An Approximate Method Evaluating the Particle Terminal Velocities in Accelerating Fluid Flows

Kenji TAKAHASHI, Meiseki KATAYAMA and Kazuo ENDOH

(Received August 26, 1992)

Abstract

An experimental investigation was performed to study the forces acting on an oscillating particle and the retarded terminal velocities in an oscillating fluid. From a correlation for energy dissipation calculated with fluid resistance, an analytical expression for the drag coefficient of a particle sinusoidally oscillating in a fluid flow was obtained. By using the drag coefficient calculated from this expression, the equation of motion for a freely falling particle in an oscillating fluid was numerically solved and the retarded particle velocities were calculated. The experimental particle velocities were compared with calculated values, and agreement was within $\pm 5\%$.

1. Introduction

The dynamic behavior of particles such as bubbles, droplets and solid particles in an accelerating continuous phase can have a pronounced effect on the hold-up and mass or heat transfer characteristics of the resulting two-phase system. Experimental investigations[3,4] have shown that vertical oscillation of fluid might retard the motion of particles or bubbles, or even make them move in a direction opposed to gravity. Some researchers, such as Jameson[11,12], Houghton[6-10], Boyadzhiev[2], Baird[1], have long been interested in the phenomena. One will fined, however, that it is not easy to calculate the particle behavior in accelerating field, since the available data for drag coefficients of particles are defined under the steady state condition[5]. It is necessary to understand that the flow patterns and fluid drag phenomena in an accelerating field are radically different from those at the steady state.

There are two problems related to the particle behavior in oscillating fluid. One is stability of particle trajectories. The other is retarded terminal velocity of particle.

Houghton[6-8] has studied stability of particle in oscillating fluids by obtaining analytic and numerical solutions to the nonlinear Langevin equation representing a superposition of forces arising from particle acceleration, displaced fluid acceleration, buoyancy and fluid drag. He showed that the Langevin equation can be transformed into a piece-wise liner Mathieu equation and that the stable particle trajectory may occur in certain range of frequency and amplitude. Schoneborn[16] studied the effect of vortex shedding on the motion of particle in an oscillating liquid. He believed that stable trajectory would be impossible in the case of sinusoidally oscillations. Boyadzhiev[2]

Dept. of Atomic Sci. and Nuclear Eng.. K. Endoh is a professor emeritus of Hokkaido Univ.

also studied this problem by using the Oseen-Tchen-Houghton force balance model and showed that in the case of asymmetrical oscillations the particle may be made quasi-stationary with a finite amplitude of oscillation. And he also suggested that under such conditions a mean movement of the particle in the upward direction is also possible.

The latter problem, retarded terminal velocity, was studied by Baird et al.[1], and Tunstall and Houghton[20]. Baird et al.[1] studied the effect of oscillation on the average terminal velocities. They found that liquid oscillation brought about a decrease of up to 28 per cent in the terminal velocity. They analysed the retardation velocity, however, the experimental retardation was greater than that expected from analytical calculation. And they concluded that the additional retardation was due to the shedding of a large wake during each oscillation. Tunstall and Houghton[20] also studied terminal velocities in oscillating fluid. In their paper, the experimental terminal velocities have been compared with the velocities predicted theoretically by the numerical solution of a nonlinear Langevin equation. For the larger spheres the experimental velocities were found to be lower than those predicted theoretically using steady state drag coefficients. They concluded that these velocity difference are primarily attributed to oscillation-induced increases in the drag coefficient, with appreciable secondary influences arising from changes in phase lag, drag exponent and virtual mass.

In the above previous studies, the Basset term in the equation of motion was neglected and the steady state drag coefficient was used. This treatment leads to underestimation of the drag force in the accelerating flow situations. The Basset term can be formulated analytically for the Stokes or viscous flow regime, its explicit form, however, is unknown for larger Reynolds numbers where the drag force is nonlinear function of relative velocity.

In this paper, we have experimentally studied the fluid force acting on particles sinusoidally oscillating in a fluid flow. Next, we propose an approximated method to predict the particle behavior based on the experimentally obtained fluid forces.

