Seasonal and inter-annual changes of volume density of Martian CO$_2$ snow from time-variable elevation and gravity

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Abstract

The Martian atmosphere seasonally exchanges CO$_2$ with the surface by repeating condensation and sublimation, causing seasonal growth and decay of the polar CO$_2$ snowcaps. These processes leave two kinds of geodetic signatures, i.e. seasonal changes of the Martian gravity field and of surface elevation of the snow-covered regions. Here we study gradual increase of the volume density of the Martian snow due to compaction, by combining these two data sets during 1999-2001 covering three Martian winters. We found that light fresh snow of \(~0.1 \times 10^3\) kg m$^{-3}$ slowly becomes denser reaching \(~1.0 \times 10^3\) kg m$^{-3}$ or more immediately before it thaw. The maximum snow density varies slightly from year to year, and between hemispheres. In the second southern winter, the density became as high as \(~1.6 \times 10^3\) kg m$^{-3}$. This might have been caused by a dust storm activity, e.g. increased mixing of silicate particles and/or enhancement of sintering.

1. Introduction

Ninety-five percent of the current atmosphere of Mars is carbon dioxide (CO$_2$). Up to one third of the total CO$_2$ is considered to solidify every Martian year, i.e. it accumulates as “snow” in the polar region of the wintry hemisphere where temperature drops below the CO$_2$ condensation point. It sublimes into atmosphere in late spring. Such seasonal mass redistribution can be measured by two independent geodetic techniques, i.e. altimetry and gravimetry.

The observations of topography by the Mars Orbiter Laser Altimeter (MOLA) on board the Mars Global Surveyor (MGS) spanned more than a full Martian year. In addition to stationary topography, they revealed changes in the surface elevation in specific high-latitude (65.5–86.5 degrees) regions due to seasonal growth and decay of the polar snowcaps. Smith et al (2001) collected over 66 million altitude data, and applied profile analysis to obtain a longitude-averaged data set with an accuracy of 5-6 cm. Increases of elevation more than a meter in winter hemispheres are clear there. Apart from that, Doppler tracking of the spacecraft revealed seasonal redistribution of CO$_2$ in terms of temporal changes of certain low degree Stokes’ coefficients of the Martian gravity field (Yoder et al., 2003; Konopliv et al., 2006).
Such seasonal altitude and gravity changes occur in phase, and we can compare the two quantities to infer average volume density of the polar snow packs. Smith et al. (2001), by comparing the MOLA surface altitude data with the changes of the Martian oblateness (the $J_2$ component of the gravity field), estimated the average density of the snow as $(0.91 \pm 0.23) \times 10^3 \text{ kg m}^{-3}$. This value is ~40 percent less than CO$_2$ ice ($\sim 1.5 \times 10^3 \text{ kg m}^{-3}$). There they assumed that the snow density remains constant throughout the year.

In the Earth, water snow density is known to change in time; light freshly-fallen snow ($\sim 0.08 \times 10^3 \text{ kg m}^{-3}$) often becomes as dense as $0.5 \times 10^3 \text{ kg m}^{-3}$ before spring thaw (e.g. Heki, 2004). Feldman et al (2003) inferred the amount of the Martian CO$_2$ snow by neutron spectrometry and compared it with the MOLA elevation results. They found that their results were often smaller than those inferred from altimetry data. Hence they suggested that snow density was, at that time, substantially lower than $0.91 \times 10^3 \text{ kg m}^{-3}$, the value they assumed after Smith et al. (2001) in order to compare their snow weight data and the MOLA elevation changes. They speculated that the Martian snow density might continuously increase due to compaction and re-crystallization. Here we try to reveal such temporal changes of snow density by combining the available elevation and gravity data sets.

2. Estimation of time-variable snow density

2-1. Gravimetric and altimetric $\Delta J_3$

We use the elevation (snow depth) changes from MOLA given in the Figure 2 of Smith et al. (2001). The differences of elevations from reference values, at 20 latitude bands from 65.5 to 86.5 degrees, are given at every 15 degrees in the solar longitudes $L_s$ ($L_s$ starts from the vernal equinox and changes by 360° in a Martian year). Fig.1 illustrates such snow depth snapshots between $L_s$ 105° and 525° expressed as the sum of zonal spherical harmonics with degrees up to 10. For the gravity data, we used the $J_3$ (a component representing “pear shape” of the planet) time series obtained by the Earth-based two-way Doppler measurements (the data shown in Fig. 10a of Konopliv et al. (2006)). Because it is difficult to discriminate low degree odd zonal coefficients, such “$\Delta J_3$” actually represents the sum of contributions from changes in all odd zonal coefficients, $\delta J_3$, $\delta J_5$, $\delta J_7$ … Konopliv et al. (2006) gives the combination for the case of MGS as

$$\Delta J_3 = \delta J_3 + 1.26 \delta J_5 + 1.31 \delta J_7 + 1.25 \delta J_9 + \ldots.$$  

