Maximum SNR Detection for Selection-Relaying Cooperative System

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ABSTRACT

The major issue of the cooperative system is how to design a system scheme whose performance is as good as that of a ML receiver at the Destination node. Here, we propose to use a threshold-selection relay and a correction weighting at the destination. With this proposed scheme, we could improve the performance during small threshold value utilization. The theoretical BER of the proposed scheme with BPSK signals is derived and shown to be tight to the simulated results. We also use the high SNR approximation to simplify the theoretical BER from which the diversity order of the proposed scheme is shown to be 1.5~2.

Index Terms—Cooperative communication, Selective relay, diversity order

1. INTRODUCTION

As we know, the MIMO system is a very popular technology for wireless communication. However, it is impossible to place 2 or more antennas in the portable device due to the limited space. In [1]-[2], the cooperative communication emerges as a promising alternative to form a virtual MIMO system and combat fading in a wireless channels. The basic idea is that users or nodes in a wireless network share their information and transmit cooperatively as a virtual antenna array, thus providing diversity without the requirement of additional antennas at each node. There are two fundamental user cooperation protocols, i.e. Amplify-and-Forward protocol and Decode-and-Forward protocol in [3]. Here, we simply discuss about the Decode-and-Forward protocol.

There have been many works carried out in this protocol. In [3], for destination receiver scheme, they find out the optimum performance can be achieved by employing a maximum likelihood (ML) receiver rather than a maximal ratio combining (MRC) receiver at the destination. Due to the complexity of ML receiver, they develop the maximum SNR receiver can reduce the complexity but suffers performance degradation of 1~2dB as a tradeoff. In [4], each relay applies a threshold value to evaluate the signal quality. According to this quality, the relay would function well or not. This scheme could improve the MRC performance and is close to the ML performance during the large threshold value utilization. But the optimum performance of selection relay with ML receiver occurs during small threshold-value utilization.

Hence, we propose a threshold-selection relay with a correction weighting at the destination. With this scheme, we improve the performance in [4] during small threshold value utilization. We derive the theoretical Bit-Error-Rate (BER) analysis for the proposed scheme with BPSK modulation. We also develop an approximation for theoretical BER which helps us know the diversity order of the proposed scheme. Simulation results validate performance improvement of the proposed scheme.

2. SYSTEM MODEL OF PROPOSED SCHEME

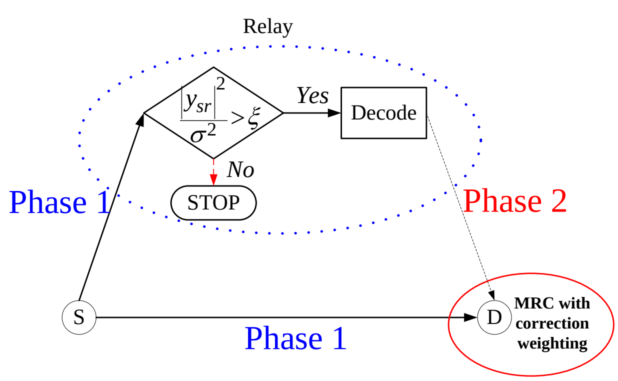

Fig. 1: Illustration of proposed cooperative system

We could use Fig. 1 to explain our proposed scheme. In Phase 1, the source (S) broadcasts the information to the destination and the relay. The received signals y_{sr} and y_{sd} can be written as

$$y_{sr} = \sqrt{P_1} h_{sr} x_s + n_{sr}, \tag{1}$$

$$y_{sd} = \sqrt{P_1} h_{sd} x_s + n_{sd} \tag{2}$$

where $x_s \in \{-1,1\}$ is a BPSK information symbol, P_1 is the transmitted power at the source, and n_{sr} and n_{sd} are additive noises. In (1) and (2), h_{sr} and h_{sd} are channel coefficients from the source to the destination and the relay respectively. In Phase 2, the relay will not work when the received signal power is lower than the threshold value. Otherwise, the relay works under the Decode-and-Forward protocol. Therefore the received signal y_{rd} can be written as

