Ranked-Based Vector Median Filter

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***Abstract*—In this paper a new class of noise attenuating and edge enhancing filters for color image processing is proposed. The new filtering design is based, like the vector median filter, on the minimization of aggregated weighted distances between pixels in a filtering window. The weights which are assigned to the sequence of the sorted distances are decreasing functions of their ranks. In this way, the largest distances are taken to the aggregated distance with smallest weight, which ensures much better noise reduction properties of the described filter. Additionally, the proposed method has a unique ability to strengthen the edges of the noisy image objects. This unique deblurring property can be useful in many applications which require reliable impulse noise reduction and efficient edge sharpening of color images.**

I. Introduction

Noise, arising from a variety of sources, is inherent to all electronic image sensors and therefore the noisy signal has to be processed by a filtering algorithm that suppresses the noise component, while preserving original image structures [1], [2].

Quite often color images are corrupted by impulse noise caused by malfunctioning sensors in the image formation pipeline, faulty memory locations in hardware, aging of the storage material or transmission errors due to natural or man-made processes [3]. Common sources of impulse noise include also atmospheric disturbances and strong electromagnetic interferences. These noise sources generate short time duration, high energy pulses which affect the regular signal, resulting in abrupt alterations of the color image samples that differ significantly from their local neighborhood in the image domain.

The most popular family of nonlinear filters used for impulsive noise removal in color images is based on the order statistics [4]. These filters perform the *vector ordering* of the set of pixels from the filtering window in order to determine the output sample.

Their output is defined as the lowest ranked vector according to a specified vector ordering technique and their main drawback lies in introducing too much smoothing which results in an extensive blurring of the output image. To remove this disadvantage, a modification of the aggregated vector ordering scheme is proposed in this paper.

The modification is based on the application of the information on the ranks of distances used for the computation of the aggregated distances used in the weighted vector median filtering scheme [5]–[10], as it enables to design efficient noise reducing filtering methods.

II. Ranked-Based Vector Median Filter

Let the color image $\boldsymbol{x}$ be defined as a mapping $Z^2 \rightarrow Z^3$ and let the set $W = \{\boldsymbol{x}_i \in Z^2; i = 1, 2, \ldots, n\}$ denotes a square filtering window consisting of n samples centered at the pixel $\boldsymbol{x}_1$.

The most popular impulsive noise reduction methods are based on a *vector ordering* scheme defined through the sorting of *aggregated distances*. The aggregated distance D_i assigned to a sample $\boldsymbol{x}_i$, for $i = 1, 2, \ldots, n$ is defined as

$$D_i = \sum_{\boldsymbol{x}_k \in W} d(\boldsymbol{x}_i, \boldsymbol{x}_k) = \sum_{k=1}^{n} d_{ik} \,, \tag{1}$$

where $d(\boldsymbol{x}_i, \boldsymbol{x}_k) = d_{ik}$ is the Euclidean distance between pixels $\boldsymbol{x}_i$ and $\boldsymbol{x}_k$.

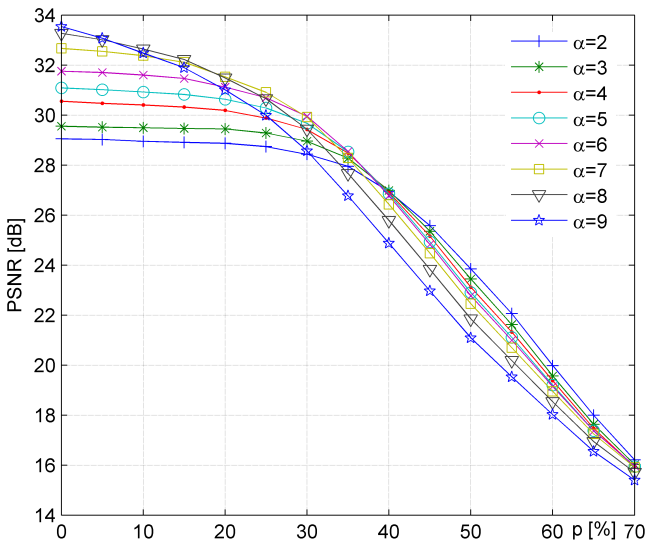


Fig. 1: Dependence of the PSNR on the α parameter of the SVMF [11] evaluated on the LENA color test images contaminated by uniform impulse noise.