Their “$\Delta J_3$” also represent the sum of contributions from all even zonal coefficients, $\delta J_2$, $\delta J_4$, $\delta J_6$ …

Both the “$\Delta J_2$” and “$\Delta J_3$” components are available as time-variable gravity coefficients in Konopliv et al. (2006). We here used only the “$\Delta J_3$” time series that are more suitable to explore asymmetric mass redistribution such as seasonal mass exchange between polar caps. Moreover, because MGS operates in a near polar orbit, odd zonal coefficients are more accurately determined,
i.e., the signal to noise ratio of “ΔJ₃” in seasonal changes is ten times as good as that of “ΔJ₂” (Konopliv et al., 2006). We took out time span between 28 February 1999 and 25 May 2001 when both elevation and gravity data are available. Because the gravity and elevation data are not synchronously given, we interpolated the elevation data to obtain those at the same epochs as the gravity data using the third order spline function. It should be noted that we may have underestimated the formal errors of the snow densities discussed in the later sections because we did not consider errors of the interpolated elevation data. In this study, we scale the formal errors of the estimated parameters with post-fit residuals to get realistic errors.

The observed elevation change can be converted into change in Jₙ (altimetric ΔJₙ) by

\[ \delta J_n = -a\rho_{\text{ave}} \int \Delta \sigma(\theta, \phi) \rho(\sin \theta) \cos \theta d\theta, \]

where \( a \) and \( \rho_{\text{ave}} \) are the Martian radius and average density, and \( \Delta \sigma \) is the surface mass (i.e. snow volume multiplied by its average density \( \rho_{\text{snow}} \)) at latitude \( \theta \) and longitude \( \phi \) over a unit area (Wahr et al., 1998). The integration is performed over the entire Martian surface, and we get “ΔJ₃” (we simply refer to this as ΔJ₃ hereafter) by combining calculated coefficients using the equation 1. There we neglected coefficients with degrees higher than 9 because their coefficients are not given in equation 1 (Konopliv et al., 2006). Although they are generally small (e.g. \( \Delta J_{11} \) is ~10 percent of ΔJ₃), they tend to have the same sign as the lower degree coefficients and this neglect may cause overestimate of the snow density by up to 20 percent if coefficients for higher terms are similar to those in equation 1. This, however, hardly affects the discussion on temporal density changes, the main target of the study.

By comparing the gravimetric ΔJ₃ and the altimetric ΔJ₃, we can estimate the average density of the snow. First, we assumed that this density does not depend on time (model 1). The blue curve in Fig. 2 shows the altimetric ΔJ₃ with the fixed density 0.91 × 10³ kg/m³ (Smith et al., 2001). It agrees fairly well with the gravimetric ΔJ₃, but the disagreements appear largely systematic, e.g. gravimetric ΔJ₃ are continuously larger than altimetric ΔJ₃ after \( L_s \approx 180^\circ \) but this reverses at \( L_s \approx 240^\circ \). Next we will show that such disagreements come from the wrong assumption of constant snow density.

2-2. Time-variable snow density

Terrestrial H₂O snow usually changes its density seasonally, i.e. it starts with a small density as freshly fallen snow, and gradually becomes denser by compaction due to its own mass as it thickens. It is due partly to sintering, combining of solid crystals that occur when temperatures approach the melting points. Heki (2004) suggested that such increase is often nearly linear in time. Here we
assume that similar compaction occurs on Mars, and allow the density $\rho_{\text{snow}}$ to increase (model 2). Then, the snow density obeys

$$\rho_{\text{snow}}(t) = \rho_0 + \alpha(t - t_0),$$

where $\rho_0$ is the density of fresh snow that starts to accumulate at time $t_0$, and $\alpha$ is the increasing rate. Instead of estimating the parameter $\alpha$, we rewrite $\rho_{\text{snow}}$ using the maximum density $\rho_{\text{max}}$ which is achieved at time $t_{\text{max}}$, i.e.

