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Tax Rate Harmonization, Renegotiation and Asymmetric Tax Competition for Profits with Repeated Interaction

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October, 2009

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Tax Rate Harmonization, Renegotiation and Asymmetric Tax Competition for Profits with Repeated Interaction

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October 13, 2009

Abstract

This paper analyzes a model of corporate tax competition with repeated interaction and with the strategic use of profit shifting within multinationals. We show that international tax coordination is more likely to prevail if the degree of asymmetry in terms of productivity differences between countries is smaller, or if concealment costs of profit shifting are larger when the tax authorities adopt grim-trigger strategies. Allowing for renegotiation in the tax harmonization process generally requires more patient tax authorities to support tax harmonization as a subgame perfect equilibrium. We find somewhat paradoxical situations where higher costs of profit shifting make international tax arrangements less sustainable under weakly-renegotiation-proof strategies.

Keywords: corporate taxation, tax coordination, multinational firms

JEL classifications: H 25, H 87, F 23

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1 Introduction

The taxation of income of multinational firms and the proper assignment of the “fiscal common” (Konrad, 2008) to individual countries has been a long-standing issue in both policy and academic debates. Concerns about the definition of a “fair” share of the corporate tax base (corporate income or profits) are strong in the European Union and elsewhere; hence design of an internationally appropriate corporate tax system is a major issue in the literature on tax competition.

Often measures of tax harmonization have been advocated to limit strategic choice of taxes by individual countries. Most of the work has characterized decentralized tax setting, measuring the effects of decentralized allocation mechanisms against the benchmark of centralized and exogenously imposed institutions, at least implicitly supposing that the latter may ultimately eliminate strategic behavior of individual countries. The key question for any particular political relevant solution, however, is whether countries will actually agree in treaty negotiations to harmonization measures, the main problem for policy makers being that each country has an incentive to free ride on others signing the tax treaty. This is so because external enforcement is usually too costly or impossible, and thus the lack of external enforcement holds for such an international agreement.

The theory of repeated games has demonstrated that repeated interactions between policy makers which allows for punishment in future periods may lead to the emergence of cooperation even in the absence of a supranational tax authority. Under such an implicit coordination mechanism, participating countries have an incentive to act in accordance with the treaty rules; namely, such a tax treaty is self-enforcing. This paper deals with the scope of long-run cooperation among noncooperative tax authorities to achieve tax harmonization when repetition is allowed but enforcement by third parties does not exist.

Existing corporate tax systems make the fundamental assumption that calculation of profits is conceptually meaningful (e.g., Meade Committee, 1978; Sinn, 1987). Thus they permit the deduction of costs from revenue to

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1Hines and Sørensen (2008) gives excellent account of the recent literature on capital tax competition.

2In politics famous examples for common-pool problems exist as the current efforts to establish international information sharing agreements between tax authorities shows (consider the OECD blacklist as a punishment device). It seems today that an agreement (implicit collusion of tax authorities) is within reach here. The EU Stability and Growth Pact was created to enforce budgetary discipline among EU member states. It is currently under renegotiation. The Kyoto Protocol aims to reduce global greenhouse gas emissions by implementing legally binding agreements.
calculate the tax base. This treatment of costs offers firms a fundamental incentive to overstate costs as a means to reduce taxable rents or profits. In federations the same economic argument creates additional challenges for operational tax design since tax planning strategies of a firm operating in several states (i.e., multinationals) not only make the sum of true economic profits private information, but also the observed profit allocation across jurisdictions is endogenous to the taxes chosen by states (see, e.g., Mintz and Smart, 2004). To limit competition for mobile tax bases and to sustain a desired level of regional cohesion or cooperation between regions, federal states as the United States, Canada and Germany traditionally implicitly distribute the tax base between jurisdictions through (apportionment) formulas, or introduced explicit interregional transfers.

It is important to keep in mind that tax competition is the process of tax policy decisions by which rational governments optimally respond to tax policy measures of foreign governments which affect the economic situation in their constituency. The arrangement we analyze is a two-country world where countries host a multinational firm. The multinational firm makes a factor employment decision and chooses the transfer price for an intermediate good to maximize net of tax profits. Indeed, there is a body of empirical research suggesting that multinationals react to international differences in tax rates to shift profits from high-tax to low-tax countries. There are many explanations for profit shifting that have been proposed in the literature; but among the tax planning strategies profit shifting attracted most attention in recent research, see Devereux and Sørensen (2005).³

We also consider tax authorities in the two countries which choose their tax rates on book profits non-cooperatively over an infinite time horizon. We abstain from symmetry assumptions to get a situation where tax authorities noncooperatively choose different tax rates.⁴ Using asymmetric countries in a multiperiod game between two tax authorities makes the analysis more complicated, but it is an empirical fact that the development of corporate tax systems in the EU and elsewhere during the last two decades exhibits both: some tendency of convergence in tax rates at a lower level and divergent tax rates.

Divergent statutory tax rates cause profit shifting. Against this background, there have in fact been attempts in the past to coordinate corporate

³Multinationals may also lower aggregate tax payments by shifting deduction of interest payments across borders (see Huizinga, Laeven, and Nicodème, 2008).
⁴Related, Wilson (1991) and Bucovetsky (1991) use a framework where capital is the only factor in a linear homogenous production technology and countries differ in endowments of labor. Countries face different incentives and in the presence of cross-ownership of firms (Huizinga and Nielsen, 1997).
income taxation in the EU, but most proposals failed since they did not get unanimous approval by the Council. See Brøchner, Jensen, Svensson and Sørensen (2006). The two most remarkable activities have been the submission of a draft directive proposal on the harmonization of the corporate income tax in 1975 and the nomination of a committee of experts (headed by Onno Ruding), which was asked to analyze the harmonization requirements for European capital taxation in December 1990. The Commission’s 1975 proposal for a harmonized corporate income tax suggested a switch to harmonized company tax bases and capital income tax rates within a rate band (45-55%) for all member countries. Given the high diversity of national corporation tax systems and tax rates in the EC-9 countries at that time, this proposal did not find unanimous consent in the Council and was finally repealed in 1990.\footnote{At the same time the Council passed two directives which had already be submitted in the late 1960s, viz. the Merger Directive and the Parent/Subsidiary Directive. Both of these measures put entrepreneurial transactions between companies residing in different EU countries on an equal footing with analogous transactions between companies residing in one country.} The Ruding committee delivered a comprehensive report in 1992, which identified a number of distortions from the interaction of uncoordinated corporate income tax systems and proposed a common EU corporation tax system as a long-term target.

The reluctance of the Commission to interfere in national company taxation seems to be at least partly influenced by its weak position in direct tax harmonization. The harmonization of capital taxation has to be based on Art. 100, which allows for a mandatory adjustment of national legislation in order to back the functioning of the internal market. The political constraints on Commission initiatives in capital taxation have not been relaxed by the Maastricht Treaty, since the room for possible intervention based on the declaration of free capital mobility as one basic liberty in the European internal market has been constrained by the simultaneous emphasis on the subsidiarity principle. The crucial question for the EU is of course, if capital tax competition and its presumed equilibrium outcome will conflict with the objectives of the EC-Treaty. In this case the EU Commission is obliged to propose appropriate coordination and harmonization measures in line with its constitutional competence.

We are interested in how the presence of profit shifting affects the likelihood of cooperation in two institutional arrangements. Especially, the analysis permits a characterization of the way that equilibria under the grim-trigger and weakly renegotiation-proof strategies of infinitely repeated games. We find that the possibility of renegotiation under the weakly renegotiation-proof strategy generally requests that tax authorities should be more patient
compared to those under grim-trigger strategies; in this sense, harmonization of tax rates is less likely to be implemented as a subgame perfect equilibrium. Therefore, not taking this effect of renegotiation into account not only yields misleading theoretical predictions, but may lead to the realization of highly inefficient outcomes. Interestingly, our comparative static analysis reveals that tax harmonization becomes more likely in economic scenarios where the multinational firm perceives higher costs of cross-border profit-shifting with grim-trigger strategies, whereas an increase in such costs reduces the likelihood to obtain harmonization as an equilibrium outcome when renegotiation is possible.

Finally, the question arises as to more related literature. It is well documented that the literature about capital tax competition characterizes decentralized allocations and contrasts resulting allocation with the efficient outcome, implicit or explicitly calling for implementation of the latter. Much less is known about the self-enforcing mechanisms that sustain tax coordination without outside enforcers such as a supernational tax authority. We are aware of Cardarelli, Taugourdeau and Vidal (2002), Catenaro and Vidal (2006) and Itaya, Okamura and Yamaguchi (2008) who use grim-trigger strategies in the repeated game of property tax competition, where regional government use source-based taxes to finance expenditures.\footnote{Chisik and Davies (2004) focus on symmetric outcomes in an interesting study of a foreign-investment game where foreign investors are taxed, but only after the investment is installed. Infinite repetition of the static three stage game yields the result that cooperation generates an outcome with high tax rates. Cooperation aggravates the time inconsistency problem.} Our focus is on profit-shifting behavior of multinational firms. We are not aware of any work that considers renegotiation, although renegotiation is definitely a key to explain the existing choices in international politics in which tax authorities can communicate and make proposals every period they want. Tax reforms causing fluctuations in both effective and statutory tax rates are well documented (see, e.g., Devereux, Griffith and Klemm, 2002; Auerbach, Devereux and Simpson, 2008), suggesting an explanatory power of attempts to model dynamic political processes.\footnote{In political science and ressource economics there is unanimous agreement that punishment and regret and the caused fluctuations in economic variables are major determinants for successful regimes in international common pool problems (Baland and Platteau 1996; Ostrom, Walker and Gardner 1992; Ostrom, 1999).}

The reminder of the paper is organized as follows. Section 2 and 3 present a simple model of asymmetric tax competition for the profits of a multinational firm and characterizes the cooperative solution as a target tax rate at which governments are coordinated. Grim-trigger strategies in tax policy
are analyzed in Section 4 and the outcome with weakly renegotiation-proof strategies is central in Section 5. Mathematical details are relegated to appendices.

