Coupling Characteristics of Multicore Photonic Crystal Fiber-Based $1 \times 4$ Power Splitters

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Abstract—A new design of multicore photonic crystal fibers (PCFs) is proposed and investigated through full-vectorial finite-element method and finite-element beam propagation method. The fiber design comprises four identical cores surrounding a central core. The optical power launched into the central core is equally divided into other neighboring four cores with a 25% of coupling ratio. The coupled-mode analysis is also carried out to understand the supermode patterns and the coupling characteristics. Through numerical simulations, it is demonstrated that the optical power can be divided equally in a 5.8-mm-long multicore PCF. The power coupling characteristics obtained through coupled-mode analysis are in very good agreement with those calculated from beam propagation method solver.

Index Terms—Fiber couplers, finite-element methods (FEMs), microstructured fibers, optical fiber devices, photonic crystal fiber, power splitter.

I. INTRODUCTION

MULTICORE optical fibers play a significant role in dividing/combining the optical power in optical fiber networks. A single power can be divided into several branches and can be routed to different locations for additional purposes. Several approaches have been adapted to process and route the optical power in $N$ branches. Among them, the most common approach is the fusion of several identical or nonidentical optical fibers together by keeping mutual contact in the fusion region. This fusion process results in the tapered region of fused optical materials, where exchange of powers occurs through proximity coupling. It has been observed that splitters using fewer than six surrounding fibers are difficult to make, as one has to use dummy fibers, e.g., it may be difficult to make $1 \times 4$ power splitters using conventional optical fibers. However, $2 \times 2$, $4 \times 4$, and $1 \times 7$ fiber couplers/splitters can be made by fusing conventional optical fibers [1]–[5]. On the other hand, with the advent of novel photonic crystal fibers (PCFs) [6] or so-called holey fibers or microstructured fibers, where the airholes are distributed in silica matrix and run down the entire fiber length, it has become possible to create multicores in a single fiber without fusion process. This makes PCFs a versatile candidate for optical power division or combining. It is rather easy to split the light into surrounding cores in multicore PCFs. Previous studies have used two/three cores PCFs to obtain coupling characteristics [7], [8], wavelength-flattened couplers [9], narrow bandpass filters [10], [11], and multicores in PCFs have been created to achieve frequency combs by four-wave mixing [12], phase-locking [13], and fiber laser array [14].

In this paper, we propose and investigate a new design of a multicore PCF that can divide a single optical power equally into four ports with 6 dB of power in each core. The design consists of four identical cores surrounding an identical central core. Each core is surrounded by small airholes, which are created in order to have a complete power transfer among the cores. To optimize the performance of the proposed device, we use a full-vectorial finite-element method (FEM) [15] and the beam propagation method (BPM) [16]. Through numerical simulations, it has been revealed that the complete power transfer takes place in a 5.8-mm-long multicore PCF having $d/\Lambda = 0.45$ and $d'/\Lambda = 0.23$, where $d$, $d'$, and $\Lambda$ are the hole diameter of the cladding, small airholes, and pitch constant of the PCF, as shown in Fig. 1. A good agreement is observed between the BPM and coupled-mode theory (CMT) results.

The paper is divided into five sections. Section I gives an overview of the previous works. In Section II, fiber design and the methodology to achieve proposed design are described. The
CMT for the proposed multicore PCF is derived in Section III, and analysis has been done on supermodes patterns. Numerical results obtained from FEM, BPM, and CMT are presented in Section IV. Finally, concluding remarks have been made in Section V.

II. FIBER DESIGN

Fig. 1 shows the proposed multicore PCF design. The airholes with diameter $d$ and pitch constant $\Lambda$ build the cladding where single missing airholes from several locations create multicore. In our design, five cores are created as marked by the numbers from 1 to 5. The core 1 is the central core, where a single optical power is launched. Cores 2 and 4 are placed horizontally, while the cores 3 and 5 are aligned vertically. The positions of the cores are selected in such a manner that the symmetry of the structure does not break up. This allows us to simplify the problem, i.e., we can use one quarter section of the proposed multicore PCF structure to evaluate the coupling characteristics. Note that the distance between the cores 1 and 2 and the cores 1 and 3 is different. Therefore, the coupling coefficients for cores 1–2 and cores 1–3 would be different. In order to have an equal transfer of the power in the neighboring cores, the horizontal and vertical coupling coefficients must be equal. This can be achieved if we increase the vertical coupling coefficient. To predict the equal coupling coefficients for horizontal and vertical coupling, we decrease the diameter of airholes (denoted by $d_f$) located just above and below of each cores, as shown by red circles in Fig. 1. The size of small airhole can be obtained and optimized when the horizontal and vertical coupling coefficients become equal. The refractive index of background material silica is 1.45 in our all numerical calculations, and we do not take into account the material dispersion of the silica in our numerical calculations as the proposed fiber splitter has small length (few mm) and focused to operate over C-band. However, for short wavelength operation (e.g., at 800 nm), one should consider the wavelength dependency of the silica.

