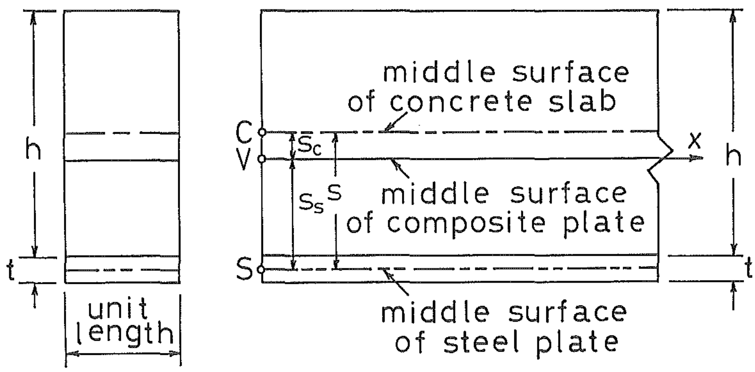

Fig. 3 Cross section of composite plate

Notations are explained in APPENDIX I. In the theory of plates it is customary to deal with internal forces and moments per unit length of the middle surface, and it should always be kept in mind that all sectional properties are defined per unit length.

3. FUNDAMENTAL DIFFERENTIAL EQUATIONS OF A COMPOSITE PLATE

Unless otherwise stated, the sectional properties of composite plates defined by Eq. (1) are used throughout this paper. And it should be remembered that interchanging the two variables, x and y, is valid.

The governing differential equation of complete composite plates expressed in terms of lateral deflection, w_v (Fig. 4), of the middle surface is given by Ref. 2 or Ref. 3

$$\nabla^2\nabla^2 w_v=\frac{p_z}{D_v} \qquad \cdots\cdots(2)$$

in which
$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2} \qquad \cdots\cdots(3)$$

is the two-dimensional Laplacian operator, D_v represents flexural rigidity of the complete composite plate, and p_z means the live load intensity. The bending moment, M_{vx}

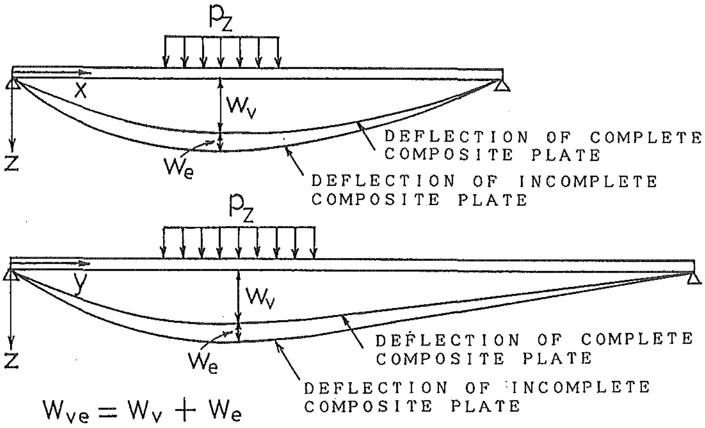

Fig. 4 Deflection of composite plate

(Fig. 5), which acts on the middle surface of complete composite plate in the x direction, is given by the following formula expressed in terms of lateral deflection, w_v :

$$M_{vx}=-D_v\left(\frac{\partial^2 w_v}{\partial x^2}+\nu\frac{\partial^2 w_v}{\partial y^2}\right) \quad \cdots\cdots(4)$$

in which ν represents Poisson's ratio of a complete composite plate. This bending moment, M_{vx}, is distributed into M_{cx}, M_{sx}, and N_{vx} (Fig. 5) ; i. e.,

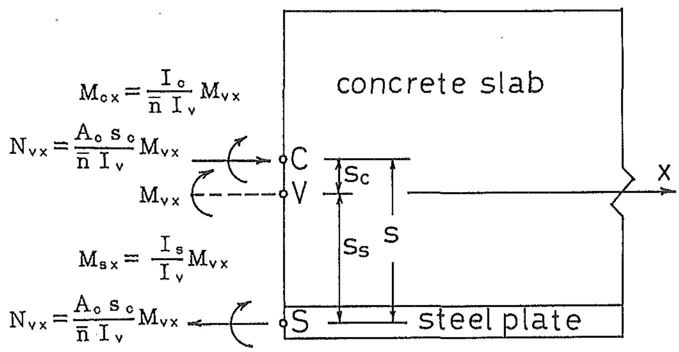

Fig. 5 Distribution of bending moment M_{vx} (complete composite plate)

$$N_{vx}=\frac{A_c s_c}{\bar{n} I_v}M_{vx} \quad \cdots\cdots(5)$$

$$M_{cx}=\frac{I_c}{\bar{n} I_v}M_{vx} \quad \cdots\cdots(6), \quad M_{sx}=\frac{I_s}{I_v}M_{vx} \quad \cdots\cdots(7)$$

in which M_{cx} and M_{sx} signify bending moments that act on the middle surface of a concrete slab and a steel plate, respectively ; N_{vx} signifies in-plane force acting on the middle surface of a concrete slab and a steel plate. It is evident that the following relationship exists :

$$M_{vx}=M_{cx}+M_{sx}+s\cdot N_{vx} \quad \cdots\cdots(8)$$

And we should mention that occasionally it might be an advantage to introduce what is called the moment-sum in a form as follows :

$$M=\frac{M_{vx}+M_{vy}}{1+\nu}=-D_v\left(\frac{\partial^2 w_v}{\partial x^2}+\frac{\partial^2 w_v}{\partial y^2}\right)=-D_v\nabla^2 w_v \quad \cdots\cdots(9)$$