Sorting the scalar quantities $D_1, D_2, \ldots, D_n$ an ordering of the corresponding vectors can be achieved, [1]

$$D_{(1)}, D_{(2)}, \ldots, D_{(n)} \Longrightarrow \boldsymbol{x}_{(1)}, \boldsymbol{x}_{(2)}, \ldots, \boldsymbol{x}_{(n)} \,, \tag{2}$$

where $\boldsymbol{x}_{(1)}$ is the vector median of the set of samples belonging to the filtering window W. This vector is minimizing the sum of distances to all other pixels contained in W, [4].

The distances between a given pixels and all other pixels belonging to W can also be ordered

$$d_{i1}, d_{i2}, \ldots, d_{in} \Longrightarrow d_{i(1)}, d_{i(2)}, \ldots, d_{i(n)}\,, \tag{3}$$

and the ranks of the ordered distances can be used for building the accumulated distances in (1). If r denotes the rank of a given distance, then $d_{i(r)}$ will denote the corresponding distance value and instead of the aggregated distances in (1) we can build a weighted sum of distances, utilizing the distance ranks

$$\Delta_i = \sum_{r=1}^{n} f(r) \cdot d_{i(r)}\,, \tag{4}$$

where $f(r)$ is a function of the distance rank r. Then, the new weighted aggregated distances Δ_i can be sorted and a new sequence of vectors is obtained

$$\Delta_{(1)}, \Delta_{(2)}, \ldots, \Delta_{(n)} \Longrightarrow \boldsymbol{x}^*_{(1)}, \boldsymbol{x}^*_{(2)}, \ldots, \boldsymbol{x}^*_{(n)}\,, \tag{5}$$

where the vector $\boldsymbol{x}^*_{(1)}$ will be the output of the proposed *Ranked-Based Vector Median Filter* (RVMF) For $f(r) = 1$, $r = 1, 2, \ldots, n$ we obtain $\Delta_i = D_i$ and $\boldsymbol{x}^*_{(1)} = \boldsymbol{x}_{(1)}$. For

$$f(r) = \begin{cases} 1 & \text{for} \quad r \leq \alpha, \;\; \alpha \leq n\,, \\ 0 & \text{otherwise}, \end{cases} \tag{6}$$

Sharpening Vector Median Filter (SVMF) is obtained, [11].

Figure 1 shows the dependence of the PSNR on the α paramater in (6) for various contamination levels using the LENA test image. As can be seen the optimal parameter α depends heavily on the noise intensity level p, which evokes the need for adaptive selection of α according to the noise intensity level. This can be seen as a drawback of the SVMF, which can be alleviated using a decreasing weighting function instead of the hard thresholding scheme in (6).

III. Experiments

In order to evaluate the effectiveness of the novel switching filter a set of test images was contaminated with uniform impulse noise defines as

$$x_{iq} = \begin{cases} \rho_{iq}\,, & \text{with probability } \pi\,, \\ o_{iq}\,, & \text{with probability } 1-\pi\,, \end{cases} \tag{7}$$

where o_{iq} denotes the q-th component of the original pixel at position i and the contamination component ρ_{iq} is a random variable in the range $[0, 255]$. The fraction of contaminated pixels is then equal to $p = 1 - (1-\pi)^3$.

TABLE I: Dependence of PSNR for the RVMF, SVMF and VMF on the noise intensity p contaminating LENA image for various weighting functions and α parameters of the SVMF.

p	$1/r$	$1/r^2$	$e^{-\left(\frac{r}{h}\right)^2}$	$e^{-\frac{r}{h}}$	$\alpha = 2$	$\alpha = 3$	$\alpha = 4$	VMF
0.1	31.62	30.30	**32.61**	32.57	28.99	29.51	30.40	32.50
0.2	30.90	29.95	**31.52**	31.46	28.81	29.37	30.11	31.04
0.3	29.86	29.53	29.88	**29.89**	28.57	29.01	29.40	28.42
0.4	27.48	**27.78**	27.61	27.68	27.12	27.08	26.93	24.73
0.5	23.86	**24.48**	24.44	**24.48**	24.01	23.48	23.20	21.16
0.6	19.98	**20.42**	20.28	20.32	19.94	19.50	19.25	17.95