$$\rho_{\text{snow}}(t) = \frac{t_{\text{max}} - t}{t_{\text{max}} - t_0} \rho_0 + \frac{t - t_0}{t_{\text{max}} - t_0} \rho_{\text{max}} \quad (t_0 < t < t_{\text{max}})$$

First, we assume that $\rho_0$ and $\rho_{\text{max}}$ take same values every winter, and estimate them by least-squares method to minimize the differences between the gravimetric and altimetric $\Delta J_3$. We used the error of $\Delta J_3$ given by Konopliv et al. (2006) to weight individual data. The values of $t_0$ and $t_{\text{max}}$ are determined by grid search so that the smallest post-fit residuals are achieved (Fig. 3).

The result drawn in green in Fig. 2 shows a much improved fit (post-fit residual significantly decreased, see Fig. 2 inset), suggesting that the compaction of the Martian snow does occur. The current data set is not sensitive enough to $\rho_0$, the density of fresh snow, to accurately constrain it because the thin early snow little disturbs the gravity field. Hence we stabilized the least-squares estimation by constraining $\rho_0$ around its a-priori value 100 kg m$^{-3}$ as strongly as ±30 kg m$^{-3}$. Actually, the fit does not vary significantly for a range of realistic values of $\rho_0$.

In the northern hemisphere, we took $t_{\text{max}}$ and $t_0$ at 60$^\circ$ and 90$^\circ$ in the solar longitude $L_s$, respectively (240$^\circ$ and 270$^\circ$ in the southern hemisphere), as determined by grid search (Fig. 3). These $L_s$ correspond to the ends of May and June in the Earth, respectively. We set up two epochs and let the density decrease from $\rho_{\text{max}}$ to $\rho_0$ over a finite duration because this resulted in a better fit than instantaneous density drop (i.e., $t_{\text{max}}$ and $t_0$ are infinitely close). This may reflect the diversity of the “crocus day” (the day when old snows disappear) at places with different latitudes and sunlight conditions. Although transitions from old to new snow may occur in one epoch at individual places, hemispheric average apparently shows such two-epoch behavior. The average of these two epochs is fairly well constrained at ~75$^\circ$, but their separation is only weakly constrained as seen in the diagonally right down pattern of the bright part in Fig. 3 (i.e., 50$^\circ$/100$^\circ$ combination provides almost as good fit as 60$^\circ$/90$^\circ$). Here we plot the results assuming the 60$^\circ$/90$^\circ$ pair assuming tentatively that the thawing takes ~30 degrees in solar longitude (~1 month).

2-3. Difference between winters
Next, we allowed the maximum snow density $\rho_{\text{max}}$ to take different values in different winters and hemispheres (model 3), and performed the least-squares estimation for four parameters in total (one northern winter and two southern winters, plus the initial density). The normalized root-mean-square of $\Delta J_3$ significantly decreased again (Fig.2 inset). We obtained similar $\rho_{\text{max}}$ in the first southern winter, $(1.22 \pm 0.05) \times 10^3$ kg m$^{-3}$, and in the northern winter, $(1.15 \pm 0.07) \times 10^3$ kg m$^{-3}$. However, the value for the second southern winter was significantly larger, i.e. $(1.64 \pm 0.07) \times 10^3$ kg m$^{-3}$ at the end of the data set ($L_s = 525$).

The snow density in the second southern winter is larger than the solid CO$_2$ ($\sim 1.5 \times 10^3$ kg m$^{-3}$), which may reflect multiple causes. One possibility is the influence of dust storms and dust particles incorporated into the snow pack. Dust storms occur more frequently when Mars is near its perihelion (Smith, 2004), and this period corresponds to spring-summer in the southern hemisphere. The two largest dust storms in the studied period occurred when $L_s$ was between $225^\circ$ and $285^\circ$ (Fig.2) (Smith, 2004), and the second one ($L_s$: $270^\circ$-285$^\circ$) occurred in the south polar region when the CO$_2$ snow in the previous winter started to be replaced by the new one. This storm may have increased the snow density by putting silicate dust particles into the growing snowcap. However, in order to bring snow pack of $1.0 \times 10^3$ kg m$^{-3}$ to $1.6 \times 10^3$ kg m$^{-3}$, we need $\sim 30$ percent of dust ($2.9 \times 10^3$ kg m$^{-3}$) and this seems too large to be real. Additional density increase might have come from enhanced sintering of the snow pack by, e.g. temporary warming by the storm or modification of physical properties of the snow pack by including dust particles.