The introduction of this moment-sum permits us to split the governing fourth-order differential equation of a complete composite plate into two second-order differential equations. Thus, we obtain

$$\nabla^2 M=\frac{\partial^2 M}{\partial x^2}+\frac{\partial^2 M}{\partial y^2}=-p_z \quad \cdots\cdots(10)$$

and

$$\nabla^2 w_v=\frac{\partial^2 w_v}{\partial x^2}+\frac{\partial^2 w_v}{\partial y^2}=-\frac{M}{D_v} \quad \cdots\cdots(11)$$

4. DERIVATION OF THE FUNDAMENTAL DIFFERENTIAL EQUATIONS OF AN INCOMPLETE COMPOSITE PLATE

In the case of incomplete composite plates, we can introduce simplifying assumptions which are practically the same as those assumed in analyzing incomplete composite girders[4]: (1) A continuous imperfect connection exists between the two separate materials, i. e., the shear connection between the concrete slab and steel plate is assumed to be continuous in all directions; (2) The amount of slip permitted by the shear connection is directly proportional to the load transmitted; (3) The distribution of strains throughout the depth of the concrete slab and steel plate is linear; and (4) The concrete slab and steel plate are assumed to deflect equal amounts at all points in all directions at all times.

Let us extend the concepts used in setting up the differential equation of an incomplete composite girder[5] to those of an incomplete composite plate. If we do, the procedures involved in setting up the differential equation of an incomplete composite plate subjected to lateral loads will be as follows:

The First Step:

Let us consider composite plates with complete interaction. The bending moment, M_{vex}, which acts on the middle surface of incomplete composite plates in the x direction, is obtained by the following equation expressed in terms of lateral deflections, w_{ve} (Fig. 4):

$$M_{vex}=-D_v\left(\frac{\partial^2 w_{ve}}{\partial x^2}+\nu\frac{\partial^2 w_{ve}}{\partial y^2}\right) \quad \cdots\cdots(12)$$

This bending moment, M_{vex}, is distributed into M_{cex}, M_{sex}, and N_{ex}; i. e.,

$$N_{ex}=\frac{A_c s_c}{\bar{n} I_v}M_{vex} \quad \cdots\cdots(13)$$

$$M_{cex}=\frac{I_c}{\bar{n} I_v}M_{vex} \quad \cdots\cdots(14), \quad M_{sex}=\frac{I_s}{I_v}M_{vex} \quad \cdots\cdots(15)$$

It is evident that the following relationship exists:

$$M_{vex}=M_{cex}+M_{sex}+s\cdot N_{ex} \quad \cdots\cdots(16)$$

The Second Step:

Let us direct in-plane compressive forces N_{2x} in the x direction on the middle surface of a concrete slab, and direct in-plane tensile forces N_{2x} in the x direction on the middle surface of a steel plate. If the concrete slab and steel plate are not interconnected, the curvatures of the deflected middle surfaces of the concrete slab and steel plate remain unchanged. Each material behaves separately, without any interaction, and this causes an abrupt change in the strain distribution on the plane of the contact surfaces. According to the definition of strain, ε_x, we can write

$$\varepsilon_x=\left\{\frac{1}{E_c'A_c}+\frac{1}{E_s'A_s}\right\}\cdot N_{2x}=\frac{\bar{n}\cdot s}{E_s'A_cs_c}N_{2x} \quad\cdots(17)$$

The bending moment due to N_{2x} can be calculated from

$$M_{2x}=s\cdot N_{2x} \quad\cdots(18)$$

Consequently, the total bending moment, M_{vx}, acting on the middle surface of a composite plate will be as follows:

$$M_{vx}=M_{vex}+M_{2x} \quad\cdots(19)$$

And the total in-plane force, N_{vex}, acting on the middle surfaces of a concrete slab and steel plate will be as follows:

$$N_{vex}=N_{ex}+N_{2x} \quad\cdots(20)$$

The translation of Eqs. (13), (14), (15), (16), (18) and (20) into (19) yields

$$M_{vx}=M_{vex}+M_{2x}=M_{cex}+M_{sex}+s\cdot N_{ex}+s\cdot(N_{vex}-N_{ex})$$
$$=\frac{\bar{n}I_s+I_c}{\bar{n}I_v}M_{vex}+(s_c+s_s)\cdot N_{vex} \quad\cdots(21)$$

Fig. 6 shows that the bending moment, M_{vx}, which acts on the middle surface of com-