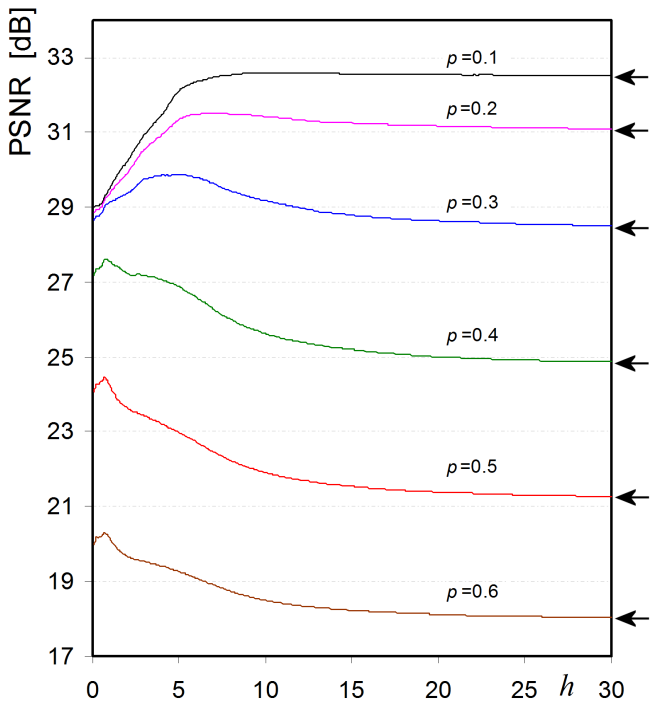


Fig. 2: Dependence of the PSNR on the h parameter of the RVMF using the f_3 weighting function for increasing noise intensity, (LENA test image). The arrows on the right of the plots indicate the values of PSNR obtained applying the VMF.

For the measurement of the impulse noise removal efficiency, the *Peak Signal to Noise Ratio* (PSNR) defined as

$$PSNR = 20\log_{10}\left(\frac{255}{\sqrt{MSE}}\right), MSE = \frac{1}{N}\sum_{i=1}^{N} \|\boldsymbol{x}_i - \boldsymbol{o}_i\|^2\,, \tag{8}$$

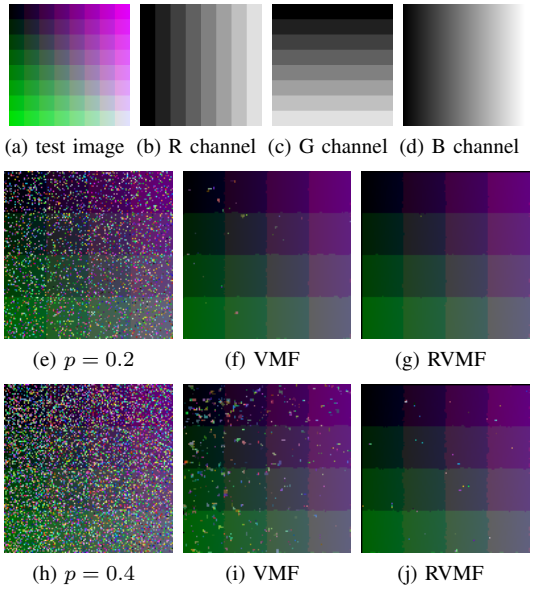
was applied, where N is the total number of image pixels, and x_{iq}, o_{iq} denote the q-th component of the noisy image pixel channel and its original, undisturbed value at a pixel position i, respectively.

The new filtering design was evaluated using the following weighting functions of the pixel ranks r

$$f_1 = \frac{1}{r}, f_2 = \frac{1}{r^2}, f_3 = \exp\left\{-\left(\frac{r}{h}\right)^2\right\}, f_4 = \exp\left\{-\frac{r}{h}\right\}\,, \tag{9}$$

where h is a smoothing parameter, which controls the influence of the ranks on the aggregated ranked distance defined in (4). The first two weighting functions do not require any tuning parameter and as can be derived from Tab. I, these functions offer similar results like the Gaussian and exponential function (f_3 and f_4) with optimal settings of the smoothing parameter h. However, the f_1 function yields good results for color images with low impulse noise pollution and the f_2 function works better for higher contamination ratios.

Figure 2 shows the dependence of the PSNR on the h smoothing parameter for increasing noise intensity p for the LENA color image. As can be noted, for low intensity levels the optimal filtering efficiency is comparable with the vector



(a) test image (b) R channel (c) G channel (d) B channel

(e) $p = 0.2$ (f) VMF (g) RVMF

(h) $p = 0.4$ (i) VMF (j) RVMF

Fig. 3: Comparison of the noise reduction efficiency of the RVMF with f_1 weighting function and VMF evaluated on an artificial test image (a) shown with its RGB components (b-d) for noise intensity $p = 0.2$ (e) and $p = 0.4$ (h). As can be observed the VMF outputs (f, i) contain much more noise than the images filtered with RVMF (g, j).