2 The Model

We consider two countries \( i = 1, 2 \) which are inhabited by a large number of investors endowed with \( k \) units of capital. In each period these investors allocate their capital internationally to finance investment of a multinational firm operating in the two countries. The multinational firm maximizes the discounted sum of profits net of the corporation tax through the choice of factor employment and strategic manipulation of declared costs over an infinite number of periods. The tax authority in each country tries to combat profit shifting through cost manipulation, but is restricted to a source tax on book profits.

2.1 Technologies

Production technologies. The multinational firm seeks \( k \) units of per capita capital and an essential service to produce output in each period. For analytical convenience, we treat the size of the essential service, such as labor inputs, fixed at unity (Riedel and Runkel, 2007). The affiliate of the multinational firm in country \( i \) has a technology described by the strictly concave, constant-returns-to-scale production function (Bucovetsky, 1991; Haufler, 1997):

\[
f_i(k_i) := (A_i - k_i) k_i,
\]

where the marginal productivity (of the first unit) of capital, \( A_i \), may differ among (asymmetric) countries. We assume throughout that the marginal productivity of capital is positive, i.e., \( A_i > 2k_i \).

Profit-shifting technology. Self selection of firms into profit-shifting and the externalities caused thereby do play a central role in this analysis. Potential for profit shifting is arising because the multinational firm has better information as to the actual costs than the tax authority. The choice of the declared cost structure between affiliated entities creates possibilities to transfer of profits between taxing jurisdictions. We shall argue that the true costs of the essential service are unity, \( s = 1 \). Thus, a choice of \( s > 1 \) implies overinvoicement and \( s < 1 \) underinvoicement of the service. To limit strategic transfer pricing it seems natural to model the costs of misdeclaration by a convex function. In the analysis of repeated interaction below it will become necessary to compare directly the levels of profits in its cooperative
and non-cooperative phases. To this end we specify that the costs of profit shifting are quadratic in the level of misdeclaration (see, e.g., Haufler and Schjelderup, 2000; Riedel and Runkel, 2007):

$$q(s) = \frac{\beta}{2}(s - 1)^2 \quad \text{with} \quad \beta \geq 0.$$ 

The lower bound $\beta = 0$ corresponds to complete or unhindered profit shifting and thus to complete or perfect spillovers of the activities of the multinational firm on the tax bases of countries.

### 2.2 Institutions

Although the multinational firms are operating over an infinite number of periods, since a supergame (=a repeated game) is defined by infinitely repetition of a one-shot game, the same book-profits of the multinational firm after the corporation tax is repeated in every period, that is:

$$\Pi := \sum_{i=1}^{2} \pi_i - q(s) = \sum_{i=1}^{2} \left\{ \left( 1 - \tau_i \right) \left[ f_i(k_i) - rk_i + (-1)^i(s - 1) \right] \right\} - q(s), \quad (1)$$

where $r$ is the world-market rental rate per unit of capital and $\tau_i$ denotes the tax rate levied on corporate profits in country $i$ (i.e., $\pi_i$).

In the stationary environment of repeated games the multinational firm chooses $k_i$ and $s$ repeatedly to maximize (1) in every period, given the tax rates $\tau_i$, $i = 1, 2$. Choices are characterized by the first-order conditions (recall $A_i > 2k_i$):

$$\frac{\partial f_i(k_i)}{\partial k_i} = A_i - 2k_i = r, \quad i = 1, 2, \quad (2)$$

$$\frac{\partial q(s)}{\partial s} = \beta(s - 1) = \tau_1 - \tau_2. \quad (3)$$

Denote in the following by $\theta := A_1 - A_2$ the difference in productivities between the affiliates of the multinational firm. Let $\theta \geq 0$ in what follows without loss of generality.

We first assume that governments maximize the discounted sum of revenue generated from profit taxation in their own constituency. In some parts

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8 Alternatively, one could assume that the costs of misdeclaration are deductible from the tax base in one of the two countries or at some convex combination (see, e.g., Stöwhase, 2005). Since all these are noteworthy alternatives Nielsen, Raimondos-Møller and Schjelderup (2009) check robustness and show that alternative modeling assumptions have an insignificant effect on outcome.
of the exiting literature this assumption is defended on the ground of political economy arguments as governments want to support the working population against the perceived wage decrease caused by globalization. Others argue that governments place a low weight on consumer or producer rents. In any case, the assumption is commonly applied in the existing literature.\footnote{E.g., Kanbur and Keen (1993), Elitzur and Mintz (1996), Mansori and Weichenrieder (2001) and Stöwhase (2005) among many others.} Tax authorities are maximizing the discounted sum of tax revenues over an infinite number of periods:

\[ V_i = \sum_{t=0}^{\infty} \delta^t R_i, \quad i = 1, 2, \]

where \( \delta \in (0, 1) \) is the common (actual) discount factors possessed by the tax authorities of the respective countries. The one-shot tax revenue in country \( i \) is

\[ R_i (r, \tau_1, \tau_2) := \tau_i \left[ f_i (k_i) - r k_i + (-1)^i (s - 1) \right], \quad i = 1, 2. \quad (4) \]

### 2.3 Capital market equilibrium

A market equilibrium in each period is a world rental rate \( r \) such that in each period equity holders choose their place of investment to maximize income and the capital market clears:

\[ k_1(r) + k_2(r) = 2\bar{k}, \quad (5) \]

where \( k_i(r) \) represents the capital demand function of the multinational’s affiliate located in country \( i \). Substituting out for \( k_i \) in (5) using (2) gives the world market rental rate for capital:

\[ r = \frac{1}{2} (A_1 + A_2 - 4\bar{k}) \quad (6) \]

Substituting (6) back into \( r \) in (2) gives the capital demand functions of the multinational firm:

\[ k_i = \frac{1}{4} \left[ 4\bar{k} - (-1)^i \theta \right], \quad i = 1, 2, \quad (7) \]

where we assume \( \theta \in [0, 4\bar{k}] \) throughout in order to ensure strictly positive investment in both countries. Then use the capital demand functions (7) and the market rental rate (6) in the profit definition (1) to get the one-shot (global) profit of the multinational firm:

\[ \Pi = \frac{(\tau_1 - \tau_2)^2}{2\beta} + \sum_{i=1}^{2} \frac{(1 - \tau_i) (r - A_i)^2}{4}. \quad (8) \]
Substituting out for \( k_i \) using (7), for \( r \) using (6) and for \( s \) using (3) in (4), the one-shot tax revenue function (4) becomes

\[
R_i := \tau_i \pi_i = \tau_i \left[ \frac{(\theta - (-1)^i 4\bar{k})^2}{16} + (-1)^i \frac{\tau_1 - \tau_2}{\beta} \right], \quad i = 1, 2, \quad (9)
\]

where \( \pi_i \) represents the tax base faced by the tax authority of country \( i \).

### 2.4 Non-cooperative phase

We first consider the tax authorities to act independently and non-cooperatively in making their policy decisions over an infinite time horizon. The solution of the stage (or one-shot) game of the repeated game is a Nash equilibrium in setting tax rates:

\[
\tau^N_i := \arg \max_{\tau_i} R_i (\tau_i, \tau^N_j), \quad i = 1, 2, \ i \neq j.
\]

Note also that the infinite repetition of the one-shot Nash equilibrium tax rates, \( \tau^N_i \), conform a subgame perfect equilibrium of the repeated game. Solving the first-order conditions for the tax rate gives

\[
\tau_i(\tau_j) = \frac{1}{32} \left[ \beta (4\bar{k} - (-1)^i \theta)^2 + 16 \tau_j \right], \quad i = 1, 2, \ i \neq j. \quad (10)
\]

Inspection of (10) shows that best responses satisfy \( \partial \tau_i(\tau_j)/\partial \tau_j < 1, \ i, j = 1, 2 \) but \( i \neq j \), implying the existence of a unique Nash equilibrium in its stage game. The solution of (10) is

\[
\tau^N_i = \frac{1}{48} \beta \left[ 48\bar{k}^2 - (-1)^i 8\bar{k} \theta + 3 \theta^2 \right], \quad i = 1, 2. \quad (11)
\]