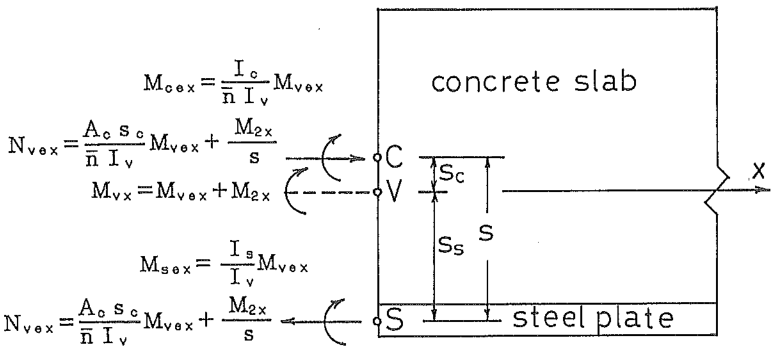

Fig. 6 Distribution of bending moment M_{vx} (incomplete composite plate)

posite plate in the x direction, is distributed into M_{cex}, M_{sex}, and N_{vex}.

The Third Step:

We have pointed out that headed stud connectors are deformed by a horizontal shear. If we assume that the amount of deformation (slip) permitted by the shear connection is directly proportional to the horizontal shear, we can define the amount of slip by using a modulus; i.e., the spring constant of the headed stud connector will be as follows:

$$H_{vex}=K\delta_x \quad\cdots(22)$$

in which δ_x represents the amount of slip of the headed stud connector in the x direction; K denotes the spring constant of the headed stud connector and can be obtained by a Push-Out Test described in Ref. 4; and H_{vex} signifies horizontal shears in the x direction. As the force H_{vex} which acts on headed stud connectors are equal to the increment of the in-plane compressive force acting on the middle surface of concrete slab, we can write

$$\frac{\partial N_{vex}}{\partial x}=H_{vex} \qquad (23)$$

And since the amount of the slip is produced by the addition of normal strains, we can write

$$\frac{\partial \delta_x}{\partial x}=\varepsilon_x \qquad (24)$$

Eqs. (12)−(24) express fundamental relationships in any consideration of deformations of headed stud connectors.

The Fourth Step:

If we use Eqs.(1)−(24), we can derive the governing differential equation of incomplete composite plates. After a derivative of Eq.(23) with respect to x, the use of Eq.(22), Eq.(24), and Eq.(17) gives

$$\frac{\partial^2 N_{vex}}{\partial x^2}=\frac{K\bar{n}}{E_s' A_c}\frac{s}{s_c}N_{2x} \qquad (25)$$

We can obtain the governing differential equation of incomplete composite plates expressed in terms of lateral deflection, w_{ve}, by the following procedure (Fig. 4). The use of Eqs.(21), (9) and (3) gives

$$\nabla^2 M\cdot(1+\nu)=\frac{\bar{n}I_s+I_c}{\bar{n}I_v}\nabla^2(M_{vex}+M_{vey})+s\cdot\nabla^2(N_{vex}+N_{vey}) \qquad (26)$$

Using Eqs.(12) and (3) gives

$$\nabla^2(M_{vex}+M_{vey})=-(1+\nu)\cdot D_v\cdot\nabla^2\nabla^2 w_{ve} \qquad (27)$$

The use of Eqs.(25), (3), and (18) gives

$$s\cdot\nabla^2(N_{vex}+N_{vey})=\frac{K\bar{n}}{E_s' A_c}\frac{s}{s_c}(M_{2x}+M_{2y}) \qquad (28)$$

After obtaining M_{2x} from Eq.(19), the use of Eqs.(4) and (12) yields

$$M_{2x}=-D_v\left(\frac{\partial^2 w_v}{\partial x_2}+\nu\frac{\partial^2 w_v}{\partial y^2}\right)+D_v\left(\frac{\partial^2 w_{ve}}{\partial x_2}+\nu\frac{\partial^2 w_{ve}}{\partial y^2}\right) \qquad (29)$$

The use of Eqs.(29) and (12) yields

$$M_{2x}+M_{2y}=-(1+\nu)\cdot D_v\cdot(\nabla^2 w_v-\nabla^2 w_{ve}) \qquad (30)$$

The translation of Eq.(30) into Eq.(28) yields

$$s\cdot\nabla^2(N_{vex}+N_{vey})=\frac{K\bar{n}}{E_s' A_c}\frac{s}{s_c}\cdot\{-(1+\nu)\cdot D_v\}\cdot(\nabla^2 w_v-\nabla^2 w_{ve}) \qquad (31)$$

After translating Eq.(27) and Eq.(31) into Eq.(26), using Eq.(10), we can write:

$$p_z=D_v\frac{\bar{n}I_s+I_c}{\bar{n}I_v}\nabla^4 w_{ve}+D_v\frac{K\bar{n}}{E_s' A_c}\frac{s}{s_c}(\nabla^2 w_v-\nabla^2 w_{ve}) \qquad (32)$$