III. COUPLED-MODE THEORY

We use standard coupled-mode equations [17] for analyzing the proposed multicore PCF power splitter. The mode coupling between the cores can be described by a simple set of equations. We define the coupling coefficients $\kappa_h$ and $\kappa_v$, which represent the coupling between horizontally placed cores (core 1-core 2 or core 1-core 4) and vertically aligned cores (core 1-core 3 or core 1-core 5), where core 1 is the central core. The “cartoon” picture of the coupling between the isolated cores is shown in Fig. 2. Note that the coupling between the adjacent outer cores (core 2 and core 3) is neglected, because the coupling coefficient $\kappa_d$ between the adjacent outer cores is sufficiently small compared with the horizontal and vertical coupling coefficients. The coupling characteristics can be described by the following coupled-mode equations:

$$\frac{da_2}{dz} + j\beta a_2 = -j\alpha_1 \kappa_h$$  \hspace{1cm} (1a)

$$\frac{da_3}{dz} + j\beta a_3 = -j\alpha_1 \kappa_v$$  \hspace{1cm} (1b)

$$\frac{da_4}{dz} + j\beta a_4 = -j\alpha_1 \kappa_h$$  \hspace{1cm} (1c)

$$\frac{da_5}{dz} + j\beta a_5 = -j\alpha_1 \kappa_v$$  \hspace{1cm} (1d)

where $a_k$ ($k = 1, 2, 3, 4, 5$) are the amplitude of the fundamental mode in core-$k$, $\beta$ is the propagation constant of the fundamental mode, $\kappa_h$ and $\kappa_v$ are the coupling coefficients between the horizontally placed cores and the vertically aligned cores, respectively. Since the cores 2 and 4 and cores 3 and 5 are identical, the mode amplitude shall be equal. Therefore, coupled-mode equations can be reduced to (2a)–(2c) as below:

$$\frac{da_1}{dz} + j\beta a_1 = -j\alpha_1 \kappa_h$$  \hspace{1cm} (2a)

$$\frac{da_2}{dz} + j\beta a_2 = -j\alpha_1 \kappa_v$$  \hspace{1cm} (2b)

$$\frac{da_3}{dz} + j\beta a_3 = -j\alpha_1 \kappa_h$$  \hspace{1cm} (2c)

with boundary conditions $a_1(0) = 1, a_2(0) = a_3(0) = 0$.

For simplicity, the coupling coefficients are assumed to be polarization independent. By making the substitution $a_k = A_k \exp(-j(\beta + \sigma)z)$ into (2a)–(2c), the characteristic equation can be written as

$$\begin{vmatrix}
\sigma & -2\kappa_h & -2\kappa_v \\
-\kappa_h & \sigma & 0 \\
-\kappa_v & 0 & \sigma
\end{vmatrix} = 0$$  \hspace{1cm} (3)

where $\sigma$ in (3) is an eigenvalue calculated by solving (3), resulting into three eigenvalues and corresponding three eigenvectors. The eigenvalues are $0$, $\sqrt{2(\kappa_h^2 + \kappa_v^2)}$, and $-\sqrt{2(\kappa_h^2 + \kappa_v^2)}$. The field at a position $z$ can be represented by the linear combination of the eigenvectors. Each eigenvector represents a mode.
of the fiber structure as shown in Fig. 3. The plus sign indicates that the phase of the mode and unit sign is the amplitude, while minus sign corresponds to out of phase. The operation of this power splitter can be understood in terms of the supermodes of the fiber-core PCF coupler. If the individual cores of the coupler are single modes, the coupler structure supports five supermodes; however, as mentioned before that the problem can be reduced in three supermodes because of the core symmetry. Through solution of coupled-mode equations (2a)–(2c), we arrive at three eigenvectors corresponding to supermodes of the structure and shown in Fig. 3. If we represent the corresponding fields by $\phi_1$, $\phi_2$, and $\phi_3$, and assume that initially all the energy is in central core 1, the combination of supermodes can be expressed in a linear combination of $\phi_2$ and $\phi_3$. However, it is difficult to excite the $\phi_2$ when a complete power transfer takes place, i.e., when $\kappa_{th} = \kappa_{ty}$.