These tax rates reveal that – given that concealment costs are positive (i.e., \( \beta > 0 \)) – the more productive country (country 1) chooses to levy the tax at a higher rate than the less productive country (country 2). The difference in taxes vanishes in the absence of a difference in productivities (i.e., \( \theta = 0 \)); in other words, the presence of the difference in productivities induces decentralized governments to set different tax rates, which in turn motivates multinational firms to engage in profit shifting. Note also that decreasing the concealment costs (i.e., decreasing \( \beta \)) reduces the difference in taxes. This is consistent with the result of Stöwhase (2005) with heterogeneous population in that profit shifting leads to convergence in tax rates. Substituting (11) into (9) gives the one-shot Nash tax revenue:

\[
R^N_i = \frac{1}{2304} \beta \left[ 48\bar{k}^2 - 8\bar{k} \theta + 3 \theta^2 \right]^2, \quad i = 1, 2. \quad (12)
\]
It is clearly straightforward that $R_1^N - R_2^N = (1/24) \beta \theta \bar{k} \left( \theta^2 + 16\bar{k}^2 \right) > 0$, implying that tax revenues are higher in country 1 where the multinational’s local affiliate has the more advanced technology, whereas the absence of such a difference (i.e., $\theta = 0$) leads to equal tax revenues. We may then characterize the market rental rate and the allocation of capital as $r^N = r$ in (2) and $k_i^N = k_i$, $i = 1, 2$, in (7). The one-shot global profits of the multinational firm are

$$
\Pi^N = \frac{\theta^2}{8} - \frac{\beta \theta^4}{128} - 2\beta \bar{k}^4 + \left( 2 - \frac{13\beta^2}{36} \right) \bar{k}^2.
$$

(13)

That $\partial \Pi^N / \partial \beta < 0$ is intuitive; higher compliance (concealment) costs decrease profits in a noncooperative situation. It is also seen from (13) that profits are increasing in $\theta$ for low values of the compliance cost parameter $\beta$ and inverse $U$ shaped in $\theta$ for higher values of $\beta$. Thus profits are decreasing in $\theta$ in situations with high compliance costs. The economic argument here is that a high marginal compliance cost combined with a difference in productivities among affiliates imply that tax rates differ between countries, giving rise to profit shifting activities at the firm level to lower the tax burden in the high-tax country. High marginal compliance costs reduce the profitability of profit shifting, allowing the high-tax country to cut into the profits of the multinational firm.

3 Cooperative phase

Although an infinite repetition of the Nash tax rates that prevail in the one-shot tax competition game constitutes a subgame perfect equilibrium in the repeated game, there is a possibility that governments may achieve a higher discounted sum of tax revenue by setting corporate tax rates in a cooperative manner. In the present model with a stationary economic environment the problem of finding the maximum of the discounted sum of joint tax revenues over an infinite time horizon simply amounts to infinite repetition of maximizing the one-shot joint tax revenue in all periods. The first-order conditions for maximizing the one-shot joint tax revenue $R_1^C + R_2^C$ are given by

$$
r_i^C = A_i - 2k_i, \quad i = 1, 2.
$$

(14)

$$
0 = \tau_1 - \tau_2.
$$

(15)

Condition (14) is the condition for the optimal use of capital. Condition (15) characterizes the tax rates needed to minimize the costs of profit shifting, i.e., minimum concealment costs. This requires equal tax rates, i.e., $\tau^C := \tau_i$,
\( i = 1, 2 \), though all periods, to eliminate a pure waste of resources associated with compliance costs. So one of the defining characteristics of harmonization measures is that taxes are set uniformly in our model. This is in accordance with much of the literature on fiscal decentralization within a system of asymmetric jurisdictions. Clearly, the sum of tax revenues, \( R^C_1 + R^C_2 \), is maximized at the upper bound of \( \tau^C \). We assume that the upper bound of \( \tau^C \) is not confiscatory (i.e., \( \tau^C < 1 \)) due to the presence of institutional and political constraints such as lobbying by domestic producers. Such an upper bound is widely observed in practice and in many countries imposed by the constitution.

Using (6) and (7) that \( \tau^C = \tau \) in (2) and \( k^C = k_i, \quad i = 1, 2 \), in (7) gives the one-shot profits of the multinational firm:

\[
\Pi^C = (1 - \tau^C) \frac{\theta^2 + 16 \bar{k}^2}{8}. \tag{16}
\]

Profits are independent of concealment costs (i.e., the absence of \( \beta \)) since there does not exist any motive for profit shifting in the absence of tax rate differentials. In contrast, profits are increasing in \( \theta \), at least for non-confiscatory rates of the corporate tax. Making use of the common tax rate \( \tau^C := \tau^C_i, \quad i = 1, 2 \), in (9) yields the one-shot tax revenue under tax harmonization as

\[
R^C_i = \frac{\tau^C}{16} \left[ \theta - (-1)^i 4 \bar{k} \right]^2, \quad i = 1, 2. \tag{17}
\]

It is seen that \( R^C_1 - R^C_2 = \tau^C \theta \bar{k} > 0 \). This implies larger tax revenues in the country where the more productive affiliate is located (i.e., country 1).

4 Subgame perfect equilibrium

Cooperation in tax policy requires that countries levy taxes at equal rates (i.e., tax harmonization) in all periods in order to avoid the resource costs of (wasteful) profit shifting activities. However, signing an agreement (or sustaining implicit collusion) that implements cooperation in tax setting is an economic activity which potentially is costly for an individual country. This means that a prerequisite (a necessary condition) to implement an agreement is the willingness of the national tax authorities to participate in the agreement.

Participation constraints. The participation constraints imply that cooperation should give a higher tax revenue compared to the outcome with non-cooperative behavior for each country in any period of time, i.e., \( R^C_i \geq R^N_i \),
\[ i = 1, 2. \] Using (17) to substitute out for \( R^C_i \) and (12) to substitute out for \( R^N_i \), and solving for the equality gives the lower bounds \( \tau^C_i \) at which a country is indifferent between tax coordination and tax competition as
\[
\tau^C_i = \frac{\beta}{144} \left( 3\theta^2 - (-1)^i 8\theta + 48\theta^2 \right)^2, \quad i = 1, 2. \tag{18}
\]
It is straightforward to check that the lower bounds of both countries are U-shaped in \( \theta \in [0, 4\bar{k}] \), where \( \tau^C_1 = \beta \bar{k}^2 \) at \( \theta = 0 \). It is also easy to confirm that \( \tau^C_1 < \tau^C_2 \) for \( \theta \in (0, 4\bar{k}) \) and \( \tau^C_1 = \tau^C_2 \) at \( \theta = 0 \). Set the coordinated tax rate \( \tau^C \) to be strictly greater than this lower bound associated with each \( \theta \):
\[
\tau^C > \tau^C_i = \max \{ \tau^C_1, \tau^C_2 \}. \tag{19}
\]
As \( \theta \) approaches the upper bound (i.e., \( 4\bar{k} \)), the lower bound \( \tau^C_2 \) goes to plus infinity. This implies that none of countries will participate in the tax harmonization when the affiliates located in different countries are highly asymmetric in technologies.

**Best-deviation tax rate.** The best-deviation tax rate \( \tau^D_i \) maximizes one-period’s tax revenue in country \( i \), given that the other country sets \( \tau^C \). Substituting out \( \tau_j \) for \( \tau^C \) in the best reply function of country \( i, i \neq j \), in (10) gives
\[
\tau^D_i := \frac{1}{32} [16\tau^C + \beta (\theta - (-1)^i 4\bar{k})^2], \quad i = 1, 2. \tag{20}
\]
Then use (20) to substitute out for \( \tau_i \) in the expression for \( R_i \) in (9), given \( \tau_j = \tau^C, i, j = 1, 2 \) but \( i \neq j \), to get the best deviation tax revenue:
\[
R^D_i := \frac{[16\tau^C + \beta (\theta - (-1)^i 4\bar{k})^2]^2}{1024\beta}, \quad i = 1, 2. \tag{21}
\]
It is straightforward to see that \( R^D_1 - R^D_2 > 0 \), so the more productive country (i.e., country 1) gets a higher level of one period’s tax revenue when deviating from cooperation. Moreover, it is easy to show that \( R^D_i > R^C_i \), \( i = 1, 2 \). Yet, the implication is not that countries always have an incentive to deviate from coordination, since deviating countries will potentially be punished by reverting to the Nash equilibrium (i.e., the punishment phase) which is accompanied by further lower tax revenues. Hence, it is important to use a relative measure for profitability of deviation, and we will introduce the minimum discount factor in the following which reflects such a relative measure.