3. Sources of error

In addition to the observation errors and the systematic overestimation of snow density due to the negligence of snow distribution expressed in higher degree zonal spherical harmonics (see section 2-1), a few factors may influence the estimation of the Martian snow density. For example, two factors, i.e. redistribution of atmospheric mass, and deformation of the Martian equipotential surface (Areoid) due to ice load, affect gravimetric $\Delta J_3$. Additionally, deformation of the Martian surface due to snow load, affects altimetric $\Delta J_3$.

Evaluation of the $\Delta J_3$ due to atmospheric mass redistribution is straightforward; we can calculate it with the equation 2 substituting snow masses with those of atmosphere. We used half-monthly snapshots of Martian atmospheric pressure distribution available in the Planetary Data System Geosciences Node (http://pds-geosciences.wustl.edu/). The root-mean-squares of the atmospheric $\Delta J_3$ in the studied period was $\sim 0.14 \times 10^{-9}$, which is $\sim 5$ percent of those by snow. Because its temporal changes are fairly random, it would little influence our conclusion. Seasonal exchange of CO$_2$ between the atmosphere and polar caps could also contribute to the seasonal change of gravity coefficients because atmospheric mass distribution is not uniform due to non-flat Areoid surface. Chao and Rubincam (1990) found that it could influence $\Delta J_2$, but its contribution to $\Delta J_3$ is negligible.
Changes of the Martian snow load cause elastic deformation of the planet, and secondary changes of gravity (or Areoid height). For example, thick and heavy snow depresses the crust, and the redistributed mass in the Martian interior modifies its gravity signature ($\Delta J_3$, in this case), i.e.

$$\Delta J_3^{\text{Obs}} = \Delta J_3^{\text{True}} (1 + k_3^1),$$

There $k_3^1$ is the Martian load Love number of degree 3 for gravity, and $\Delta J_3^{\text{True}}$ is the value to be observed if Mars is perfectly rigid. Normally, $k_3^1$ is negative (the deformation reduces the amplitude of $J_3$ changes), and is about $-0.194$ for the Earth (Han and Wahr 1995). Although it has never been measured for Mars, we have good reasons to consider it significantly smaller than the Earth. From Doppler shift observations of MGS, Yoder et al. (2003) suggested that the Martian tidal Love number of degree 2 ($k_2^1$) is $-0.15$. This is nearly a half of $k_2^1$ of the Earth. Métivier et al. (2008) calculated the Martian load Love numbers assuming a plausible internal structure with a liquid outer core. There $k_3^1$ is inferred to be between $-0.05$ and $-0.08$. Thus, the elastic deformation of Mars due to surface load might cause systematic underestimation of $\Delta J_3$ (and snow density, too). However, it would not exceed 10 percent, and this would partly cancel the overestimation of the snow density by neglecting coefficients with degrees >9 in calculating altimetric $\Delta J_3$.

Martian surface deformation due to snow load might also affect snow depth measurements by MOLA. Because the snow depresses the Martian surface, the orbital laser altimeter may underestimate the true snow depth. The relationship between vertical displacement $\Delta u$ and the observed $\Delta J_3$ is

$$\Delta u(\theta) = R \left( \frac{h_3^1}{1 + k_3^1} \right) \Delta J_3^{\text{Obs}} P_{3,0}(\sin \theta),$$

where $P_{3,0}(\sin \theta)$ is the normalized spherical harmonics with degree 3 and order 0, and $\theta$ is the latitude (e.g. Davis et al., 2004). There $h_3^1$ is the load Love number of degree 3 for vertical, and Métivier et al. (2008) suggested it to be between $-0.21$ and $-0.30$. Thus $\Delta u$ would remain smaller than a centimeter under the observed amplitudes of $\Delta J_3$. This is much smaller than the elevation errors (Smith et al., 2001), and would not influence our conclusion.

Systematic error in MOLA data due to orbital uncertainty could also cause errors in snow density. MOLA data in Smith et al. (2001) show that the maximum elevation change is slightly more than 1 meter in the north and slightly less than 1 meter in the south. Aharonson et al. (2004) reprocessed the MOLA crossover data at latitudes 86° north and south, and showed that the snow depth variation reach ~1.5 meter in the north and ~2.5 meter in the south, values much larger and more asymmetric than Smith et al. (2001). Here we see how this revision may influence our results.