2. Forces on a oscillating particle in a fluid flow

In this section, we study the in-line forces on a sinusoidally oscillating particle in a fluid flow based on the so-called Morison equation[13,15]. In this equation, the force is composed of two components, drag and inertia. The drag component of the force is assumed to be proportional to the squared velocity of the body while the inertia component is assumed to be proportional to the acceleration of the particle.

2. 1 Experimental apparatus and method

The flow system and the oscillation apparatus are shown in **Fig. 1.** The test section was a cylindrical duct 1.35 m long with an inner diameter of 0.19 m. The velocity of the fluid was controlled by an inverter connected with a pump. The test body, a sphere, was fastened to a transducer by a supporting rod. The transducer was a leaf spring of phosphorbronze on which two pairs of strain gauges were attached to form a Wheatstone bridge sensitive to unidirectional forces. Ranges of experimental variables are shown in **Table 1.** The in-line force on an oscillating body was estimated based on the Morison equation[13,15,17].

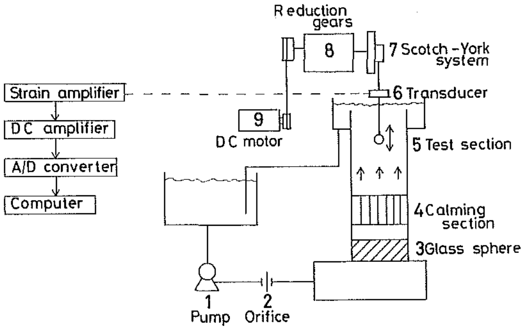


Fig. 1 Experimental apparatus

Table 1 Experimental conditions

Diameter d [m]	0.038
Amplitude a [m]	0.02—0.04
Frequency f [HZ]	0.5—1.5
Flow velocity U_0 [m/s]	0.05—0.15
Reynolds number $U_0 d/\nu$ [−]	2100—6300
Reynolds number $d^2\omega/\nu$ [−]	3980—9950

$$F = -M_0 C_m (dv/dt) - (1/2)\rho S_0 C_d |v| v \tag{1}$$

Where v is the oscillating velocity and M_0 and S_0 are the mass of displaced fluid and the projected area of the body respectively. C_m and C_d are the virtual mass and the drag coefficients. These coefficients can be calculated by using the Fourier-Transfor analysis[15,17]. When the particle is oscillating in a fluid flow, the relative velocity of an oscillating particle to the fluid stream is expressed by the following equation.

$$v = -a\omega \sin(\omega t) - U_0 \tag{2}$$

Though C_m and C_d are function of the time-dependent angular displacement, we assume that the coefficients C_m and C_d are constant irrespective of length of period. The fluctuating in-line force was sampled with the aid of a computer through the A/D converter. By subtracting both the inertia forces due to the effective mass of the body-transducer system and the sinusoidal buoyancy forces due to the supporting rod from the measured instantaneous force, the fluid resistance were obtained at each phase of the oscillating cycle.

2. 2 Results and Discussion

The first term on the right-hand side of Eq.(1) are eliminated by multiplying each side of Eq.(1) by the oscillating velocity, Eq.(2), and integrating over a period of one cycle.

$$1/(2\pi)\int_0^{2\pi} Fvd(\omega t) = 1/(2\pi)\int_0^{2\pi} (-1/2)\rho C_d S_0 V^2 |v| d(\omega t) \tag{3}$$

Since (force)x(velocity) is work per unit time, this equation represents time- averaged energy dissipation. We define the energy dissipations E_v, E_f and E_{fv} as follows.

When the velocity of the steady fluid stream is equal zero, the average energy dissipation for an oscillating particle in a liquid at rest, E_v, is

$$E_v = (1/2\pi)\int_0^{2\pi} -(1/2)\rho S_0 C_{dv} (a\omega \sin \omega t)^2 |-a\omega \sin \omega t| d(\omega t)$$
$$= (2/3\pi)\rho S_0 C_{dv} (a\omega)^3 \tag{4}$$

where C_{dv} is the drag coefficient in a liquid at rest.

While the oscillating velocity is equal zero, the energy dissipation for a stationary particle in a fluid stream, E_f, becomes

$$E_f=(1/2)\rho S_0 C_{df} U_0^3 \tag{5}$$

where C_{df} is the drag coefficient for a stationary particle.