$$y_{rd} = \begin{cases} \sqrt{P_2} h_{rd} \hat{x} + n_{rd}, & |y_{sr}|^2 / \sigma^2 > \xi \\ n_{rd}, & \text{others} \end{cases} \tag{3}$$

where $\hat{x} = x$ if the relay decodes the transmitted signal correctly. Otherwise, $\hat{x} \neq x$ i.e. $\hat{x} = -x$ in case of BPSK signal. Hence, we use the method in [3] to overcome the error propagation phenomenon. We assume $\hat{x}$ could be shown as

$$\hat{x} = x_s + e \tag{4}$$

where e is a random variable that accounts for the effect of relay decision errors. With some calculations, we know

$$E[e \mid x_s] = \begin{cases} -2p & \text{if } x_s = 1 \\ 2p & \text{if } x_s = -1 \end{cases} \tag{5}$$

$$\sigma_e^2 = 4p, \tag{6}$$

where p represents the probability of relay decision error. According to (4), we know the constellation of relay transmitted signal has been changed to

$$\begin{aligned} \hat{x}_1 &= (1-p)x_1 + px_{-1} = 1-2p \\ \hat{x}_{-1} &= (1-p)x_{-1} + px_1 = -1+2p \end{aligned} \tag{7}$$

With the result of (7), we could rewrite (4) as $\hat{x} = \tilde{x}_s + \tilde{e}$, where $\tilde{x}_s = x_s + E[e \mid x_s]$ and $\tilde{e} = e - E[e \mid x_s]$. Based on these results, the received signal y_{rd} could be rewritten as

$$y_{rd} = \begin{cases} \sqrt{P_2} h_{rd} (\tilde{x}_s + \tilde{e}) + n_{rd}, & |y_{sr}|^2 / \sigma^2 > \xi \\ n_{rd}, & \text{others} \end{cases} \tag{8}$$

With (2) and (8), the weightings for relay received power large than threshold value can be shown to be

$$w_{sd} = \frac{\sqrt{P_1} h_{sd}^*}{\sigma^2} \quad w_{rd}^{'} = \frac{\sqrt{P_2} h_{rd}^* (1-2p)}{P_2 |h_{rd}|^2 \sigma_e^2 + \sigma^2} \tag{9}$$

Compared to the traditional MRC method, we could know $w_{rd}^{'}$ consists of traditional w_{rd} followed by the additional correction weighting α, i.e.

$$w_{rd} = \frac{\sqrt{P_2} h_{rd}^*}{\sigma^2} \qquad \alpha = \frac{(1-2p)}{\gamma_{rd} \sigma_e^2 + 1}. \tag{10}$$

In (10), it could be seen as a weighted-MRC method. The weighting w_{rd} would change to 0 when the relay received power is lower than threshold value. According to the above receiver weighting results, the destination receiver scheme could be shown in the Fig. 2.

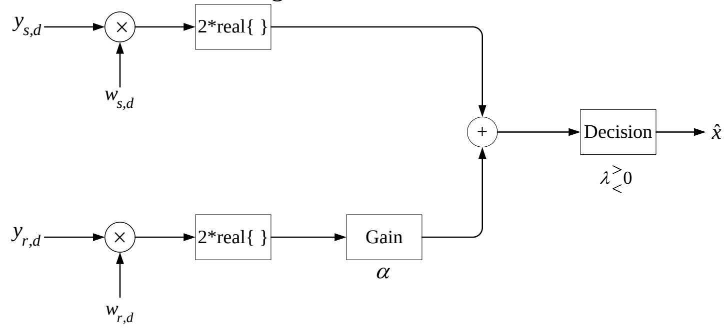

Fig. 2 The weighted-MRC receiver scheme

3. BER ANALYSIS

[4] has performed the BER analysis of the threshold selection relay with traditional MRC receiver in destination. Here, we try to analyze the BER of our proposed system by following the similar methods in [4] and [6].