**Minimum discount factors.** Let us assume that the tax authorities in both countries adopt the grim-trigger strategy in setting their tax rates; that
is, country \( i \) sets its capital tax at some predetermined level denoted by \( \tau^C_i \) at the beginning of the game onwards as long as country \( j (j \neq i) \) maintains \( \tau^C \), in the previous period. If the tax authority of some country deviates from \( \tau^C \) in, say, period \( t \), then cooperation collapses and triggers punishment which results in a Nash equilibrium from period \( t + 1 \) to forever thereafter. Accordingly, the conditions to sustain cooperation are given by

\[
\frac{1}{1 - \delta} R^C_i \geq R^D_i + \frac{\delta}{1 - \delta} R^N_i, \quad i = 1, 2, \tag{22}
\]

which are equivalent to

\[
\delta_i \geq \frac{R^D_i - R^C_i}{R^D_i - R^N_i}, \quad i = 1, 2. \tag{23}
\]

The minimum values of both countries’ discount factors which sustain cooperation are obtained as follows. For country 1 we use (17) to substitute out for \( R^C_1 \), (21) to substitute out for \( R^D_1 \) and (12) for \( R^N_1 \) on the right-hand side of (23). For country 2 we use (17) for \( R^C_2 \), (21) for \( R^D_2 \) and (12) for \( R^N_2 \) on the right-hand side of (23) to get

\[
\hat{\delta}_i := \frac{R^D_i - R^C_i}{R^D_i - R^N_i} \tag{24}
\]

\[
= \frac{9[\beta (\theta + (1/4k)^2 - 16\tau^C)]^2}{9[16\tau^C + \beta (\theta + (1/4k)^2)]^2 - 4\beta^2 [3\theta^2 - (1/8k\theta + 48k^2)]^2} \quad i = 1, 2.
\]

The cooperative tax rate \( \tau^C \) is sustainable as a subgame perfect equilibrium of the repeated game only in situations where the actual (common) discount factor of both countries, \( \delta \), is larger than the threshold discount factor defined by \( \delta^* = \delta_2 = \max\{\hat{\delta}_1, \hat{\delta}_2\} \). The minimum discount factor of country 2 to sustain cooperation is always greater than that of country 1, except at \( \theta = 0 \) or \( \beta = 0 \) where \( \hat{\delta}_1 = \hat{\delta}_2 \), which is confirmed by inspection of (24).

**Comparative statics.** We are now ready to analyze the effect of a change in some principle parameters on the likelihood of cooperation to sustain tax harmonization. To see this, we differentiate \( \delta^* \) with respect to \( \theta \) to get (see Appendix A)

\[
\frac{\partial \delta^*}{\partial \theta} = \frac{\partial \hat{\delta}_2}{\partial \theta} > 0. \tag{25}
\]

To understand the economic mechanisms underlying (25), we need to know how an increase in the degree of productivity difference affects the one-shot
tax revenue of the respective countries at all phases of the repeated game. To this end, we first differentiate $R^C_2$ in (17) with respect to $\theta$. This gives\footnote{Note that $\partial R^C_1/\partial \theta = (\tau^C/8)(\theta + 4\bar{k}) > 0$ for the high-productivity country 1.}

$$\frac{\partial R^C_2}{\partial \theta} = \tau^C \frac{\partial \pi^C_2}{\partial \theta} = \frac{\tau^C}{8} (\theta - 4\bar{k}) < 0,$$ \hspace{1cm} (26)

for $\theta \in [0, 4\bar{k})$, where $R^C_2 := \tau^C \pi^C_2$ and $\partial \pi^C_2 / \partial \theta \equiv (\theta - 4\bar{k})/8 < 0$. To explain this result notice that an increase in the difference of productivities has opposite effects on the tax revenues of the two countries given by (17). A higher $\theta$ induces the multinational firm to expand (shrink) its production by making more (less) investments in the high- (low-) productivity country. As a result, the high-productivity country 1 enjoys larger profits, while the low-productivity country 2 does lower profits. There is no incentive for the multinational firm to shift profits across countries due to the common tax rate (i.e., there is no "tax-rate effect"). Hence, an increasing asymmetry in productivities between countries has a negative effect on the tax base (i.e., the negative "tax-base effect") in country 2 in the cooperative phase without any tax-rate effect and this explains the observation that country 2 ends up collecting less tax revenue compared to that of country 1. This means that cooperation becomes less attractive for the low-productivity country 2.

The effect of an increase in $\theta$ on the tax revenue of country 2 in the deviation phase can be decomposed into tax-rate and tax-base effects as follows: \footnote{Note that $\partial R^D_1/\partial \theta = (1/256)[16\tau^C + \beta (\theta + 4\bar{k})^2] (\theta + 4\bar{k}) > 0$ for the high-productivity country 1.}

$$\frac{\partial R^D_2}{\partial \theta} = \pi^D_2 \frac{\partial \tau^D_2}{\partial \theta} + \tau^D_2 \frac{\partial \pi^D_2}{\partial \theta} = \frac{[16\tau^C + \beta (\theta - 4\bar{k})^2]}{256} (\theta - 4\bar{k}) < 0,$$ \hspace{1cm} (27)

where $R^D_2 := \tau^D_2 \pi^D_2$, $\partial \tau^D_2 / \partial \theta = (\beta/16)(\theta - 4\bar{k}) < 0$ and $\partial \pi^D_2 / \partial \theta = (1/16)(\theta - 4\bar{k}) < 0$. An increase in $\theta$ causes both tax-rate (i.e., $\partial \tau^D_2/\partial \theta < 0$) and tax-base effects (i.e., $\partial \pi^D_2/\partial \theta < 0$) in the deviation phase. The tax-rate effect is negative since a higher $\theta$ leads to a decrease in $\tau^D_2$ from (20) (recall $\theta < 4\bar{k}$). The tax-base effect works through two channels; that is, changes in capital demand and the amount of profit shifting. The tax-base effect arising from variations in capital demand is negative since (7) implies that a higher degree of country-specific asymmetries measured by $\theta$ causes a reduction of the tax base in country 2 (i.e., $\pi^D_2$) as a result of the lower capital demand associated with country 2’s lower marginal productivity, whereas the amount of profit shifting to country 2 is increased due to a larger tax differential $\tau^D_2 - \tau^C$. It follows from $\partial \pi^D_2 / \partial \theta < 0$ that the first (negative) tax-base effect dominates...
the second (positive) tax-base effect and so their sum is negative. Taken together, an increase in the asymmetries between countries makes deviation less profitable for the low-productivity country 2, which is consistent with the negative sign of (27).

The effect of an increase in \( \theta \) on country 2’s tax revenue in the Nash equilibrium phase can also be decomposed into tax-rate and tax-base effects:

\[
\frac{\partial R^N_2}{\partial \theta} = \pi^N_2 \frac{\partial \tau^N_2}{\partial \theta} + \tau^N_2 \frac{\partial \pi^N_2}{\partial \theta} = \frac{\beta \left[ 3\theta^2 - 8\bar{k}\theta + 48\bar{k}^2 \right] (3\theta - 4\bar{k})}{576} < 0, \tag{28}
\]

where \( \frac{\partial \tau^N_2}{\partial \theta} = (\beta/24)(3\theta - 4\bar{k}) \gtrless 0 \) and \( \frac{\partial \pi^N_2}{\partial \theta} = (1/24)(3\theta - 4\bar{k}) \gtrless 0 \).

Tax revenue in the Nash equilibrium phase is increasing in \( \theta \) for \( 3\theta > 4\bar{k} \) and decreasing in \( \theta \) for \( 3\theta < 4\bar{k} \) provided \( \beta \neq 0 \). The intuition for the result is as follows. A higher level of \( \theta \) always decreases the capital demand of country 2 (i.e., the negative tax-base effect), whereas the larger tax differential \( \tau^N_1 - \tau^N_2 \) makes the amount of profit shifting to the low-productivity country 2 larger (i.e., the positive tax-rate effect), thus leading to an ambiguous effect on the tax base \( \pi^N_2 \). In addition, it follows from (11) that an increase in \( \theta \) has an ambiguous effect on \( \tau^N_2 \). These results together indicate that the whole effect on the tax revenue in the Nash equilibrium phase depends on the size of \( \theta \).

The above results reveal the change of incentives caused by an increase in asymmetries between countries in different phases of the repeated game. This serves as a basis of the following discussion which attempts to shed more light on the economic mechanisms which establish cooperation as an equilibrium of the repeated game. The problem of the tax authority is to choose whether or not to maintain tax harmonization over an infinite number of periods; it compares the immediate gain from its unilateral deviation with the opportunity cost when reverting to the Nash equilibrium in all the subsequent periods. To simplify the exposition, suppose that the actual discount factor of country 2 happens to coincide with the minimum discount factor \( \delta^* \) in (23). Subtracting \( R^C_2 \) from the resulting equality gives

\[
\frac{\delta^*}{1 - \delta^*} \left( R^C_2 - R^N_2 \right) = R^D_2 - R^C_2. \tag{29}
\]

The left-hand-side of (29) represents the discounted future (opportunity) costs from country 2’s unilateral deviation, while its right-hand side is the immediate gain from deviating. For ease of exposition, we further decompose the discounted future costs on the left-hand-side of (29) into two components: the discount factor component, \( \delta^*/(1 - \delta^*) \), and the opportunity cost incurred

\[12\text{ Note that } \frac{\partial R^N_1}{\partial \theta} = (\beta/576) \left[ 3\theta^2 + 8k\theta + 48\bar{k}^2 \right] (3\theta + 4k) > 0 \text{ for the high-productivity country } 1.\]
by country 2, \( R_C^2 - R_N^2 \). Comparing (26) with (27), and with (28) reveals that the effect on the immediate gain from deviation, \( \partial(R_D^2 - R_C^2)/\partial \theta \), is positive, while that on the future loss, \( \partial(R_C^2 - R_N^2)/\partial \theta \), is negative. Put together, the gain is larger than the loss, so that country 2 has a stronger incentive to deviate. This is because the negative effect of increasing \( \theta \) on \( R_C^2 \) should be much larger than that on \( R_N^2 \) or \( R_D^2 \) in absolute value, which stems from the fact that there is no tax-rate effect on \( \partial R_C^2/\partial \theta \) which may counteract the tax-base effect. Hence, the minimum discount factor \( \delta^* \) should be higher so as to resort to the equality in (29) and exactly this economic intuition is confirmed by (25).