The energy dissipation for the oscillating particle in a fluid flow is obtained from Eq.(3) and is defined as follows.

$$E_{fv}=(1/2\pi)\int_0^{2\pi}\{\rho C_d S_0(a\omega \sin \omega t+U_0)^2|-a\omega \sin t-U_0|\}d(\omega t) \tag{6}$$

The values of energy dissipation E_{fv} divided by E_f are shown in **Fig. 2**, plotted against the energy dissipation ratio E_f/E_v. When the values of E_f/E_v are smaller than 0.1, E_{fv}/E_f is roughly inversely proportional to the energy dissipation ratio, E_f/E_v, which means that the E_{fv} values are proportional to E_v in this range. We found the following 2/3-power low relation[17] from Fig. 2.

$$E_{fv}^{2/3}=E_f^{2/3}+E_v^{2/3} \tag{7}$$

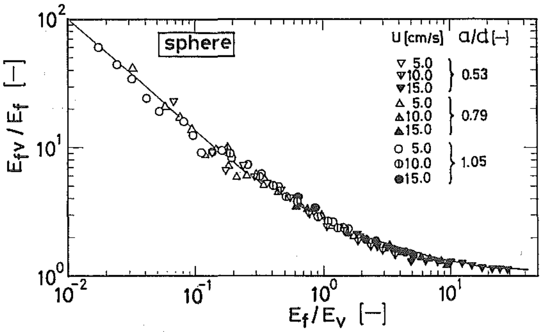


Fig. 2 A relation for energy dissipation for an oscillating particle in a fluid flow

Insertion of the expressions for E_{fv}, E_f and E_v, Eq.(4)-(6), into Eq.(7) then gives

$$\{(1/2\pi)\int_0^{2\pi}-(1/2)\rho C_d S_0(a\omega \sin \omega t+U_0)^2|-a\omega \sin \omega t-U_0|d(\omega t)\}^{2/3}$$
$$=\{(1/2)\rho S_0 C_{df} U_0^3\}^{2/3}+\{(2/3\pi)\rho S_0 C_{df}(a\omega)^3\}^{2/3} \tag{8}$$

By integrating the left-hand side of Eq.(8) we find that:

$$C_d=[\{K(U_0/a\omega)\}^{2/3}+\{E(U_0/a\omega)\}^{2/3}]^{3/2} \tag{9}$$

where $K(U_0/a\omega)$ $E(U_0/a\omega)$ and are given by the following expressions:

(i) When $U_0<a\omega$

$$K(U_0/a\omega)=\frac{2\pi C_{df}}{-\frac{4(1-R^2)^{3/2}}{3R^3}+\left(\frac{4}{R^3}+\frac{6}{R}\right)\sqrt{1-R^3}+\left(\frac{3}{R^2}+2\right)(2\arcsin R-\pi)+\frac{3\pi}{R^2}+2\pi}$$

$$K(U_0/a\omega)=\frac{\frac{8}{3}C_{dv}}{-\frac{4(1-R^2)^{3/2}}{3}+\left(4+\frac{6}{R^2}\right)\sqrt{1-R^2}+(3R+2R^3)(2\arcsin R-\pi)+3\pi R+2\pi R^3} \tag{10}$$

(ii) When $U_0>a\omega$

$$K(U_0/a\omega)=\frac{\frac{C_{df}}{3}}{2\left(\frac{1}{R}\right)^2+1}$$

$$K(U_0/a\omega)=\frac{\frac{4C_{dv}}{3\pi}}{\frac{3}{2}R+R^3} \tag{11}$$

where $R=U_0/a\omega$.

As is evident from Eqs.(9), (10) and (11), C_d values can be expressed as a function of C_{df}, C_{dv} and the fluid-velocity-to oscillating-velocity ratio $U_0/a\omega$. Accordingly, the drag coefficient for the oscillating bodies in a fluid flow is of the form:

$$C_d=\mathcal{F}(C_{df},\ C_{dv},\ U_0/a\omega) \tag{12}$$

Some sample profiles of the drag coefficient calculated from Eq.(9) are plotted in **Fig. 3.** It can be seen from Fig. 4 that the curves calculated from Eq.(9) are in fairly close agreement with the experimental results.