The governing differential equation of incomplete composite plates expressed in terms of lateral deflection w_{ve} is obtained by using the following form after dividing both sides of Eq.(32) by $\frac{\bar{n}I_s+I_c}{\bar{n}I_v}$

$$D_v\nabla^4 w_{ve}-D_v\kappa^2\nabla^2 w_{ve}=-D_v\kappa^2\nabla^2 w_v+\frac{\bar{n}I_v}{\bar{n}I_s+I_c}p_z \qquad (33)$$

in which

$$\kappa^2=\frac{\bar{n}I_v}{\bar{n}I_s+I_c}\frac{K\bar{n}}{E_s' A_c}\frac{s}{s_c} \qquad (34)$$

Application of Eq.(3) to Eq.(33), the use of Eq.(2) yields

$$D_v\nabla^6 w_{ve} - D_v\kappa^2\nabla^4 w_{ve} = -\kappa^2 p_z + \frac{\bar{n}I_v}{\bar{n}I_s + I_c}\nabla^2 p_z \quad \cdots\cdots(35)$$

Either Eq. (33) or Eq. (35), both of which can be derived from this paper, will be the governing differential equation of incomplete composite plates expressed in terms of lateral deflection, w_{ve}.

On the other hand, the thickness of the steel plate in Ref. 1 is small compared with that of the concrete slab, and is of a negligible order of magnitude.
A comparison of the sectional properties of composite plates according to both theories is given in Table 1.

Table 1 Comparison of Sectional Properties of Composite Plates According to Both Theories

Sectional Properties (1)	Present Theory (2)	Clarke's Theory (3)
A_c	h	h
A_s	t	t
A_v	$A_s + \frac{A_c}{\bar{n}}$	$\frac{A_c}{\bar{n}}$
s	$0.5(h+t)$	$0.5h$
s_c	$\frac{0.5(h+t)\times t}{A_v}$	0
s_s	$\frac{0.5(h+t)\times h}{\bar{n}A_v}$	$0.5h$
I_c	$\frac{h^3}{12}$	$\frac{h^3}{12}$
I_s	$\frac{t^3}{12}$	0
I_v	$I_s + \frac{I_c}{\bar{n}} + \frac{A_c s_c s}{\bar{n}}$	$\frac{I_c}{\bar{n}} + t\times 0.5h\times 0.5h$
D_v	$E_s' I_v$	$E_c' I_c(1+3C)$

After dividing both sides of Eq.(35) by D_v, substituting $(1+4C)\frac{K}{E_s' t}$, $\frac{1}{E_c' I_c}\frac{K}{E_s' t}(1+C)$ and $\frac{1}{E_c' I_c}$ instead of κ^2, $\frac{\kappa^2}{D_v}$ and $\frac{\bar{n}I_v}{\bar{n}I_s+I_c}\frac{1}{D_v}$ and multiplying both sides by $E_c' I_c E_s' t$, and dividing by $K(1+C)$ gives

$$\frac{E_c' h^3}{12(1+C)}\left[\frac{E_s' t}{K}\nabla^6 w_{ve} - (1+4C)\nabla^4 w_{ve}\right] = -p_z + \frac{E_s' t}{K(1+C)}\nabla^2 p_z \quad \cdots\cdots(36a)$$

in which $C = \frac{E_s'}{E_c'}\frac{t}{h}$ $\cdots\cdots(36b)$

Eq.(36) is the theory of the incomplete composite plates given on p. 564 of Ref. 1. It is obvious that Eq. (35) (the theory offered here) includes Eq. (36). Let us consider a condition similar to that of an incomplete composite girder. As w_v represents the lateral deflection of a complete composite plate, it must satisfy Eq.(2); i. e., Eq.(38a), and as w_{ve} represents lateral deflection of an incomplete composite plate, it must satisfy Eq.(33). If w_e is defined as the difference between w_{ve} and w_v, we can obtain the governing differential equation expressed in terms of lateral deflection, w_e, by adjudging the difference between Eq.(33) and Eq.(2). Therefore, the differential equation expressed in terms of, w_e, is given in the following form:

$$D_v\nabla^4 w_e - D_v\varkappa^2\nabla^2 w_e = \frac{A_c s_c s}{\bar{n}I_s + I_c}p_z \quad \cdots\cdots (37)$$

Consequently, Eq.(33) consists of the two following differential equations; i. e., Eqs. (38a) and (38b):

$$\begin{cases} \nabla^4 w_v = \dfrac{p_z}{D_v} & \cdots\cdots (38a) \\ \nabla^4 w_e - \varkappa^2 \nabla w_e = \dfrac{p_z}{D_e} & \cdots\cdots (38b) \end{cases}$$

in which

$$D_v = E_s' I_v, \quad D_e = D_v \frac{\bar{n}I_s + I_c}{A_c s_c s} \quad \cdots\cdots (38c)$$

The sectional properties expressed in Eq.(38) have already been explained in Eq.(1). D_v defined in Eq.(38a) is identical with that of the two layered plates given on p. 391 of Ref. 2 and p. 5 of Ref. 6. It is interesting to note that Eq.(38b) has a form similar to that of the partial differential equation with respect to the plate under lateral loads and an in-plane force $(D_e \cdot \varkappa^2)$.