We could formulate the BER of the proposed scheme by analyzing the receiver scheme in Fig. 2. Its BER depends on the relay received signal power. If the received signal power is larger than the relay threshold value, the relay would transmit the decoded signal to the destination. Therefore, the destination receiver scheme is the same as Fig. 2. On the contrary, the destination receiver scheme only works in the upper part. With these two phenomena, the BER of the proposed scheme is written as

$$P_e = P(\phi_1) P_{e|\phi_1} + P(\bar{\phi}_1) P_{e|\bar{\phi}_1} \tag{11}$$

where $P(\phi_1)$ means the probability of the event that the received signal power falls below the relay threshold value, $P_{e|\phi_1}$ means the error probability of this event. $P(\bar{\phi}_1)$ and $P_{e|\bar{\phi}_1}$ have similar meanings with $\bar{\phi}$ denoting the event that the received signal power falls above the relay threshold value.

We use (1) to derive the occurrence of these two events. Because h_{sr} and n_{sr} are complex Gaussian random variables, the received signal power $|y_{sr}|^2$ is exponentially distributive. Hence, the $P(\phi_1)$ is

$$P(\phi_1) = P\left[|y_{sr}|^2 \le \xi\sigma^2\right] = 1 - e^{-\frac{\xi\sigma^2}{P_1\sigma_{sr}^2 + \sigma^2}} \tag{12}$$

Now we proceed to derive the error probability of ϕ_1, i.e. $P_{e|\phi_1}$. Since the received signal power is lower than relay threshold value, the destination receiver only works in the upper part of Fig. 2. Hence, $P_{e|\phi_1}$ could be derived by taking the conditional expectation on the channel effect first and then averaging the effect in the end. From [7], we know

$P_{e|\phi_1}$ is known to be

$$P_{e|\phi_1} = \frac{1}{2}\left(1-\sqrt{\frac{\overline{\gamma}_{sd}}{1+\overline{\gamma}_{sd}}}\right). \quad (13)$$

Before derivation of $P_{e|\overline{\phi}_1}$, we should know the components of $P_{e|\overline{\phi}_1}$. Due to the relay error decision, the transmitted signal would have a probability p to represent the relay error decision. Hence,

$$P_{e|\overline{\phi}_1} = (1-p)P_{b1} + pP_{b2} \quad (14)$$

The derivation of p is similar to $P_{e|\phi_1}$ with different parameter γ_{sr}. The only different part occurs when averaging the channel effect. From (1), the received signal power is $P_1|h_{sr}|^2+\sigma^2$. Under the condition that the relay received power is larger than the threshold value, γ_{sr} needs to satisfy the condition $\gamma_{sr} \geq \xi - 1$. The lower limit would be shown as

$$\text{lower\_limit} = \begin{cases} 0 & \text{if } \xi \leq 1 \\ \xi - 1 & \text{if } \xi > 1 \end{cases} \quad (15)$$

With the result of (15), we could derive p as

$$\begin{aligned} p &= \frac{1}{2}\left(1-\sqrt{\frac{\overline{\gamma}_{sr}}{1+\overline{\gamma}_{sr}}}\right) \quad \xi \leq 1 \\ &= \frac{1}{2}\left[\frac{erf(\sqrt{a(b+1)})}{\sqrt{b+1}} + e^{-ba} erfc\left(\sqrt{a}\right) - \frac{1}{\sqrt{b+1}}\right] \quad \xi > 1 \end{aligned} \quad (16)$$

where $a = \xi - 1$ and $b = (\overline{\gamma}_{sr})^{-1}$. From (16), we could derive P_{b1} and P_{b2} individually. The conditional $P_{b1}^{h_{sd},h_{rd}}$ is similar to $P_{e|\phi_1}^{h_{sd}}$ with different parameters, i.e.

$$P_{b1}^{h_{sd},h_{rd}} = Q\left(\frac{\sqrt{2}(\gamma_{sd}+\alpha\gamma_{rd})}{\sqrt{\gamma_{sd}+\alpha^2\gamma_{rd}}}\right) \quad (17)$$