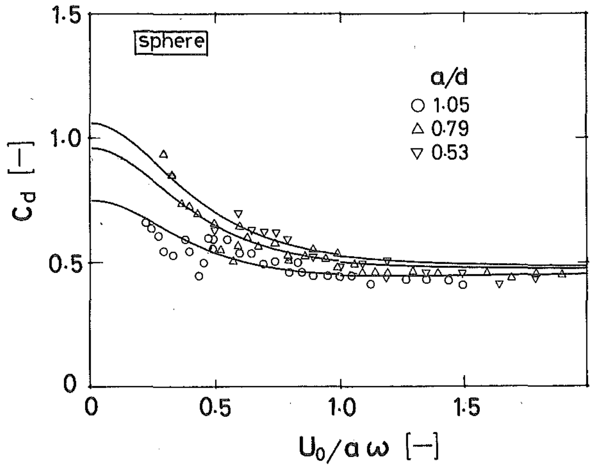


Fig. 3 Variation of drag co-efficient with $U_0/a\omega$

3. Retarded Velocity of a Particle in an Oscillating Fluid

In this chapter we study the time averaged falling velocities of particle in an oscillating fluid, and try to estimate the particle falling velocities based on the fluid drag obtained in the previous chapter.

3.1 Experimental apparatus and method

Experimental apparatus was shown in **Fig. 4.** The test section was a cylindrical duct 2.0 m long with an inner diameter of 0.06 m. The fluid oscillation was produced by a Scotch-York system. The time averaged velocities of a particle was so measured that the time interval of two pulse signals produced when a particle throughout sensors was recorded in a digital recorder. Distance between two sensors was 1.2 m. Nylon

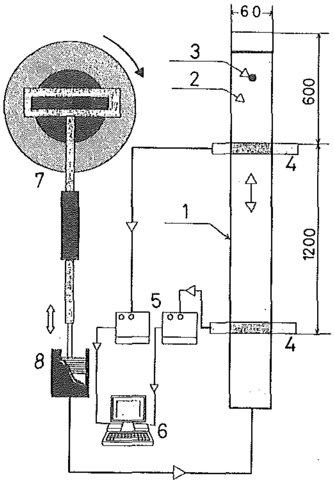


Fig. 4 Schematic diagram of experimental apparatus.
1. test section; 2. tap water; 3. test particle; 4. sensor; 5. sensor amplifier; 6. digital recorder; 7. Scotoch-York; 8. piston and cylinder

Table 2 Experimental conditions

Diameter of particle d [m]	0.80, 1.27, 1.59
Density of particle ρ_p [Kg/m^3]	1380, 1350, 1250
Frequency of oscillation f [Hz]	0.8 —2.2
Arplitude of oscillation a [cm]	1.38—4.06

spheres were used as a test particle. Experimental conditions were listed in **Table 2.**

3. 2 Results

3. 2. 1 Terminal velocity in a liquid at rest

Measured velocities in a liquid at rest and standard deviation of measured velocities were listed in **Table 3.** The Reynolds number based on both the measured velocity and the diameter of particle was in the range of $2310 < Re < 5160$, hence the low of fluid drag was in the so-called Newton's low region. The values of v^* and v^{**} in Table 3 were calculated values from proposed equations by Turton[21] and Zigrang[22], respectively. The measured velocities were almost the same as v^*.

Table 3 Terminal velocities of test particles in a liquid at rest

d [cm]	ρ_p [kg/m^3]	v_t [m/s]	v^* [m/s]	v^{**} [m/s]	$v_t d/\nu$ [−]	σ/v [−]
0.80	1.380	0.292	0.289	0.270	2310	0.051
1.27	1350	0.353	0.358	0.342	4435	0.061
1.59	1250	0.328	0.339	0.326	516	0.053

3. 2. 2 Time averaged velocity in an oscillating fluid

Measured particle velocities in an oscillating fluid, standard deviation, velocity ratio were listed in **Table 4.** It is evident from table 4 that the time averaged velocities in an oscillating fluid is smaller than the velocities in a fluid at rest.