The lateral deflection, w_{ve}, of the middle surface of incomplete composite plates can be obtained by the following equations.

$$w_{ve} = w_v + w_e \quad \cdots\cdots (39)$$

By using Eqs.(12) and (4), we can obtain the bending moment, M_{vex}, from the following formula:

$$M_{vex} = -D_v\left(\frac{\partial^2 w_{ve}}{\partial x^2} + \nu\frac{\partial^2 w_{ve}}{\partial y^2}\right) = -D_v\left(\frac{\partial^2 w_v}{\partial x^2} + \nu\frac{\partial^2 w_v}{\partial y^2}\right)$$
$$-D_v\left(\frac{\partial^2 w_e}{\partial x^2} + \frac{\partial^2 w_e}{\partial y^2}\right) = M_{vx} + \frac{D_v}{D_e}M_{eex} \quad \cdots\cdots (40)$$

in which

$$M_{vx} = -D_v\left(\frac{\partial^2 w_v}{\partial x^2} + \nu\frac{\partial^2 w_v}{\partial y^2}\right); \text{ and } M_{eex} = -D_e\left(\frac{\partial^2 w_e}{\partial x^2} + \nu\frac{\partial^2 w_e}{\partial y^2}\right)$$

By using Eqs.(21), (14), (15), and I_v in Eq.(1), we can obtain the in-plane force, N_{vex}, from the following formula (Fig. 6):

$$N_{vex} = \frac{1}{s}(M_{vx} - M_{cex} - M_{sex}) = \frac{A_c s_c}{\bar{n}I_v}(M_{vx} - M_{eex}) \quad \cdots\cdots (41)$$

5. ANALYSIS OF THE SIMPLY SUPPORTED RECTANGULAR COMPOSITE PLATES

Let us determine the lateral deflections of an incomplete composite plate for a simply supported rectangular composite plate subjected to lateral loads. Since the deflection and the bending moment along the boundary are zero, the formulation of this type of boundary condition involves statements concerning displacements and forces. For simply supported rectangular plates, Navier's solution offers considerable mathematical advantages, since the solution of the governing fourth-order partial differential equation is reduced to a solution of an algebraic equation. The boundary conditions of rectangular plates, for which Navier's solution is applicable, are

$$(w_v)_{x=0,x=a}=0, \quad (M_{vx})_{x=0,x=a}=0$$
$$(w_v)_{y=0,y=b}=0, \quad (M_{vy})_{y=0,y=b}=0 \qquad \cdots\cdots(42a)$$

and

$$(w_e)_{x=0,x=a}=0, \quad (M_{eex})_{x=0,x=a}=0$$
$$(w_e)_{y=0,y=b}=0, \quad (M_{eey})_{y=0,y=b}=0 \qquad \cdots\cdots(42b)$$

which represent simply supported edge conditions at all edges. The solutions of the governing differential equations of the plate (Eq.(38)) subjected to a lateral loading are obtained by Navier's method as follows:

1. The deflections are expressed by a double sine series,

$$w_v=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} {}_vW_{mn}\cdot\sin\frac{m\pi x}{a}\cdot\sin\frac{n\pi y}{b} \qquad \cdots\cdots(43a)$$

$$w_e=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} {}_eW_{mn}\cdot\sin\frac{m\pi x}{a}\cdot\sin\frac{n\pi y}{b} \qquad \cdots\cdots(43b)$$

which satisfy all the above-stated boundary conditions. In Eq.(43) the coefficients of expansion ${}_vW_{mn}$ and ${}_eW_{mn}$ are unknown.

2. The lateral load p_z may also be expanded into a double sine series:

$$p_z=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} P_{mn}\cdot\sin\frac{m\pi x}{a}\cdot\sin\frac{n\pi y}{b} \qquad \cdots\cdots(44)$$

The coefficients P_{mn} of the double Fourier expansion of the load can be calculated:

$$P_{mn}=\frac{4}{ab}\int_0^a\int_0^b p_z\cdot\sin\frac{m\pi x}{a}\cdot\sin\frac{n\pi y}{b}\,dxdy \qquad \cdots\cdots(45)$$

3. If we translate Eqs.(43) and (44) into the governing differential equations; i.e., Eqs.(38a) and (38b), we obtain algebraic equations from which the unknown ${}_vW_{mn}$ and ${}_eW_{mn}$ can be readily calculated. Thus, for specific m and n values, Eqs.(38a) and (38 b) become

$${}_vW_{mn}\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]^2\cdot\sin\frac{m\pi x}{a}\cdot\sin\frac{n\pi y}{b}=\frac{1}{D_v}P_{mn}\cdot\sin\frac{m\pi x}{a}\cdot\sin\frac{n\pi y}{b} \qquad \cdots\cdots(46a)$$