We use the same structure of arguments to discuss the economic consequences of an increase in the marginal compliance costs \( \beta \) on the incentives of tax authorities to implement tax harmonization. By differentiating \( \delta^* \) with respect to \( \beta \) we obtain (see Appendix A)

\[
\frac{\partial \delta^*}{\partial \beta} = \frac{\partial \delta_2}{\partial \beta} < 0. \tag{30}
\]

Similarly, differentiating the corresponding tax revenues with respect to \( \beta \) yields

\[
\frac{\partial R_C^2}{\partial \beta} = 0, \tag{31}
\]

\[
\frac{\partial R_D^2}{\partial \beta} = \frac{\partial r_2^D}{\partial \beta} \pi_2^D + \tau_2^D \frac{\partial \pi_2^D}{\partial \beta} = \tau_2^D \frac{\beta(\theta - 4k)^2 - 16\tau_2^D}{16\beta^2} < 0, \tag{32}
\]

\[
\frac{\partial R_N^2}{\partial \beta} = \pi_2^N \frac{\partial \tau_2^N}{\partial \beta} = \frac{1}{2304} [48k^2 - 8k\theta + 3\theta^2]^2 > 0. \tag{33}
\]

An increase in \( \beta \) has no effect on the productivities in the two countries so that the change has a second-order effect on capital input decisions, i.e., there is no tax-base effect through variations in capital demand. Consequently, there remains the tax-base effect through changes in the amount of profit shifting since the multinational firm only adjusts the level of profit shifting activities in response to the tax differentials.

Interestingly, higher levels of compliance costs reduce both the gains and the losses from deviation. To see this compare (31) with (32) and with (33). This comparison reveals that the immediate gain from deviation is decreasing in \( \beta \) (i.e., \( \partial(R_D^2 - R_C^2)/\partial \beta < 0 \)), so does the future loss, (i.e., \( \partial(R_C^2 - R_N^2)/\partial \beta < 0 \)). Nevertheless, the reduction in \( R_D^2 \) is in absolute value larger than the increase in \( R_N^2 \) (i.e., \( |\partial(R_D^2 - R_C^2)/\partial \beta| > |\partial(R_C^2 - R_N^2)/\partial \beta| \)). This explains the result from (30) in that the incentive for deviation is weakened.
The result is quite intuitive but stands in stark contrast to some results derived in the following. An increase in the barriers for profit shifting means that it becomes relatively unattractive for the multinational firm to shift profits across borders. A lower mobility of profits reduces the potential of tax authorities to cut into the profits created by firms for any given choice of the tax rate in the neighboring country. Stated differently, two asymmetric countries perceive a lower degree of tax base mobility, implying that countries find it easier to cooperate.

**Proposition 1** In a repeated tax competition game with grim-trigger strategies

(i) there is a subgame perfect equilibrium in which the cooperative equal tax rates (i.e., the revenue-maximizing tax harmonization) are sustained if both countries are sufficiently patient;

(ii) the less productive country has a stronger incentive to deviate from the revenue-maximizing tax harmonization;

(iii) an increase in the difference in productivities among countries makes the revenue-maximizing tax harmonization more difficult; and

(iv) an increase in the concealment cost of profit shifting makes the revenue-maximizing tax harmonization easier.

5 **Renegotiation**

The trigger strategy postulates that countries can be deterred from short-run opportunism by threats of continued future retaliation. It seems counter-intuitive that once punishment is activated, it continues forever.

We can briefly describe the concept of weakly renegotiation-proof (WRP in what follows) strategies which support the cooperative tax rate as a subgame perfect equilibrium of the repeated game when renegotiation is allowed as follows. In each period, each country chooses $\tau^C$, provided the other country chooses $\tau^C$ in the previous period. If country $i$ alone deviates from the coordinated tax rate $\tau^C$ by choosing its best-deviation tax rate $\tau^D_i$ in some period, then country $j$ starts to punish $i$ by choosing $\tau^N_j$ in the next period onwards. Defector $i$ has two options to react. She can either accept the punishment and choose $\tau^C$ for a finite sequence of $m$ periods (repentance phase), or not to give in and continue with defection, thus reverting to the Nash equilibrium (retaliation phase). In the first case defector $i$ resumes cooperation after the punishment has been implemented for finite periods, while in the second case the punishment (non-cooperative choice of taxes) is prolonged.
The sequence of the coordinated tax rate $\tau^C$ thus constitutes a WRP strategy in an infinitely repeated game between the two tax authorities if the following four conditions are satisfied (see Farrel and Maskin, 1989, page 335):

\begin{align}
R^D_i + \frac{\delta}{1 - \delta} R^N_i &\leq \frac{1}{1 - \delta} R^C_i, \\
R^D_i + \frac{1 - \delta^m}{1 - \delta} R_i(\tau^C_i, \tau^D_j) &\leq \frac{1 - \delta^{m+1}}{1 - \delta} R^C_i, \\
\frac{\delta^m}{1 - \delta} R^C_i + \frac{1 - \delta^m}{1 - \delta} R_i(\tau^C_j, \tau^D_j) &\geq \frac{1}{1 - \delta} R^N_i, \\
\frac{1 - \delta^m}{1 - \delta} R^D_j + \frac{\delta^m}{1 - \delta} R^C_j &\geq \frac{1}{1 - \delta} R^N_j, \quad i = 1, 2, i \neq j.
\end{align}

Condition (34) means that the payoff from deviation, $R^D_i$, and anticipated realization of Nash revenues, $R^N_i$, in the retaliation phase must not exceed payoff under cooperation. Condition (35) requires that defection in one period and cooperation resumed over $m$ punishment (repentance) periods gives a lower payoff compared to cooperation over $m + 1$ periods. Condition (36) requires that tax revenues from being punished over $m$ periods and cooperation resumed after $m$ punishment (repentance) periods must not fall short of that from playing non-cooperatively. Condition (37) ensures that punishment is credible in the sense that the punisher has no incentive to renegotiate on the preassigned punishment. For this the punisher must benefit from implementing the preassigned punishment scenario compared to returning to cooperation without punishment. Since $R^D_j \geq R^C_j$ implies (37), this condition is trivially satisfied and thus imposes no additional restriction. Furthermore, since it can easily be verified that conditions (36) and (37) together imply (34), we can drop (34) also.

**Minimum discount factors under WRP strategies.** For analytical convenience, assume that the punishment length is restricted to one period (i.e. $m = 1$) so that conditions (35) and (36) further simplify to

\begin{align}
(1 + \delta) R^C_i &\geq R^D_i + \delta R_i(\tau^C_i, \tau^D_j), \\
(1 - \delta) R_i(\tau^C_i, \tau^D_j) + \delta R^C_i &\geq R^N_i, \quad i = 1, 2, i \neq j.
\end{align}

To get the *minimum* values of the discount factors of both countries in the retaliation phase and repentance phases we rewrite (38) and (39), respectively,
$$\delta \geq \delta_{NR}^i := \frac{R_i^D - R_i^C}{R_i^C - R_i(\tau^C, \tau^D)}; \quad i = 1, 2,
$$

$$\delta \geq \delta_{PN}^i := \frac{R_i^N - R_i(\tau^C, \tau^D)}{R_i^C - R_i(\tau^C, \tau^D)}, \quad i = 1, 2, i \neq j; \quad (41)$$

where $R_i^D$ is given by (21) and $R_i^C$ is given by (17).

The tax revenue when being punished $R_i(\tau^C, \tau^D)$ in the repentance phase is obtained as follows; setting $\tau_i$ equal to $\tau^C$ and using (20) to substitute out $\tau_j$ for $\tau^D$ in the expression for $R_i(\tau^C, \tau^D)$ in (9) gives

$$R_i(\tau^C, \tau^D) = \frac{\tau^C [\beta (3\theta^2 - (-1)^i 8\bar{k}\theta + 48\bar{k}^2) - 16\tau^C]}{32\beta}. \quad (42)$$

It is straightforward that $R_i^N - R_i(\tau^C, \tau^D) \geq 0$, since $R_i^N < R_i(\tau^C, \tau^D)$ would economically mean that repentance becomes meaningless. Solving for the equality of $R_i^N - R_i(\tau^C, \tau^D) \geq 0$ gives the lower bound of the harmonized tax rate, $\tau^C$, under WRP strategies:

$$\tau^C = \frac{1}{24} [3\theta^2 - (-1)^i 8\bar{k}\theta + 48\bar{k}^2] \geq 0, \quad i = 1, 2, \quad (43)$$

implying, from (18), that $\tau^C > \tau^C$. Thus, the lower bound of the cooperative tax rate under WRP strategies is always higher than that under grim-trigger strategies. Clearly, this indicates that renegotiation weakens the potential to achieve tax harmonization since it takes the edge off the thread.