Table 4 Experimental results of particle velocities in an oscillating fluid

a [cm]	f [Hz]	d [cm]	$\bar{v}$ [m/s]	$\bar{v}/v_t$ [−]	$\sigma\bar{v}$ [−]	d [cm]	$\bar{v}$ [m/s]	$\bar{v}/v_t$ [−]	$\sigma/\bar{v}$ [−]	d [cm]	$\bar{v}$ [m/s]	$\bar{v}/v_t$ [−]	$\sigma/\bar{v}$ [−]
1.36	0.8	0.80	0.286	0.977	0.018	1.27	0.349	0.991	0.029	1.59	0.321	0.980	0.040
	1.0		0.284	0.972	0.021		0.353	0.999	0.021		0.329	1.005	0.038
	1.2		0.286	0.980	0.018		0.347	0.983	0.021		0.331	1.009	0.023
	1.4		0.287	0.984	0.019		0.347	0.983	0.026		0.318	0.970	0.030
	1.6		0.288	0.988	0.024		0.343	0.970	0.021		0.316	0.962	0.031
	1.8		0.283	0.969	0.022		0.346	0.948	0.025		0.305	0.931	0.042
	2.0		0.269	0.923	0.031		0.333	0.945	0.027		0.297	0.905	0.040
	2.2		0.257	0.879	0.026		0.319	0.903	0.033		0.285	0.868	0.026
2.36	0.8	0.80	0.289	0.989	0.033	1.27	0.359	1.019	0.049	1.59	0.325	0.990	0.031
	1.0		0.287	0.981	0.019		0.356	1.008	0.034		0.324	0.986	0.028
	1.2		0.286	0.979	0.022		0.351	0.994	0.037		0.314	0.959	0.035
	1.4		0.281	0.961	0.025		0.335	0.949	0.065		0.304	0.926	0.049
	1.6		0.275	0.942	0.022		0.313	0.886	0.034		0.297	0.906	0.044
	1.8		0.264	0.903	0.035		0.301	0.852	0.051		0.279	0.853	0.043
	2.0		0.257	0.878	0.024		0.289	0.819	0.027		0.273	0.833	0.041
	2.2		0.247	0.845	0.035		0.283	0.802	0.032		0.254	0.773	0.033
4.08	0.8	0.80	0.289	0.989	0.041	1.27	0.358	1.014	0.044	1.59	0.320	0.977	0.027
	1.0		0.293	1.003	0.046		0.351	0.994	0.054		0.313	0.954	0.032
	1.2		0.285	0.974	0.082		0.340	0.964	0.037		0.319	0.972	0.050
	1.4		0.277	0.946	0.034		0.331	0.937	0.069		0.294	0.897	0.045
	1.6		0.271	0.929	0.026		0.296	0.835	0.044		0.274	0.834	0.052
	1.8		0.246	0.839	0.062		0.289	0.820	0.018		0.267	0.815	0.053
	2.0		0.237	0.811	0.058		0.275	0.779	0.049		0.259	0.788	0.027
	2.2		0.226	0.774	0.055		0.262	0.742	0.051		0.247	0.752	0.048

3. 3 Discussion

Forces acting on a particle in an oscillating fluid could be expressed by the drag force, the buoyancy force due to the pressure gradient in the fluid surrounding the particle caused by fluid acceleration, the force accelerating the virtual mass of the particle, the acceleration history force which is so-called Basset force, and the gravitational force. The Basset force, however, can be formulated for only viscous flow regime, hence the mathematical analysis of the Basset force in the high Reynolds number regime is difficult in general. However, from the study by Torbin and Gauvin[19], the Basset force is small under the high Reynolds number conditions. As a consequence, the equation of motion can be written as follows[18].

$$\begin{aligned}\rho_p V_p dv/dt = &-(1/2)\rho A_p C_d|v-u|(v-u)+\rho V_p C_m(dv/dt-du/dt)\\ &-\rho V_p(g-du/dt)+\rho_p V_p g \qquad (13)\end{aligned}$$

where v is particle velocity, u fluid velocity, C_m and C_d are virtual mass and drag coefficients.