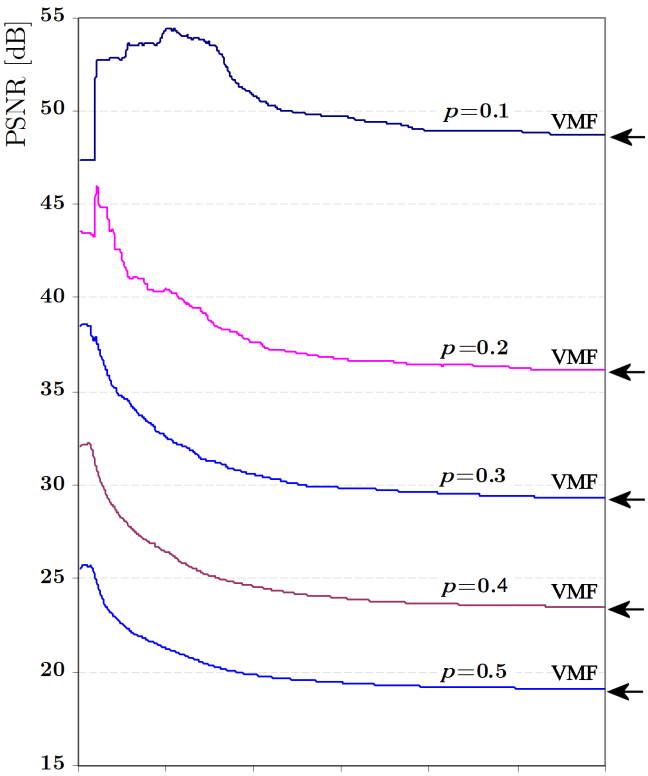
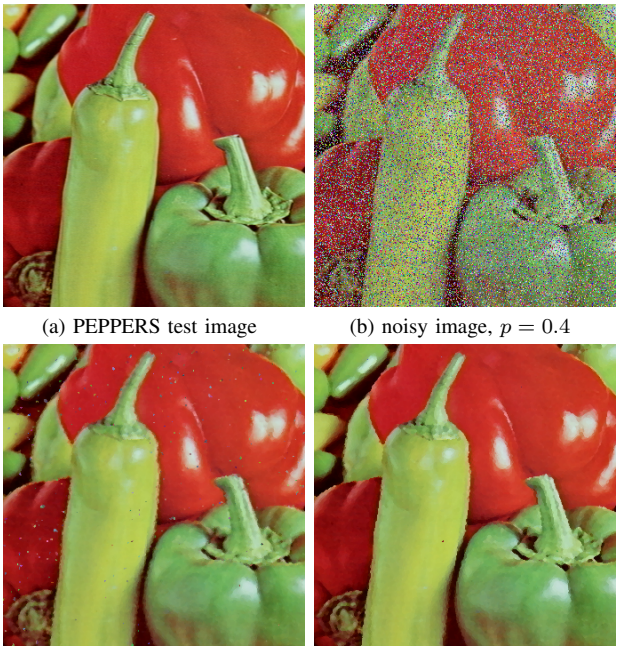


Fig. 4: Dependence of the PSNR of the proposed RVMF on the h parameter in the f_3 weighting function for the artificial test image shown in Fig. 3a with noise intensity levels p ranging from 0.1 to 0.5. The arrows indicate the values of PSNR obtained when using the VMF.



(a) PEPPERS test image (b) noisy image, $p = 0.4$

(c) VMF (d) RVMF

Fig. 5: Efficiency of RVMF as compared with VMF: (a) part of the PEPPERS image, (b) noisy test image, $p = 0.4$, (c) and (d) VMF and RVMF output, (two iterations).

median filter, (see also Tab. I). For higher contaminations, the RVMF excels significantly over the VMF. However, the apparently lower effectiveness of the proposed filtering scheme for low contamination levels is due to its edge enhancing properties, which are treated by the PSNR measure as a distortion, but in fact the edge enhancing effect is very beneficial and enhances the restoration quality of the new filter [12].

The edge sharpening effect makes that the PSNR measure poorly describes the noise reduction ability of the RVMF for low noise intensity levels. For the evaluation of the efficiency of the new filter an artificial image has been generated (Fig. 3a). The test image does not change after the filtering with VMF and RVMF and the edges are not affected by both filters. In this way, contaminating and restoring this test image, the real noise reducing properties of the RVMF can be evaluated. As can derived from the plots in Fig. 4, the proposed filtering solution offers much better performance than the VMF. For the simulations, the Gaussian weighting function was used, but similar filtering efficiency can be obtained using the f_1 function for low contaminations and f_2 function for higher noise intensity.