To yield a characterization of the minimum discount factors in the retaliation and repentance phases for the two countries, rewrite (40) and (41) using $R_i^D$ from (21), $R_i^C$ from (17) and $R_i^P$ from (42):

$$\delta_{NR}^i = \frac{[\beta (\theta - (-1)^i 4\bar{k})^2] - 16\tau^C]^2}{32\tau^C[16\tau^C - \beta (\theta + (-1)^i 4\bar{k})^2]^2}, \quad i = 1, 2, \quad (44)$$

$$\delta_{PN}^i = \frac{[24\tau^C - \beta (3\theta^2 - 8(-1)^i 4\bar{k}\theta + 48\bar{k}^2)] [48\tau^C - \beta (3\theta^2 - 8(-1)^i 4\bar{k}\theta + 48\bar{k}^2)]}{72\tau^C [16\tau^C - \beta (\theta + 4(-1)^i 4\bar{k})^2]^2}. \quad (45)$$

**Minimum discount factor.** If the actual (common) discount factor for both countries exceed the minimum values in the respective phases, the two countries find it in their interests to cooperate. Let $\delta^{**} := \max\{\delta_{NR}^1, \delta_{NR}^2, \delta_{PN}^1, \delta_{PN}^2\}$. 

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Inspection of (45) shows that \( \hat{\delta}_2^{PN} > \delta_1^{PN} \), and (44) implies that \( \hat{\delta}_2^{NR} > \delta_1^{NR} \), except at \( \beta = 0 \) or \( \theta = 0 \) where \( \hat{\delta}_2^{PN} = \delta_1^{PN} \) and \( \hat{\delta}_2^{NR} = \delta_1^{NR} \).

Furthermore, it is seen from (44) and (45) that \( \lim_{\tau_C \to -\infty} \hat{\delta}_2^{PN} = 1/2 \) and \( \lim_{\tau_C \to -\infty} \hat{\delta}_2^{PN} = 1 \); clearly also \( \tau_C > 1/16\beta (4k + \theta)^2 \) for all \( \beta > 0 \). This means that the discontinuity in (44) and (45) at \( \tau_C = 1/16\beta (4k + \theta)^2 \) does not occur in the relevant domain of \( \tau_C \).\(^\text{13}\) At \( \tau_C = \overline{\tau}_C \) we further find

\[
\left. \frac{\hat{\delta}_2^{NR}}{(\tau_C - \overline{\tau}_C)} \right|_{\tau_C = \overline{\tau}_C} = \frac{(3\theta + 4k)^2 (\theta + 12\overline{k})^2}{36\theta^4 + 896\overline{k}^2\theta^2 + 9216\overline{k}^4},
\]

\[
\left. \frac{\hat{\delta}_2^{PN}}{(\tau_C - \overline{\tau}_C)} \right|_{\tau_C = \overline{\tau}_C} = \frac{48\overline{k} (\theta + 4k)^2}{9\theta^4 + 224\overline{k}^2\theta^2 + 2304\overline{k}^4},
\]

so that \( \hat{\delta}_2^{NR} \left|_{\tau_C = \overline{\tau}_C} > \hat{\delta}_2^{PN} \right|_{\tau_C = \overline{\tau}_C} \). To complete the identification of \( \delta^{**} \), we solve \( \hat{\delta}_2^{NR} = \hat{\delta}_2^{PN} \) to get \(^\text{14}\)

\[
\tau_C = \frac{1}{48} (6\beta \theta^2 + 96\beta \overline{k}^2) + \frac{1}{48} \sqrt{\beta^2 \left[ 9\theta^4 + 16\overline{k} (3\theta^3 + 2\overline{k} (19\theta^2 + 24 \overline{k} (\theta + 3\overline{k})) \right]} > 0. \quad (46)
\]

Then

\[
\delta^{**} (\tau_C) \begin{cases} 
\hat{\delta}_2^{NR} & \text{if } \tau_C \in (\overline{\tau}_C, \tau_C), \\
\hat{\delta}_2^{PN} & \text{if } \tau_C \in (\tau_C, 1],
\end{cases}
\]

whose graphs are depicted in Figure 1.

As before, we investigate the effects of a change in the principle model parameters \( \theta \) and \( \beta \) on the likelihood to implement tax coordination. To interpret results in a way outlined in the previous section, we rewrite conditions (40) and (41), respectively, as

\[
R_2^D - R_2^C = \delta^{**} \left[ R_2^C - R_2(\tau_C, \tau_1^D) \right], \quad (47)
\]

\[
R_2^N - R_2(\tau_C, \tau_1^D) = \delta^{**} \left[ R_2^C - R_2(\tau_C, \tau_1^D) \right], \quad (48)
\]

\(^\text{13}\)Note that \( \lim_{\tau_C \to -\beta/16(4\overline{k} + \theta)^2} \hat{\delta}_2^{NR} = \infty \) and \( \lim_{\tau_C \to -\beta/16(4\overline{k} + \theta)^2} \hat{\delta}_2^{PN} = \infty \) for \( 3\theta - 4\overline{k} > 0 \) and \( = -\infty \) for \( 3\theta - 4\overline{k} < 0 \). Hence, depending on the value of \( \theta \), the minimum discount factors (44) and (45) may have a local minimum for values of \( \tau_C \) in the interval \( (\beta/16) (4\overline{k} + \theta)^2, \overline{\tau}_C \) which, in any case, lies outside the domain \( (\overline{\tau}_C, 1] \).

\(^\text{14}\)The second solution (not reported here for brevity) violates the participation constraint as it has a value smaller than \( (\tau_C, 1] \).
Figure 1: An increase in $\theta$ shifts the $\delta^N_{2}$ and the $\delta^P_{2}$-curve up; note that $\partial \tau^C / \partial \theta > 0$ from (46).

whose left-hand sides represent the immediate gains to deviate from the preassigned scenarios, and whose right-hand sides represent the discounted (one-period) opportunity costs from deviation.

Let us first discuss the economic consequences of an increase in the asymmetry measured by $\theta$. As in the previous section, $\partial (R^D_2 - R^C_2) / \partial \theta > 0$, while it turns out from (28) and inspection of

$$
\frac{\partial R_2(\tau^C, \tau^D_1)}{\partial \theta} = \frac{\tau^C (6\theta - 8\bar{k})}{16} > 0,
$$

that $\partial (R^N_2 - R_2(\tau^C, \tau^D_1)) / \partial \theta \geq 0$. It is also seen that it becomes less attractive for the low-productivity country 2 to trigger cooperation through repentance since $\partial (R^N_2 - R_2(\tau^C, \tau^D_1)) / \partial \theta < 0$, whose sign follows from (26) and (49). This discussion reveals that an increase in asymmetry of productivities between countries reduces the likelihood to obtain cooperation as an equilibrium outcome. This intuition is confirmed by differentiating (44) and (45) with respect to $\theta$. This gives (see Appendix B)

$$
\frac{\partial \delta^N_{2}}{\partial \theta} > 0 \text{ and } \frac{\partial \delta^P_{2}}{\partial \theta} > 0.
$$

Figure 1 illustrates that the range of the coordinated tax rate $\tau^C$, which is less than one and consistent with (43), becomes more narrow in response to the increase in $\theta$, implying that it is harder to sustain tax coordination. This result is the same as that under grim-trigger strategies.

In the previous section we obtained the result that an increase in the concealment costs parameter $\beta$ unambiguously increases the likelihood to
Figure 2: An increase in $\beta$ shifts the $\hat{\delta}_2^{NR}$-curve up and the $\hat{\delta}_2^{PN}$-curve down; note that $\partial \tau_C / \partial \beta > 0$ from (46).

sustain cooperation as an equilibrium outcome. This is not the case here. Straightforward differentiation yields $\partial (R_2^D - R_2^C) / \partial \beta < 0$, which confirms the intuition that higher concealment costs reduce the profitability of deviating behavior as before. It is also intuitive that a higher $\beta$ reduces the costs of being punished in the repentance phase; that is, $\partial (R_2^N - R_2(\tau_C, \tau_D^P)) / \partial \beta < 0$.

Interestingly, however, the increase in $\beta$ not only reduces the costs of non-cooperation but also the benefit from cooperation as $\partial (R_2^C - R_2(\tau_C, \tau_D^P)) / \partial \beta < 0$. To evaluate the overall effect thus requires to quantitatively weigh counteracting effects in (44) and (45). It follows that (see Appendix C)

$$\frac{\partial \hat{\delta}_2^{NR}}{\partial \beta} > 0 \text{ and } \frac{\partial \hat{\delta}_2^{PN}}{\partial \beta} < 0.$$  

(51)

Figure 2 illustrates that the relevant range of the coordinated tax rate $\tau_C$ becomes more narrow in response to higher $\beta$ for lower values of $\tau_C$, whereas it becomes wider for higher values of $\tau_C$.

**Proposition 2** In a repeated tax competition game with weakly renegotiation-proof strategies

(i) there is a subgame perfect equilibrium in which the cooperative equal tax rates (i.e., the revenue-maximizing tax harmonization) are sustained if both countries have sufficiently high discount factors;

(ii) an increase in the difference of productivities among countries makes the revenue-maximizing tax harmonization more difficult; and

(iii) an increase in the concealment cost of profit shifting makes the revenue-maximizing tax harmonization easier if the coordinated tax rate is higher than $\bar{\tau}_C$, vice versa if the coordinated tax rate is lower than $\bar{\tau}_C$. 