In the chapter 2 we have studied the force acting on an oscillating particle in a fluid flow. The relation for the drag coefficient expressed by Eq.(9) may apply to this phenomena. Then, we solved Eq.(13) numerically with assuming $C_m=0.5$, and calculated the time-averaged particle velocity. By comparing the calculated velocity with the measured one, the drag coefficient which satisfies both Eq.(13) and the measured velocity was obtained. The one of the results was shown in **Fig. 5.** By comparing obtained C_d and the relation expressed by Eq.(9), we found that the drag coefficients C_{dv} in Eq. (9) are $C_{dv}=1.8$, 1.2, 1.0 corresponding to the fluid amplitudes $a=1.36$, 2.72, 4.08 cm. The similar calculations were done for other conditions, and resulting C_{dv} were shown in **Fig. 6**. From the figure C_{dv} can be expressed by a function of amplitude ratio as follows.

$$C_{dv}=0.7+(a/d)^{-1} \tag{14}$$

$0.85<a/d<5.1$, $330<d^2\omega/\nu<1600$, $1250<\rho_p<1380$

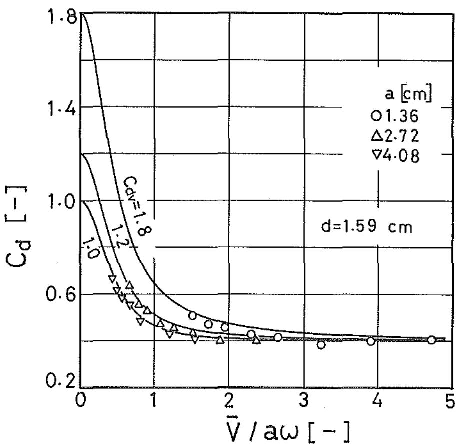


Fig. 5 Experimental and calculated drag coefficients. Solid lines show Eq.(9)

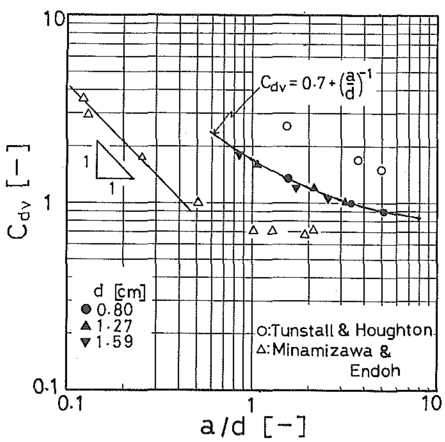
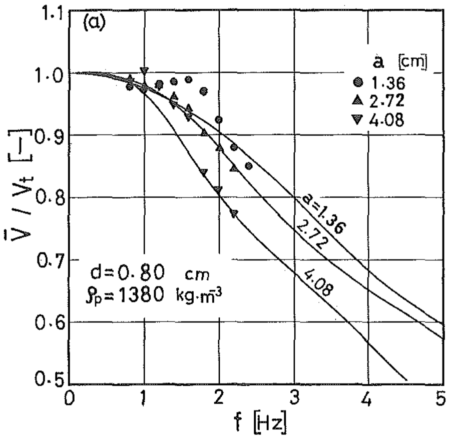
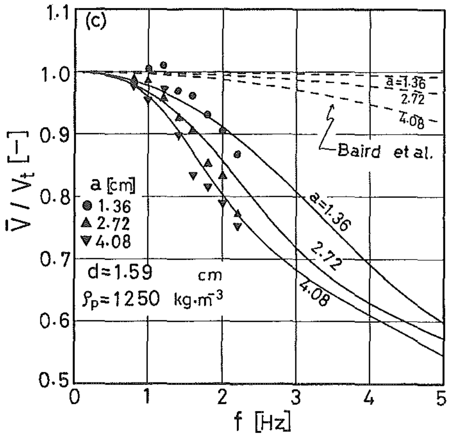


Fig. 6 Correlation between C_{dv} and amplitude ratio

The equation of motion, Eq.(13) was solved numerically by using the drag coefficients obtained from Eq.(14) and Eq.(9), and the calculated velocities were shown in **Fig. 7 (a), (b), (c)** by solid lines. Keys plotted in Fig. 7 are the measured velocities listed in Table 4. The calculated values agree with the measured one within ±5% error. The analytical solutions by Baird et al.[1] who assumed the steady state drag coefficients in the calculation were plotted by dotted lines in Fig. 7(c). It is clear that their method can not predict the experimental values.