First we extrapolated the snow depth data at 86° north/south (Aharonson et al., 2004) to lower
latitude regions assuming that the depth linearly decreases to zero at 65°. We then followed the same procedure as in section 2, and obtained $\rho_{\text{max}}$ of $(0.44 \pm 0.05) \times 10^3$, $(0.56 \pm 0.1) \times 10^3$, and $(0.61 \pm 0.01) \times 10^3$ kg m$^{-3}$ for the first southern winter, the northern winter, and the second southern winter (the value at the end of the data set, i.e. $L_s = 525$), respectively. Although the absolute values of the density decreased by half, the essence of the present study remains the same, i.e. density increase toward the end of winter and the larger density in the second southern winter. In fact, time-variable density gives much better fit than the constant density of $0.5 \times 10^3$ kg m$^{-3}$ given by Aharonson et al. (2004).

4. Conclusion

We compared the gravimetric $\Delta J_3$, observed by Doppler tracking, and altimetric $\Delta J_3$, inferred from the snow depth measured by MOLA, and estimated the average volume density of the snow pack. In doing so, we found that they agree better by allowing the snow density to increase in time, i.e. freshly sublimated light snow gets denser until it thaw in late spring and early summer. From analogy to terrestrial H$_2$O snow (Maeno and Kuroda, 1986), we suggest possible densification mechanisms such as gravity-driven compaction and/or sintering of CO$_2$ crystals.

Next we discussed the difference in the maximum snow densities among the three winters. The second southern winter showed maximum density significantly larger than the others. We speculate that it reflects dust storm activities during the growth of the snow pack, i.e. incorporation of significant amount of dust particles into snow, and enhanced sintering related to dust storms. Evaluation of systematic errors caused by elastic deformation of Mars and by the neglect of snow distribution expressed by spherical harmonics of degrees $>10$, suggested that the obtained snow density might be overestimated by 10 percent (or a little more) in total. As suggested by revised estimate of snow depth by Aharonson et al. (2004), improved measurements of snow depths in future Mars exploration missions may call for major modifications of absolute values of the density.

Acknowledgments

We thank Alex Konopliv, JPL, for sending gravity data, Sander Goossens, NAO, for comments, and Akira Kouchi and Masatsugu Odaka, Hokkaido University, Ben Chao, National Central University, Taiwan, for their advises on Martian snow and atmosphere. Critical reviews by two anonymous referees also improved the manuscript.

Reference


Fig.1. Time series of changes in latitudinal profiles of snow depth at every 15 degrees of solar longitude $L_s$ between 105° and 525° expressed as the sum of zonal spherical harmonics with degrees 2-10. The left and right panels show south and north regions, respectively. Snow depth attains the largest value (~1.5 m in north and ~1.0 m in south regions, respectively) from late winter to early spring.
Fig. 2. (Upper panel) Time series of changes in gravity coefficient $J_3$ ($\Delta J_3$). The black dots with 1-$\sigma$ error bars show gravimetric $\Delta J_3$ observed by MGS (Konopliv et al., 2006). The three curves show altimetric $\Delta J_3$ calculated from MOLA snow depth data (Smith et al., 2001), where the average density of snow pack is assumed constant (model 1, blue), time-variable without inter-annual difference (model 2, green), and time-variable with inter-annual difference (model 3, red). Improvement of normalized root-mean-squares (NRMS) of post-fit residuals is shown in the inset (their colors correspond to those of the curves). (Lower panel) Time series of average snow density in the northern (solid lines) and southern hemispheres (dashed lines). Three colors correspond to the three models shown in the upper panel. The two gray bars indicate regional dust storms (Smith, 2004), and the second one that started at $L_s \approx 270^\circ$ in the south polar region may have increased the snow density in the second southern winter (see text).
Fig. 3. Variation of NRMS between the gravimetric and altimetric $\Delta J_3$ by assuming different epochs (in $L_s$) for the maximum and minimum average densities of snow packs. Brighter colors show better fits. Here, the search was performed in 5 degree steps with the model 3 (red curve in Fig. 2). We adopted 60° ($t_{\text{max}}$) and 90° ($t_0$) to draw Fig. 2.