and

$${}_eW_{mn}\left\{\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]^2+\varkappa^2\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]\right\}\cdot\sin\frac{m\pi x}{a}\cdot\sin\frac{n\pi y}{b}$$
$$=\frac{1}{D_e}P_{mn}\cdot\sin\frac{m\pi x}{a}\cdot\sin\frac{n\pi y}{b} \qquad \cdots\cdots(46b)$$

hence

$${}_vW_{mn}=\frac{P_{mn}}{D_v\mu_{mn}^4} \quad \cdots\cdots(47a), \qquad {}_eW_{mn}=\frac{P_{mn}}{D_e(\mu_{mn}^4+\varkappa^2\mu_{mn}^2)} \quad \cdots\cdots(47b)$$

in which

$$\mu_{mn}^2=\frac{m^2\pi^2}{a^2}+\frac{n^2\pi^2}{b^2} \qquad \cdots\cdots(48)$$

If we translate ${}_vW_{mn}$ in Eq.(47a) into w_v in Eq.(43a), we obtain an analytical solution, w_v, for the deflection in Eq.(43a). In a similar way, we can also obtain w_e. D_v or D_e is the flexural rigidity of the strip of a plate of unit width. It is obvious that the lateral deflections vary inversely as D_v or D_e. Upon the introduction of symbols and the use of Eq.(1), the ratio, $\frac{w_e}{w_v}$,

becomes

$$\gamma=\frac{w_e}{w_v}=\frac{D_v}{D_e}\cdot\beta=\frac{A_cs_cs}{\bar{n}I_s+I_c}\cdot\beta=\frac{A_cs_cs}{\bar{n}I_s+I_c}\cdot(1-\alpha) \quad\cdots\cdots(49)$$

in which

$$\beta=\frac{\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{P_{mn}}{\mu_{mn}{}^4+\kappa^2\mu_{mn}{}^2}\cdot\sin\frac{m\pi x}{a}\cdot\sin\frac{n\pi y}{b}}{\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{P_{mn}}{\mu_{mn}{}^4}\cdot\sin\frac{m\pi x}{a}\cdot\sin\frac{n\pi y}{b}} \quad\cdots\cdots(50)$$

We can then obtain the lateral deflection, w_{ve}, of incomplete composite plates

$$w_{ve}=w_v+w_e=w_v\left(1+\frac{w_e}{w_v}\right) \quad\cdots\cdots(51)$$

If we translate γ in Eq.(49) into w_{ve} in Eq.(51), we find

$$w_{ve}=w_v(1+\gamma)=w_v\left(1+\frac{I_{csv}}{I_{cs}}\cdot\beta\right)$$

$$=w_v\frac{I_{cs}+\beta\cdot I_{csv}}{I_{cs}}$$

$$=\frac{(I_{cs}+\beta\cdot I_{csv})}{E_s'\cdot(I_{cs}+I_{csv})\cdot I_{cs}}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{P_{mn}}{\mu_{mn}{}^4}\cdot\sin\frac{m\pi x}{a}\cdot\sin\frac{n\pi y}{b} \quad\cdots\cdots(52)$$

in which $I_v=I_{cs}+I_{csv}$; $I_{cs}=I_s+\frac{I_c}{\bar{n}}$; and $I_{csv}=\frac{A_cs_cs}{\bar{n}}$ $\quad\cdots\cdots(53)$

The subscripts "v" and "ve" of w_v and w_{ve} in Eq.(51) denote complete and incomplete composite plates, respectively.

It should be remembered that the composite plates will be subdivided into the following three major categories, based on their structural action and depending on the value of β defined by Eq. (50).

$\beta=0$, i. e., $\alpha=1$; $\gamma=0$, signifies complete composite plates.

$0<\beta<1$, i. e., $\alpha=1-\beta$, $\gamma=\frac{A_cs_cs}{\bar{n}I_s+I_c}\beta$, signifies incomplete composite plates.

$\beta=1$, i. e., $\alpha=0$, $\gamma=\frac{A_cs_cs}{\bar{n}I_s+I_c}$, signifies individual plates which do not interract.

6. THE ESSENTIAL FEATURES OF LATERAL DEFLECTION CHARACTERISTICS

Let us find the essential features of the lateral deflection characteristics for a simply supported rectangular plate subjected to a uniformly distributed load. The expansion coefficient P_{mn} defined by Eq.(45) will be $\frac{16p_z}{\pi^2 mn}$. Here, we consider ten terms of the double Fourier series. Fig. 7 shows the relationship between α and κa at (0.5a, 0.5b). In Fig. 7, α is given by the ordinate and κa is given by the abscissa. α is defined by Eq. (49), while κ, defined by Eq. (34), is an important constant and multiplying "a" by κ becomes a dimensionless constant.