**IV. Numerical Results**

To compute the coupling coefficients of the proposed multicore PCF coupler, we use FEM [15] and later the power splitting is verified by the BPM solver [16]. The geometrical parameters of the multicore PCF are $d'/\Lambda = 0.45, d'/\Lambda = 0.23, \Lambda = 2.64 \mu m$, and $\Lambda = 1550$ nm. The BPM results state the normalized power propagation as a function of the propagation distance $z$ are depicted in Fig. 6, where the horizontally polarized fundamental mode is launched into the central core 1 at a coupler input. The solid black curve corresponds to the power variation in the central core 1, solid blue curve corresponds to the coupled power in the horizontal core 2, and the diamond markers stand for the coupled power in the vertical core 3. It can be clearly seen from these numerical results that the powers in the vertical core increase gradually and at a distance of 5.8 mm, there is a complete transfer of the power from the central core to neighboring cores 2, 3, 4, and 5. This verifies the predicted pitch constant and the small-hole-diameter values and also demonstrates numerically the power splitting functionality.

In Fig. 7, we show the modal field distribution of the $x$ component of the electric field ($E_x$) at the initial stage, where the power is launched into the central core and at the final stage, where the complete power is equally divided into the outer cores. Fig. 7(a) exhibits the modal field distribution at 1550 nm wavelength at $z = 0$. After launching the modal field into the central core, the coupling starts and transfer of the power takes place with an equal splitting into neighboring outer cores, as shown in Fig. 7(b). The power propagation in the multicore PCF ($d'/\Lambda = 0.45, d'/\Lambda = 0.23$, and $\Lambda = 2.64 \mu m$) is also obtained by CMT, and the results are demonstrated in Fig. 8. It
can be seen from the results that the power get transferred into surrounding cores at a distance of 5.8 mm, which is in good agreement with the results calculated through BPM. From these results, we can say that the assumption that the cross-coupling coefficient $\kappa_{cd}$ is sufficiently small in comparison to $\kappa_h$, and $\kappa_v$ is valid in the proposed PCF shown in Fig. 1.

Fig. 9 establishes the scaling rules for the proposed multicore PCF structure when the cladding hole diameter $d$ and pitch $\Lambda$ of fiber are varied, keeping fixed cladding hole diameter $d/\Lambda$. The solid curves correspond to $x$-polarization state, whereas the dashed curves correspond to $y$-polarization state. The $d'/\Lambda$ is obtained at every pitch value of the corresponding multicore PCF with fixed $d/\Lambda$ and $\kappa_h = \kappa_v$. For small value of the pitch constant, the $d'/\Lambda$ is large and it decreases as we increase the pitch constant. For large cladding hole diameter, the $d'/\Lambda$ is large, and it reduces as the cladding hole diameter gets smaller. It can be clearly seen from the graph that the proposed multicore PCF structure shows the polarization dependency. For $x$-
and \(y\)-polarization states, the coupling coefficients are different, resulting into different values of small hole diameter.

V. CONCLUDING REMARKS

The power coupling characteristics of a newly designed multicore PCFs were demonstrated numerically by employing accurate FEM and BPM solvers. The coupled-mode equations have also derived to compare the power transfer among the neighboring cores with 25\% of coupling ratio. The proposed multicore PCF can split a single input power into four ports with equal power in each four cores, showing the capability to acts as a 1 × 4 power splitter. Through numerical simulations, it has been revealed that the power can be divided into four cores in a 5.8-mm-long multicore PCF having \(d/\Lambda = 0.45\) and \(d'/\Lambda = 0.23\), and \(\Lambda = 2.64 \mu m\) at 1550 nm wavelength. We also investigate the effect of displacement of surrounding cores on the power splitting characteristics. During the fabrication process, there may be a slight shift in the location of the surrounding cores, and this will result into different coupling coefficient. We vary the coupling coefficient by ±2\% and observe the power splitting characteristics of a 5.8-mm-long multicore PCF. Through numerical simulations, we find that the power splitting ratio decreases to 24.97\% from 25\%. This indicates that even if the cores are slightly displaced, the splitting power ratio does not change so much, and we can divide the incident light into other four cores by an equal amount.

Scaling rules are established for the proposed multicore PCF. It has been found that \(d'/\Lambda\) decreases as the pitch constant increases. For large values of cladding hole diameter, a large \(d'/\Lambda\) is required. However, if we select large-cladding hole diameter, the fiber may no longer be single mode. It has also been shown that the proposed multicore PCF design exhibits strong polarization dependency. This polarization dependency arises mainly by the small hole diameters \(d'\) in the cladding. The proposed multicore PCF power splitter can be fabricated by following the same guidelines for fabricating regular PCF structures. After drawing few meters, the desired length of the proposed PCF power splitter can be cut. Furthermore, we are working on an alternative design of the proposed multi-core PCF power splitter, which can be a polarization independent and easier to fabricate. Such study would be published elsewhere.
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REFERENCES

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