Similarly, $P_{b2}^{h_{sd},h_{rd}}$ could be shown to be

$$P_{b2}^{h_{sd},h_{rd}} = Q\left(\frac{\sqrt{2}(\gamma_{sd}-\alpha\gamma_{rd})}{\sqrt{\gamma_{sd}+\alpha^2\gamma_{rd}}}\right) \quad (18)$$

After averaging the channel effect of (17) and (18), P_{b1} and P_{b2} could be derived. With the result (12), (13), (16) and averaging result of (17) and (18), BER in (11) can be shown to be

$$\begin{aligned} P_e = &\left(1-e^{-\frac{\xi\sigma^2}{P_1\sigma_w^2+\sigma^2}}\right)\left(\frac{1}{2}\left(1-\sqrt{\frac{\overline{\gamma}_{sd}}{1+\overline{\gamma}_{sd}}}\right)\right) + \\ &e^{-\frac{\xi\sigma^2}{P_1\sigma_w^2+\sigma^2}}\left(\begin{aligned}&(1-p)E\left[Q\left(\frac{\sqrt{2}(\gamma_{sd}+\alpha\gamma_{rd})}{\sqrt{\gamma_{sd}+\alpha^2\gamma_{rd}}}\right)\right] + \\ &pE\left[Q\left(\frac{\sqrt{2}(\gamma_{sd}-\alpha\gamma_{rd})}{\sqrt{\gamma_{sd}+\alpha^2\gamma_{rd}}}\right)\right]\end{aligned}\right) \end{aligned} \quad (19)$$

4. HIGH-SNR APPROXIMATION OF BER

We could simplify (11) to get the diversity order by assuming the system has a high SNR. It is well known that the diversity order is defined as the negative exponent of the average BER plotted in log-log scale. Let us parameterize three SNRs first, $\overline{\gamma}_{sd} = c_1\overline{\gamma}_s$, $\overline{\gamma}_{rd} = c_2\overline{\gamma}_s$ and $\overline{\gamma}_{rd} = c_3\overline{\gamma}_s$ where $\overline{\gamma}_s$ denotes the averaged SNR. We also assume the relay is close to the source. With this assumption, p is almost equal to zero. Therefore, the correction weighting α is very close to 1. With these approximations, (11) becomes

$$P_e \approx P(\phi_1)P_{e|\phi_1} + (P_{b1} + pP_{b2}) \quad (20)$$

By the Taylor expansion and some calculations, we have

$$\begin{aligned} P_e \approx &\frac{\xi}{(1+\overline{\gamma}_{sr})(4\overline{\gamma}_{sd})} + \\ &\left(\begin{aligned}&\left(\frac{1}{2(\overline{\gamma}_{sd}-\overline{\gamma}_{rd})}\left[-\frac{\overline{\gamma}_{sd}^{1.5}}{\sqrt{\overline{\gamma}_{sd}+1}} + (\overline{\gamma}_{sd}-\overline{\gamma}_{rd}) + \frac{\overline{\gamma}_{rd}^{1.5}}{\sqrt{\overline{\gamma}_{rd}+1}}\right]\right) \\ &+pE\left[Q\left(\frac{\gamma_{sd}-\gamma_{rd}}{\sqrt{\gamma_{sd}+\gamma_{rd}}}\right)\right]\end{aligned}\right) \end{aligned} \quad (21)$$

From (21), we only know the diversity of first term is 2 but cannot know the diversity order in second term. Therefore, we use a similar approximation method in [6] to derive the second term. With these assumptions and the Chernoff bound, P_{b1} is shown to be

$$P_{b1} \leq \frac{1}{1+\overline{\gamma}_{sd}}\frac{1}{1+\overline{\gamma}_{rd}} \propto (\overline{\gamma}_s)^{-2} \quad (22)$$