Let us explain the manner in which the diagram which is shown in Fig. 7. Firstly, we can determine the value of κ defined by Eq. (34) and can multiply "a" by κ. Secondly, we can find α graphically by plotting the value of κa. Therefore, $\beta=1-\alpha$ can also be obtained. Thirdly, γ can be calculated by $\frac{A_cs_cs}{\bar{n}I_s+I_c}\beta$. Finally, if we translate γ into its equivalent in Eq.(49), we can obtain lateral deflections of an incomplete com-

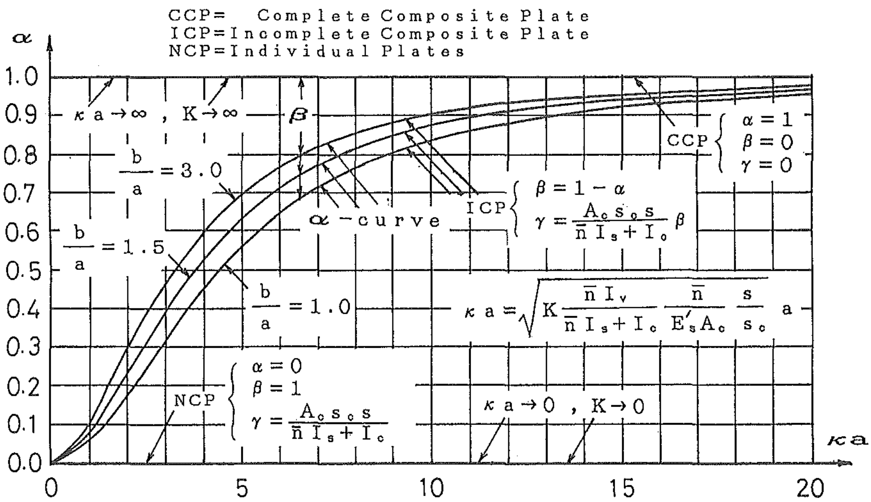

Fig. 7 α-curve (in static analysis)

posite plate by using w_{ve} in Eq. (52). Let us illustrate the practical use of this operation by numerical calculations. The general dimensions of the incomplete composite plate are as follows (Figs. 2 and 3) : Rectangular plate of size $a \times b = 2\,m \times 3m$, $E_s = 206010$ MPa, $I_{cs} = 2.481 \times 10^{-5} m^4/m$, $I_{csv} = 2.069 \times 10^{-5} m^4/m$, $K = 98.1$ MN/m/m (assumed to be extremely small), $t = 0.006$ m, $h = 0.13$ m, $\bar{n} = 7.385$, $\nu = 0.3$(assumed), $\nu_s = 0.3$, $\nu_c = 0.2$, $s = 0.068$ m, $s_c = 0.0173$ m, $s_s = 0.0507$ m, $A_c = 0.13\,m^2/m$, $A_s = 0.006\,m^2/m$, $A_v = 0.0236\,m^2/m$, $I_c = 1.831 \times 10^{-4} m^4/m$, $I_s = 1.8 \times 10^{-8} m^4/m$, $I_v = 4.55 \times 10^{-5} m^4/m$, $D_v = 1.03005 \times 10^4\,kN \cdot m^2/m$, $p_z = 9.81 \times 10^5 N/m/m$. Ten terms of the double Fourier series are again considered in this case. By using these general dimensions, we can find $\kappa = 4.21425/m$, $\kappa a = 8.4285$, $\beta = 0.1589$, $\alpha = 0.8411$, and $\gamma = 0.1325$. The lateral deflection $w_v = 0.01177$ m in the case of a complete composite plate can again also be found. The lateral deflection $w_{ve} = 0.01333$ m in the case of an incomplete composite plate can therefore be obtained from multiplying $w_v = 0.01177$ m by $(1+\gamma)$; i. e., 1.1325. Since we can calculate the lateral deflection w_{ve} of incomplete composite plates without too much difficulty by the introduction of dimensionless parameters α, β, and γ, the present diagram is very convenient for most practical purposes as a means of determining the lateral deflection.

7. CONSIDERATION BY NUMERICAL CALCULATIONS OF BOTH THEORIES

In this section, in order to compare both theories of incomplete composite plates by means of numerical calculations, let us determine lateral deflections at (0.5a, 0.5b) for a simply supported rectangular plate subjected to a uniformly distributed load. ${}_a w_{ve}$ is calculated by Eq.(36a), and ${}_e w_{ve}$ is calculated by Eq.(38). For simply supported rectangular plates, Navier's solution is available. K is variable($0 \sim \infty$). Ten terms of the double Fourier series are once more considered in this case, likewise. In Fig. 8, the ratio of lateral deflections between both theories is given by the ordinate while $C\left(=\frac{E'_s}{E_c'}\right.$

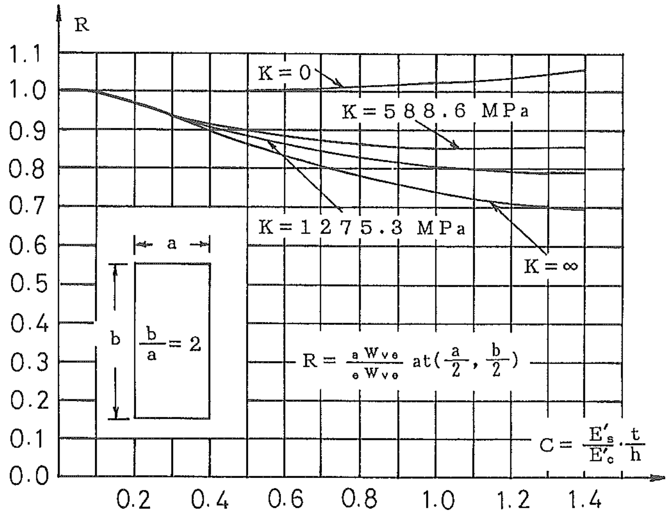

Fig. 8 Comparison by numerical calculation of both theories

$\left.\frac{t}{h}\right)$ is given by the abscissa. Fig. 8 shows that the greater C becomes, the greater grows the difference between the two theories.