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It might be surprising at first sight that an increase in concealment costs does not unambiguously increase the chances for implementing tax harmonization, the intuition being here that establishing the cooperative outcome through renegotiation requires a transfer of resources in the repentance phase. Such a transfer would be difficult at low tax rates, reducing the possibility for implicit collusion.

6 Conclusion

This analysis has shown not only that countries can cooperate with repeated interaction to achieve a revenue-maximizing tax harmonization; and identified economic environments where countries are willing to cooperate in the presence of profit shifting within multinationals. When tax authorities interact only once, they often have an incentive to deviate from cooperation, since a cooperation outcome is not a Nash equilibrium because of the lack of punishment in future periods. The crucial condition is a high discount factor (i.e., sufficiently close to one). Although the discount factor may not be directly observable, it should be larger when the frequency of interaction is high (i.e., the period between interaction is short). An empirically testable implication of our analysis thus is that measures of tax harmonization are more likely to happen between economically well integrated countries. However, the possibility to establish tax harmonization as a subgame perfect equilibrium will be reduced either if countries become asymmetric or if the concealment cost of multinationals for profit shifting is lower.

Another important insight of the paper is that the political will to implement tax harmonization depends on the precise institutional setup of the international negotiations. In practice, what constitutes cooperation in tax setting is often fuzzy or complicated. This constitutes an argument to rely on temporary punishment, i.e., to accept renegotiation. After all, tax authorities may well get together after a deviation and agree to ‘let bygones be bygones’. This makes tax harmonization self-defeating. Hence, renegotiation surely reduces the possibility to establish measures of international tax harmonization.

Renegotiation also causes a remarkable difference in the effects of a variation in the costs of profit shifting, e.g., though an international standardization of rules for arms length pricing on the firm level. An increase in such costs might reduce profit-shifting and, at the same time, distorts the incentives of tax authorities to establish tax harmonization. This result sheds new light on the fact that political restrictions are crucial determinants for successful cooperation (e.g., whether renegotiation is allowed). We have demon-
strated that increasing firms costs (through the introduction of standardization) may reduce profit-shifting activities and, at the same time, reduce the chances to achieve cooperation in tax policy; the present analysis suggests that the outcome depends on the precise level of the envisaged harmonized tax rate. This insight may explain the historical failure of proposals to introduce measures of tax harmonization in the European Union and elsewhere. Nevertheless, much more econometric and theoretical work are needed to gain more knowledge about the dynamics in international treaty (re)negotiations.

The model used in this paper certainly is highly stylized and rests on some strongly simplifying assumptions. A central assumption is that the fiscal authorities are limited in their capacity to calculate the profits of the multinational. The model has ignored any possibility that governments share information about tax bases across borders. Moreover, countries not only compete in tax rates, but also in other parameters of the tax system (e.g., depreciation allowances). Another rather obvious model extension would be to introduce welfare maximizing governments although tractability then would certainly dictate the use of numerical methods to solve for outcomes with asymmetric countries; and one could relax the demand for strict tax rate equalization by considering that tax treaties often set a floor (bandwidth) of tax rates (i.e., a common minimum tax rate). The principle effects we have characterized will, however, remain robust determinants of outcomes in all these model extensions.

Finally, we note that many observers in practice may dislike our attempt to stabilize tax rate harmonization through implicit collusion or self-enforcing agreement on the ground of political economy arguments. This potential opposition would not be justified. Depending on the precise formulation of the political economy model, tax rate harmonization would potentially not be welcomed from the perspective of the average voter exactly because it eliminates competition. Taking this position, we have learned from the present paper that renegotiation makes competition more likely an equilibrium outcome. We would claim then that harmonization on a high tax rate is preferable because this increases the potential to achieve implicit collusion or self-enforcing agreement between tax authorities. To embed political economy arguments into the present model is without doubt a fruitful task and such an analysis is on our agenda for future research. Nevertheless, the principle mechanisms we characterized in this paper are certainly robust, although implicit collusion through tax harmonization might no longer be a preferred outcome.


A Appendix

Differentiating (24) with respect to \( \theta \) yields

\[
\frac{\partial s^*}{\partial \theta} = -9 \left[ -16\tau C + \beta (-4\bar{k} + \theta)^2 \right] \left[ -8\beta^2 (-8\bar{k} + 6\theta) (48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2)^2 \right] \\
\left[ -4\beta^2 (48\bar{k}^2 - 8\theta + 3\theta^2)^2 + 9 \left( 16\tau C + \beta (-4\bar{k} + \theta)^2 \right)^2 \right]^2 \\
+ \frac{36\beta (-4\bar{k} + \theta) \left[ -16\tau C + \beta (-4\bar{k} + \theta)^2 \right]}{-4\beta^2 (48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2)^2 + 9 \left( 16\tau C + \beta (-4\bar{k} + \theta)^2 \right)^2}.
\]  
(A.1)

Taking a common denominator and then ignoring the denominator yields

\[
-9 \left[ -16\tau C + \beta (-4\bar{k} + \theta)^2 \right] \left[ -8\beta^2 (-8\bar{k} + 6\theta) (48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2)^2 \right] \\
+ 36\beta (-4K\bar{k} + \theta) \left( 16\tau C + \beta (-4\bar{k} + \theta)^2 \right) + 36\beta (-4\bar{k} + \theta) \left[ -16\tau C + \beta (-4K + \theta)^2 \right].
\]  
(A.2)

Ignoring the common factor \( 36 \left[ -16\tau C + \beta (-4\bar{k} + \theta)^2 \right] \) in (A.2) yields

\[
- \left[ -16\tau C + \beta (-4\bar{k} + \theta)^2 \right] \left[ -4\beta^2 (-4\bar{k} + 3\theta) (48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2)^2 \right] \\
+ 9\beta (-4\bar{k} + \theta) \left( 16\tau C + \beta (-4\bar{k} + \theta)^2 \right) + \beta (-4\bar{k} + \theta) S,
\]

where \( S := -4\beta^2 (48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2)^2 + 9 \left[ 16\tau C + \beta (-4\bar{k} + \theta)^2 \right]^2 \). Rearranging the above expression gives:

\[
\frac{16\tau C - \beta (-4\bar{k} + \theta)^2}{-4\bar{k} + \theta} \left[ -4\beta^2 (16\bar{k}^2 - 16\theta\bar{k} + 3\theta^2) (48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2)^2 \right] + 9 \left( 16\tau C + \beta (-4\bar{k} + \theta)^2 \right) \left( 16\tau C + \beta (-4\bar{k} + \theta)^2 \right) - 144\tau C \left( 16\tau C + \beta (-4\bar{k} + \theta)^2 \right)
\]

\[
+ \beta (-4\bar{k} + \theta) S.
\]

Collecting terms, we have

\[
\frac{16\tau C}{-4\bar{k} + \theta} S - \left[ 16\tau C - \beta (-4\bar{k} + \theta)^2 \right] \left[ 32\beta^2 \bar{k} (48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2) \right] \\
- \frac{144\tau C}{-4\bar{k} + \theta} \left[ 16\tau C - \beta (-4\bar{k} + \theta)^2 \right] \left[ 16\tau C + \beta (-4\bar{k} + \theta)^2 \right].
\]
Finally, we have

$$
= \left[ S - 9 \left( 16\tau^C - \beta \left( -4\bar{k} + \theta \right)^2 \right) \left( 16\tau^C + \beta \left( -4\bar{k} + \theta \right)^2 \right) \right] \frac{16\tau^C}{-4\bar{k} + \theta} \\
- \left[ 16\tau^C - \beta \left( -4\bar{k} + \theta \right)^2 \right] \frac{16\tau^C}{-4\bar{k} + \theta}.
$$

(A.3)

By definition \( \hat{\delta} < 1 \) in (24), implying that the first component of the first term in (A.3) is negative, and, moreover, its second term of (A.3) is clearly negative. Taking into account \(-16\tau^C + \beta \left( -4\bar{k} + \theta \right)^2 < 0\), we have (25).

On the other hand, differentiating (24) with respect to \( \theta \) yields

$$
\frac{\partial \delta^s}{\partial \beta} = - \frac{9 \left[ -16\tau^C + \beta \left( -4\bar{k} + \theta \right)^2 \right]^2 \left[ -8\beta \left( 48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2 \right)^2 \right]}{-4\beta^2 \left( 48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2 \right)^2 + 9 \left( 16\tau^C + \beta \left( -4\bar{k} + \theta \right)^2 \right)^2} + \frac{18 \left( -4\bar{k} + \theta \right)^2 \left[ -16\tau^C + \beta \left( -4\bar{k} + \theta \right)^2 \right]}{-4\beta^2 \left( 48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2 \right)^2 + 9 \left( 16\tau^C + \beta \left( -4\bar{k} + \theta \right)^2 \right)^2}.
$$

Taking a common denominator and then ignoring the denominator yields

\[-9 \left[ -16\tau^C + \beta \left( -4\bar{k} + \theta \right)^2 \right]^2 \times
\left[ -8\beta \left( 48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2 \right)^2 + 18 \left( -4\bar{k} + \theta \right)^2 \left( 16\tau^C + \beta \left( -4\bar{k} + \theta \right)^2 \right) \right]
\]

\[+ 18 \left( -4\bar{k} + \theta \right)^2 \left[ -16\tau^C + \beta \left( -4\bar{k} + \theta \right)^2 \right] S.
\]