The experimental results by Tunstall and Houghton[20] who studied the particle velocity falling in an oscillating fluid under the high frequency conditions were rearranged, and were shown in **Fig. 8.** The solid lines in Fig. 8 are calculated values from Eq. (9) and Eq.(13) as $C_{dv}=2.5$, 1.7, 1.5 in Eq.(9) corresponding to the amplitude $a=$





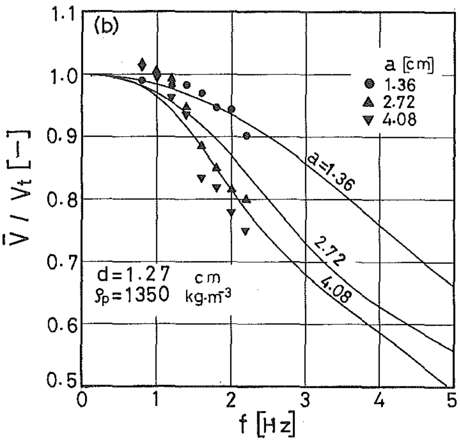


Fig. 7 Predicting the retardation velocities. Solid lines show calculated values.

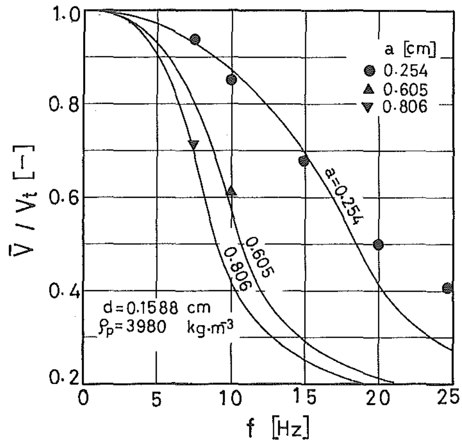


Fig. 8 Predicting a particle velocity on high frequency conditions

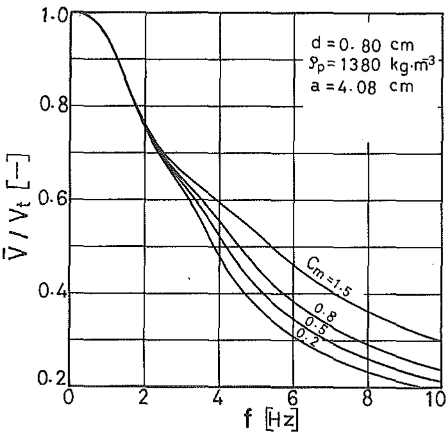


Fig. 9 Effect of virtual mass coefficient on particle falling velocity

0.254, 0.605, 0.806 cm. These C_{dv} values were shown in Fig. 6. The drag coefficients for an oscillating sphere in a liquid at rest obtained by Minamizawa and Endoh[14] were also shown in Fig. 6 for comparison. These relations for the drag coefficients were different from each other. This is because the fluid amplitude used in correlation was a virtual values as to a free particle in an oscillating fluid.

In the above discussion we assumed $C_m=0.5$ as a virtual mass coefficient. However, the effect of the virtual mass must be studied to predict the velocity of particle explicitly. **Fig. 9** shows the calculated results for the effect of virtual mass coefficient on retardation velocities of particle. When a frequency of fluid oscillation smaller than 2 Hz, there is no effect of the virtual mass on the particle velocity. When the frequency is greater than 2 Hz, the particle velocities are retarded with decreasing of the virtual mass coefficients.

4. Conclusions

We studied fluid forces acting on an oscillating particle in a fluid flow and proposed an approximate method evaluating a retarded velocities of a particle in an oscillating fluid.

A relation for the drag coefficient for an oscillating particle was obtained analytically based on a experimental relation of energy dissipation calculated from the measured fluid force. The drag coefficient was expressed by three terms as follows.

$$C_d=\mathscr{F}(C_{dv},\ C_{df},\ U_0/a\omega)$$

Time-averaged falling velocities of particle have been measured in vertically oscillating fluid. Drag coefficients were calculated, based both on a force balance equation of a free particle in an oscillating fluid and using a measured particle velocity. The calculated drag coefficient was predicted by the above relation of drag coefficient for an oscillating particle in a fluid flow. The experimental particle velocities were compared with the numerically calculated values, and agreement was within ±5% error.