From (22), the diversity of P_{b1} is 2. We derive the diversity of p by using $Q(x) \leq \frac{1}{2}e^{-\frac{x^2}{2}}$. It is shown to be

$$p \leq \frac{1}{1+\overline{\gamma}_{sr}}e^{-d} \propto (\overline{\gamma}_s)^{-1} \quad (23)$$

where $d = 0$ or $\xi - 1$. From (23), the diversity order is 1. We also use this Q-function property to derive diversity order of P_{b2}. With $P_{b2} \le E\left[\frac{1}{2}e^{-\left(\gamma_{sd}+\gamma_{rd}-\frac{4\gamma_{sd}\gamma_{rd}}{\gamma_{sd}+\gamma_{rd}}\right)}\right]$ and the decreasing property of the exponential term, we change $\frac{4\gamma_{sd}\gamma_{rd}}{\gamma_{sd}+\gamma_{rd}}$ to $2\sqrt{\gamma_{sd}\gamma_{rd}}$. With some calculations and approximations,

$$P_{b2} \le \frac{1}{(1+\overline{\gamma}_{sd})(1+\overline{\gamma}_{rd})} + \frac{(\overline{\gamma}_{sd}\overline{\gamma}_{rd})^{1/2}}{(\overline{\gamma}_{sd}+\overline{\gamma}_{rd}+1)^{3/2}} \propto (\overline{\gamma}_s)^{-1/2} \quad (24)$$

From (24), the diversity order is 0.5. From (21)-(24), we could conclude the diversity order of the (20) is 1.5~2. This result agrees with that in [5], i.e. for the uncoded relay, the diversity is 1.5~2 under decode-and-forward protocol.

5. SIMULATION RESULTS

Computer simulations are performed to validate our derivations. Here we assume the relay is placed in the middle. With this assumption, the channel coefficient's variance we used is the same, i.e. $\sigma_{sr}^2 = \sigma_{sd}^2 = \sigma_{rd}^2 = 1$. The additive noise is zero mean with $\sigma^2 = 1$. The total power for system is $P = P_1 + P_2$. Both transmitted power P_1 and P_2 are set to equal. We also assume the relay is placed close to source. With this assumption, we change the channel coefficient h_{sr} to 10. From Fig. 3, we know the performance of proposed scheme greatly improves the threshold-selection relay scheme and also is close to the destination with ML receiver without using any non-linear mapping function. Besides, the curve based on the theoretical BER matches closely with the simulated curve. From Fig. 4, we know the curve of the simplified BER matches those of the simulated and theoretical BERs.

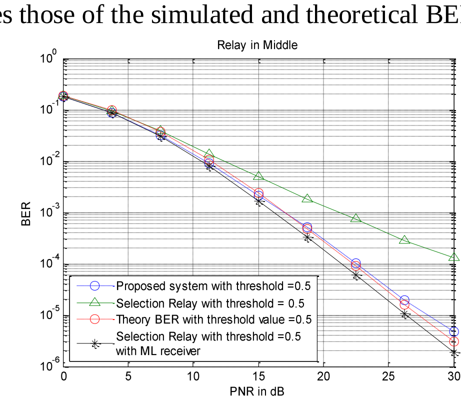

Fig. 3 Performance comparison with theoretical BER

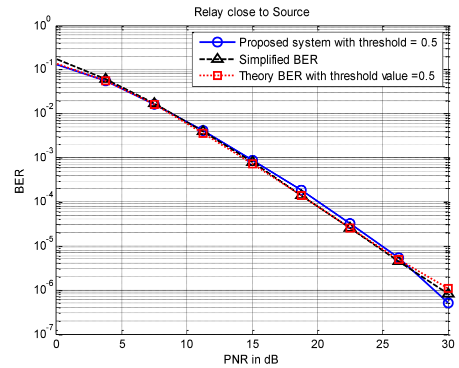

Fig. 4 Performance comparison of theoretical and approximated BERs

6. CONCLUSIONS

In this paper, we propose a threshold-selection relay with correction weighting at the destination. It improves the performance during small threshold value utilization and successfully reduces the threshold value. The optimum threshold value is 0.5 when the relay is placed in the middle. BER performance is derived theoretically and the diversity order is shown be 1.5~2.

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