Ignoring the common factor \( 18 \left[ -16\tau^C + \beta \left( -4\bar{k} + \theta \right)^2 \right] \) and rearranging yields

\[= \left[ 16\tau^C - \beta \left( -4\bar{k} + \theta \right)^2 \right] \frac{1}{\beta} \left[ -4\beta^2 \left( 48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2 \right)^2 + 9 \left( 16\tau^C + \beta \left( -4\bar{k} + \theta \right)^2 \right) \right] \times \left( 16\tau^C + \beta \left( -4\bar{k} + \theta \right)^2 \right) - 144\tau^C \left( 16\tau^C + \beta \left( -4\bar{k} + \theta \right)^2 \right) + \left( -4\bar{k} + \theta \right)^2 S.
\]

Combining terms yields

\[= \frac{1}{\beta} \left[ 16\tau^C - \beta \left( -4\bar{k} + \theta \right)^2 \right] S
\]

\[- \frac{144\tau^C}{\beta} \left[ 16\tau^C - \beta \left( -4\bar{k} + \theta \right)^2 \right] \left[ 16\tau^C + \beta \left( -4\bar{k} + \theta \right)^2 \right] .
\]
Ignoring the common factor $16\tau C/\beta$ and further rearranging leads to

$$\begin{align*}
&\left[ -4\beta^2 (48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2)^2 + 9 \left( 16\tau C + \beta (-4\bar{k} + \theta)^2 \right)^2 \right] \\
&- 9 \left[ 16\tau C - \beta (-4\bar{k} + \theta)^2 \right] \left[ 16\tau C + \beta (-4\bar{k} + \theta)^2 \right] \\
&\geq \left[ -4\beta^2 (48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2)^2 + 9 \left( 16\tau C + \beta (-4\bar{k} + \theta)^2 \right)^2 \right] \\
&- 9 \left[ 16\tau C + \beta (-4\bar{k} + \theta)^2 \right]^2 > 0.
\end{align*}$$

The last inequality follows from the fact that $1 > \delta$ in (24), which proves (30).

**B Appendix**

The effect of a change in $\theta$ in the NR phase of a weakly renegotiating-proof equilibrium, we differentiate (44) with respect to $\theta$ to get

$$\frac{\partial \hat{\delta}^{NR}}{\partial \theta} = \frac{\beta (4\bar{k} + \theta) \left[ -16\tau C + \beta (-4\bar{k} + \theta)^2 \right]^2}{16\tau C \left[ 16\tau C - \beta (4k + \theta)^2 \right]^2} + \frac{\beta (-4\bar{k} + \theta) \left[ -16\tau C + \beta (-4\bar{k} + \theta)^2 \right]}{8\tau C \left[ 16\tau C - \beta (4k + \theta)^2 \right]} > 0,$$

whose positive sign immediately follows from $-4\bar{k} + \theta < 0$ and the participation constrain $\tau^C > \tau > 1/16\beta (4\bar{k} + \theta)^2$.

In order to identify the effect of a change in $\theta$ in the PN phase, we differentiate (45) with respect to $\theta$ to get

$$\frac{\partial \hat{\delta}^{PN}}{\partial \theta} = -\frac{\beta (8\bar{k} - 6\theta) \left[ 24\tau C + \beta (-48\bar{k}^2 + 8\bar{k}\theta - 3\theta^2) \right]}{72\tau C \left[ -16\tau C + \beta (4k + \theta)^2 \right]} - \frac{\beta (8\bar{k} - 6\theta) \left[ 48\tau C + \beta (-48\bar{k}^2 + 8\bar{k}\theta - 3\theta^2) \right]}{72\tau C \left[ -16\tau C + \beta (4k + \theta)^2 \right]}$$

$$+ \frac{2\beta (4\bar{k} + \theta) \left[ 24\tau C + \beta (-48\bar{k}^2 + 8\bar{k}\theta - 3\theta^2) \right] \left[ 48\tau C + \beta (-48\bar{k}^2 + 8\bar{k}\theta - 3\theta^2) \right]}{72\tau C \left[ -16\tau C + \beta (4k + \theta)^2 \right]^2}.$$  

(B.1)

Ignoring the denominator of (B.1) and then collecting the first and second
terms yields
\[
\beta (8\bar{k} - 6\theta) \left[ 16\tau^C - \beta \left( 4\bar{k} + \theta \right)^2 \right] \frac{72\tau^C + 2\beta (-48\bar{k}^2 + 8\bar{k}\theta - 3\theta^2)}{72\tau^C + 2\beta (-48\bar{k}^2 + 8\bar{k}\theta - 3\theta^2)} \\
+ 2\beta (4\bar{k} + \theta) \left[ 24\tau^C + \beta (-48\bar{k}^2 + 8K\bar{k}\theta - 3\theta^2) \right] \frac{[48\tau^C + \beta (-48\bar{k}^2 + 8\bar{k}\theta - 3\theta^2)]}{[48\tau^C + \beta (-48\bar{k}^2 + 8\bar{k}\theta - 3\theta^2)]}. 
\]

\begin{align*}
((B.2))
\end{align*}

Dividing the above expression by \( 2\beta \) and then dividing the first term into two terms:
\[
= -4\theta \left[ 16\tau^C - \beta \left( 4\bar{k} + \theta \right)^2 \right] \frac{36\tau^C - \beta (48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2)}{36\tau^C - \beta (48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2)} \\
+ (8\bar{k} - \theta) \left[ 16\tau^C - \beta \left( 16\bar{k}^2 - 8\bar{k}\theta + \theta^2 \right) \right] \frac{[36\tau^C - \beta (48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2)]}{[36\tau^C - \beta (48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2)]} \\
+ (4\bar{k} + \theta) \left[ 24\tau^C - \beta (48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2) \right] \frac{[48\tau^C - \beta (48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2)]}{[48\tau^C - \beta (48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2)]}. 
\]

\begin{align*}
(B.2)
\end{align*}

Since \( 48\tau^C - \beta (48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2) < 0 \), which implies that \( 36\tau^C - \beta (48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2) < 0 \) and \( 24\tau^C - \beta (48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2) < 0 \). Note further that \( 16\tau^C - \beta \left( 4\bar{k} + \theta \right)^2 > 0 \) from the definition of \( \delta^2_{PN} \) in (45). Taken together, the first, second and third terms in (B.2) are all positive, so does (B.2), which proves (51).

\section*{C Appendix}

In order to get the effect of a change in \( \beta \) on the minimum discount factor \( \hat{\delta}^2_{NR} \) in the NR phase, we differentiate (44) with respect to \( \beta \) to get
\[
\frac{\partial \hat{\delta}^2_{NR}}{\partial \beta} = \frac{(4\bar{k} + \theta)^2 \left[ -16\tau^C + \beta (-4\bar{k} + \theta)^2 \right]^2}{32\tau^C \left[ 16\tau^C - \beta \left( 4\bar{k} + \theta \right)^2 \right]^2} + \frac{\beta (-4\bar{k} + \theta)^2 \left[ -16\tau^C + \beta (-4\bar{k} + \theta)^2 \right]}{16\tau^C \left[ 16\tau^C - \beta \left( 4\bar{k} + \theta \right)^2 \right]} > 0,
\]
whose positive sign immediately follows from \( -4\bar{k} + \theta < 0 \) and the participation constrain \( \tau^C > \tau^C > 1/16\beta (4\bar{k} + \theta)^2 \).

In order to get the effect of a change in the PN phase we differentiate
(45) with respect to $\beta$ to get
\[
\frac{\partial \hat{\delta}^{P_N}}{\partial \beta} = - \frac{(-48\bar{k}^2 + 8\bar{k}\theta - 3\theta^2) [24\tau^C + \beta (-48\bar{k}^2 + 8\bar{k}\theta - 3\theta^2)]}{72\tau^C \left[-16\tau^C + \beta (4\bar{k} + \theta)^2\right]} - \frac{(-48\bar{k}^2 + 8\bar{k}\theta - 3\theta^2) [48\tau^C + \beta (-48\bar{k}^2 + 8\bar{k}\theta - 3\theta^2)]}{72\tau^C \left[-16\tau^C + \beta (4\bar{k} + \theta)^2\right]} + \frac{(4\bar{k} + \theta)^2 [24\tau^C + \beta (-48\bar{k}^2 + 8\bar{k}\theta - 3\theta^2)] [48\tau^C + \beta (-48\bar{k}^2 + 8\bar{k}\theta - 3\theta^2)]}{72\tau^C \left[-16\tau^C + \beta (4\bar{k} + \theta)^2\right]^2}.
\]

(C.1)

Taking a common denominator and ignoring the denominator yields
\[
(48\bar{k}^2 - 8\bar{k}\theta + 3\theta^2) \left[-16\tau^C + \beta (4\bar{k} + \theta)^2\right] [72\tau^C + 2\beta (-48\bar{k}^2 + 8\bar{k}\theta - 3\theta^2)] + (4\bar{k} + \theta) [24\tau^C + \beta (-48\bar{k}^2 + 8\bar{k}\theta - 3\theta^2)] [48\tau^C + \beta (-48\bar{k}^2 + 8\bar{k}\theta - 3\theta^2)] > 0.
\]

(C.2)

Since (C.2) is positive, (C.1) turns out to be negative.

References