Nomenclature

A_p	= projected area of particle	[m^2]
a	= amplitude of particle or amplitude of fluid	[m]
a^*	= relative amplitude of particle oscillation to fluid oscillation	[m]
C_d	= drag coefficient of oscillating particle in a fluid flow	[−]
C_{df}	= drag coefficient of stationary particle in a steady fluid flow	[−]
C_{dv}	= drag coefficient of oscillating particle in a fluid at rest	[−]
C_m	= virtual mass coefficient	[−]
d	= diameter of particle	[m]
E	= energy dissipation	[W]
F	= fluid force	[N]
f	= frequency of oscillation	[Hz]
g	= gravitational acceleration	[m/s^2]
M_0	= mass of displaced fluid	[kg]
S_0	= projected area of particle	[m^2]
t	= time	[s]
U_0	= velocity of steady fluid flow	[m/s]

u	= velocity of oscillating fluid	[m/s]
V_p	= particle volume	[m^3]
v	= particle velocity	[m/s]
$\bar{v}$	= time averaged particle velocity in oscillating fluid	[m/s]
v_t	= terminal velocity of particle in a fluid at rest	[m/s]
ν	= kinematic viscosity of fluid	[m^2/s]
ρ	= fluid density	[kg/m^3]
ρ_p	= particle density	[kg/m^3]
σ	= standard deviation of measured particle velocity	[m/s]
ω	= angular frequency of fluid oscillation	[rad/s]

Literature Cited

1) Baird, M. H. I, M. G. Senior and R. J. Thompson: Chem. Eng. Sci., **22,** 551 (1966)
2) Boyadzhiev, L.: J. Fluid Mech., **57-3,** 545 (1973)
3) Buchanan, R. H., G. Jameson and D. Oedjoe: Ind. Eng. Chem., Fund., **1,** 82 (1962)
4) Deng, Y. and M. Kwauk: Chem. Eng. Sci., **45**, 483 (1990)
5) Endoh, K.: Funtai Kogaku Kenkyu kaishi, **6**, 247 (1969)
6) Houghton, G.: Proc. Roy. Soc., **A772**, 33 (1963)
7) Houghton, G.: Can. J. Chem. Eng., **44**, 90 (1966)
8) Houghton, G.: Can. J. Chem. Eng., **46**, 79 (1968)
9) Houghton, G.: Nature, **201,** 568 (1964)
10) Houghton, G.: Nature, **204,** 447 (1964)
11) Jameson, G. J. and J. F. Davidson: Chem. Eng. Sci., **21,** 29 (1966)
12) Jameson, G. J.: Chem. Eng. Sci., **21**, 35 (1966)
13) Keulegen, G. H. and L. H. Carpenter: J. Res. Nat. Bur. Stand., **60,** 423 (1958)
14) Minamizawa, M. and K. Endoh: Kagaku Kogaku Rinbunsyu, **11,** 603 (1985)
15) Morison, J. R., M. P. O'Brien, J. W. Johnson and Schaaf: J. Petro. Tech., AIME., **189**, 149 (1950)
16) Schonebeorn, P. R.: Int. J. Multiphase Flow, **2,** 307 (1975)
17) Takahashi, K., H. Tsuruga and K. Endoh: J. Chem. Eng. Japan, **21,** 405 (1988)
18) Takahashi,K., H. Miyazaki, S. Mori and A. Tanimoto: Kagaku Kogaku Ronbunshu, **17,** 1161 (1991)
19) Torobin, L. B. and W. H. Gauvin: AIChE. J., **7,** 615 (1961)
20) Tunstall, E. B. and G. H. Houghton: Chem. Eng. Sci., **23,** 1067 (1968)
21) Turton, R. and N. N. Clark: Powder Tech., **53,** 127 (1987)
22) Zigrang, D. J. and N. D. Sylvester: AIChE. J., **27,** 1043 (1981)