The computations were carried out with double precision by using a personal computer (NEC PC-9801VM2, made in Japan).

8. CONCLUSION

The major conclusions of this investigation can be summarized as follows:

(1) This paper considers the factor of steel plate thickness in its presentation of partial differential equations in static analysis of incomplete composite plates.

(2) This paper demonstrates that its theory of incomplete composite plates subsumes the theory of the incomplete composite plates given on p. 564 of Ref.1, and that it can be applied to the formulation of a system of equations which govern the elastic bending of incomplete composite plates which consist of two layers of isotropic materials.

(3) This paper offers a mathematical analysis of the relationship of lateral deflections among complete composite plates, incomplete composite plates, and individual plates which do not interact, as applied to simply supported rectangular composite plates subjected to a uniformly distributed load.

$\beta=0$, i. e., $\alpha=1$, $\gamma=0$, signifies complete composite plates. $0<\beta<1$, i. e., $\alpha=1-\beta$, $\gamma=\frac{A_c S_c S}{\bar{n} I_s+I_c}\beta$, signifies incomplete composite plates. $\beta=1$, i. e., $\alpha=0$, $\gamma=\frac{A_c S_c S}{\bar{n} I_s+I_c}$, signifies individual plate which do not interact. Dimensionless parameters α, β, and γ can also be defined in the case of continuous incomplete composite plates and other types of load. The present method can therefore also be applied to the finding of lateral deflections of continuous incomplete composite plates and other types of load.

(4) This paper submits a diagram that represents the essential features of the lateral deflection characteristics of simply supported rectangular incomplete composite plates that will be found suitable for most design purposes. Lateral deflections w_{ve} can be obtained by the use of the diagram without too much difficulty.

9. ACKNOWLEDGMENT

The writer wishes to acknowledge his deep gratitude to emeritus Professor Doctor Noboru Watanabe, Department of Civil Engineering, Hokkaido University, Sapporo, Japan, for his suggestions and encouragement during the preparation of this manuscript.

REFERENCES

1) J. L. Clarke and C. T. Morley: Steel-Concrete Composite Plates with Flexible Shear Connectors, Proc. Inst. Civ. Engrs, Vol. 53, Part 2, pp. 557-568, Dec., 1972.
2) R. Szilard: Theory and Analysis of Plates,Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1974.
3) S. P. Timoshenko and S. Woinowsky-Krieger: Theory of Plates and Shells, 2nd ed., McGraw-Hill Book Company Inc., New York, 1970.
4) N. M. Newmark, C. P. Siess and I. M. Viest: Tests and Analysis of Composite Beams with Incomplete Interaction, Proc. of the Society for Experimental Analysis, Vol. 9 No. 1, pp. 75-93, 1951.
5) S. Shimada and R. Takagi, Theory and Design of Composite Girders (in Japanese), Sankaido, 1986.
6) K. S. Pister and S. B. Dong: Elastic Bending of Layered Plates, J. Engrg. Mech., ASCE, Vol. 85, No. 4, pp. 1-10, 1959.

APPENDIX I

The following symbols are used in Eq.(1):

A_c, A_s = sectional areas of concrete slab and steel plate per unit length, respectively;

$A_v = A_n + \dfrac{A_c}{\bar{n}}$: sectional area of composite plate per unit length, defined by transforming a concrete part of the section into an equivalent steel plate;

$D_v = E_s' I_v$: flexural rigidity of composite plate;

E_c, E_s = Young's moduli of concrete and steel, respectively;

E_c', $E_s' = \dfrac{E_c}{1-\nu_c^2}$ and $\dfrac{E_s}{1-\nu_s^2}$, respectively;

h = thickness of concrete slab;

I_c, I_s = moments of inertia per unit length about the middle surface of concrete slab and steel plate, respectively;

$I_v = I_s + \dfrac{I_c}{\bar{n}} + A_v s_c s_s$: moment of inertia per unit length about the middle surface of composite plate by transforming a concrete part of the section into an equivalent steel plate;

$\bar{n} = \dfrac{E_s'}{E_c'}$;

s = distance between the middle surface of concrete slab and steel plate;

$s_c = \dfrac{A_s}{A_v} s$: distance between the middle surface of composite plate and concrete slab;

$s_s = \dfrac{A_c}{\bar{n} A_v} s$: distance between the middle surface of composite plate and steel plate;

t = thickness of steel plate; and

ν_c, ν_s = Poisson's ratios of concrete and steel, respectively.