The edge enhancing effect is visible in Fig. 5, which depicts the results of the filtering of the PEPPERS color test image distorted by 40% of impulse uniform noise. The comparison with the output of the VMF shows that the noise is better suppressed and image details are better preserved. Moreover, the edges are much sharper, which can help in further stages of image processing like edge detection or segmentation. The same effect can be observed in Fig. 6 which depicts a blurry

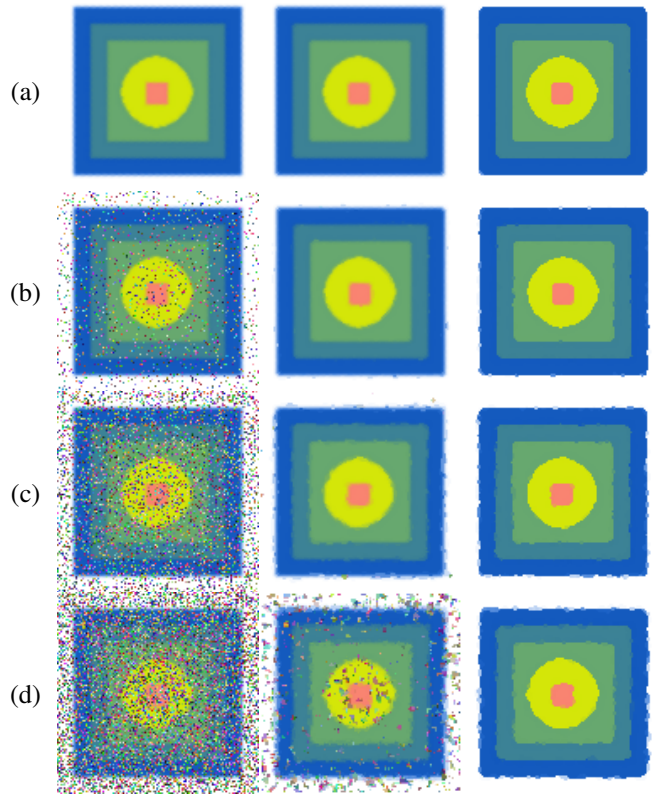


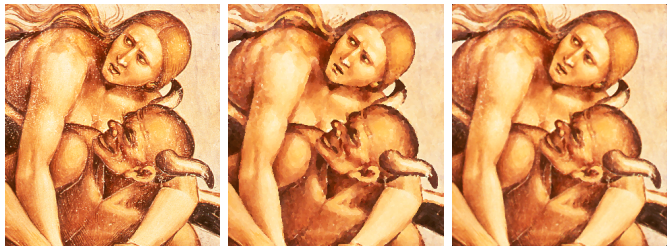
Fig. 6: Noise reducing and edge enhancing properties of the RVMF: left column shows the test image (a), contaminated by 10% (b), 30% (c) and 50% (d) impulse noise. The middle column shows the output of the VMF, whereas the right column shows the images restored with RVMF.

artificial image contaminated with increasing noise intensity and restored with the VMF and RVMF. As can be seen, again the RVMF better copes with the impulses injected to the image by the noise process and produces sharp edges. Figure 7 shows that the proposed algorithm can be used not only for noise reduction but also for image sharpening. As can be noticed the processed image has much sharper edges without overshots typical for linear edge enhancing methods. Another example of the efficiency of the proposed filtering design is depicted in Fig. 8, which shows the results of the denoising of a small part of an image of a fresco "The Condemned in Hell" by Luca Signorelli. As can be observed, the proposed filter removes efficiently the impulsive noise while sharpening image edges.

IV. Conclusions

In this paper a novel filtering design utilizing the information of the pixels ranks in the ordered sequence of distances between a pixel and its neighbors has been proposed. The described method is an extension and improvement of the well known Vector Median Filter. Extensive simulations revealed very good noise attenuation properties of the proposed filtering scheme combined with its unique ability to sharpen image edges. As a result, color image noise removal is combined with

Fig. 7: Edge enhancing property of the proposed filter: blurry test image (left) and it sharpened version (right) obtained with the RVMF with f_1 weighting function.



(a) original (b) RVMF (c) VMF

Fig. 8: Results of virtual restoration of an image of of a small part of a fresco "The Condemned in Hell" by Luca Signorelli.

edge enhancement. Further work will focus on the adaptive adjustment of the ranked based weighting function to obtain the optimal filtering results.

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