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Amplify-and-Forward Relaying Based Multi-Relay Wireless Network Coding

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***Abstract*—This paper extends the amplify-and-forward based wireless network coding (AFNC) for single-relay network to multi-relay network with a chain topology. Direct application of the relaying protocol utilized in the conventional single-relay AFNC to the multi-relay scenario causes two major problems: the phenomenon of back stream of data, which we call *data reflection*, and inter-hop interference. We propose a distributed relaying protocol for the multi-relay AFNC, which can cope with the data reflection by using locally available information. Moreover, we introduce the technique of frequency domain equalization (FDE) for the transmission and the detection of signals in order to utilize the interference as informative signal. Computer simulation results show that, with the proposed method, the multi-relay AFNC can be realized and the interference is successfully utilized to improve the throughput performance.**

***Index Terms*—Amplify-and-Forward Relaying, Network Coding, Multi-Relay, FDE**

I. Introduction

Network Coding (NC) [1] is a packet level coding scheme performed by relay nodes in the network and has been gaining much attention both in wired and wireless communication research communities. Originally, NC has been studied for multicast communications of wired network, where it has been shown that there exists NC scheme such that the minimum value of maximum flow of each point-to-point communication can be achieved as the maximum flow of the multicast communications in arbitrary network [2]. The benefit from NC depends on the network topology or the path of packet transmission [3], therefore, path design to maximize the effect of NC is the key issue in the wired network.

On the other hand, taking advantage of the broadcast nature of wireless channel, the idea of NC has been successfully introduced to wireless communications systems [4]-[6]. For a single-relay wireless model, two simple NC methods, namely, Amplify-and-Forward based NC (AFNC) and Decode-and-Forward based NC (DFNC), have been proposed, and it has been shown that the throughput of bi-directional communications can be greatly improved with the methods.

In this paper, we consider to extend the conventional single-relay AFNC to multi-relay network with a chain topology. Direct application of the relaying protocol for single-relay network to multi-relay system causes the back stream of the data signal, which we call *data reflection*, and this makes it very difficult to recover the original signal from the received signal. In order to overcome the problem, we propose a distributed relaying protocol for the multi-relay AFNC, which can cope with the data reflection by using locally available information. Moreover, the inter-hop interference is also one of the multi-relay network specific problems, therefore, we introduce the technique of frequency domain equalization (FDE) [7], [8] for the interference problem. Computer simulations show that the proposed relaying algorithm enable us to realize multi-relay AFNC, and that the proposed AFNC with the FDE technique can achieve better throughput performance than the conventional method without interference for a wide range of signal-to-noise power ratio (SNR) in spite of the requirement of one redundant signal block for the cyclic prefix.

II. Conventional Single-Relay AFNC

In this section, we explain the conventional single-relay AFNC algorithm proposed in [5]. Assume that two terminal nodes S_1 and S_2 exchange signals via a relay node R_1. Let $\mathbf{x}_u(m) = (x_u[1](m), \cdots, x_u[K](m))^T$ denote a signal vector of K symbols transmitted by node $u \in \{\mathrm{S}_1, \mathrm{S}_2\}$ in the m-th time slot with each symbol having transmit power of P. In the AFNC, S_1 and S_2 transmit their signals simultaneously, therefore, the received signal at R_1 is given by

$$\mathbf{r}_{\mathrm{R}_1}(m) = h_1\mathbf{x}_{\mathrm{S}_1}(m) + h_2\mathbf{x}_{\mathrm{S}_2}(m) + \mathbf{z}_{\mathrm{R}_1}(m), \tag{1}$$

where $\mathbf{z}_u(m)$, $u \in \{\mathrm{S}_1, \mathrm{S}_2, \mathrm{R}_1\}$ denotes a zero mean additive white Gaussian noise (AWGN) with covariance matrix of $\sigma_u^2\mathbf{I}_K$. h_1 and h_2 are channel coefficients between S_1 and R_1, and between S_2 and R_1, respectively. Hereafter, all the channels are assumed to be reciprocal and time-invariant. In the $(m+1)$-th time slot, R_1 amplifies the previously received signal $\mathbf{r}_{\mathrm{R}_1}(m)$ by a factor β and broadcasts it to both S_1 and S_2. Thus, the transmitted signal by R_1 is written as

$$\mathbf{t}_{\mathrm{R}_1}(m+1) = \beta\mathbf{r}_{\mathrm{R}_1}(m), \tag{2}$$

where β is an amplification factor given by

$$\beta = \sqrt{\frac{P}{|h_1|^2P + |h_2|^2P + \sigma_{\mathrm{R}_1}^2}}. \tag{3}$$

By the factor β, the transmitted power from the relay node is adjusted to be P. Then, the received signals at S_1 is given by

$$\begin{aligned}\mathbf{r}_{\mathrm{S}_1}(m+1) =& \beta h_1^2\mathbf{x}_{\mathrm{S}_1}(m) + \beta h_1 h_2\mathbf{x}_{\mathrm{S}_2}(m) \\ &+ \beta h_1\mathbf{z}_{\mathrm{R}_1}(m) + \mathbf{z}_{\mathrm{S}_1}(m+1).\end{aligned} \tag{4}$$

Assuming that S_1 knows β, h_1, and h_1h_2, S_1 can eliminate the first term of the right hand side in (4) because it is his

Fig. 1. Multi-relay network with a chain topology

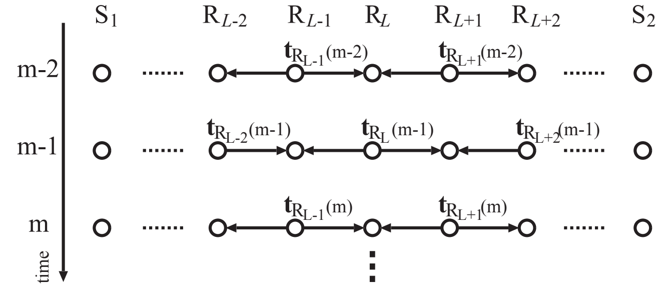

Fig. 2. Relaying schedule for multi-relay AFNC

transmitted signal. The estimated signal of $\mathbf{x}_{\mathrm{S}_2}(m)$ at S_1 is obtained as

$$\hat{\mathbf{x}}_{\mathrm{S}_2}(m) = \frac{\mathbf{r}_{\mathrm{S}_1}(m+1) - \beta h_1^2 \mathbf{x}_{\mathrm{S}_1}(m)}{\beta h_1 h_2}, \quad (5)$$

where $(\cdot)^*$ denotes the complex conjugate. In the same way, S_2 can obtain the estimate of $\mathbf{x}_{\mathrm{S}_1}(m)$. With this procedure, S_1 and S_2 can exchange their signals in 2 time slots.

III. Proposed Relaying Protocol for Multi-Relay AFNC

Assume that two terminal nodes S_1 and S_2 exchange signals via N (≥ 1) relay nodes, which are connected in line, i.e., chain topology network, as shown in Fig. 1, where R_i denotes the i-th relay node from S_1. For simplicity, we assume N to be an odd number so that the model becomes symmetric with respect to the center relay node R_L $(L = (N+1)/2)$, however, it is straight forward to apply the proposed method to the network with even number relays. h_i denotes the channel coefficient between R_{i-1} and R_i except for h_1 and h_{N+1}, which are channel coefficients between R_1 and S_1, S_2 and R_N, respectively.

Fig. 2 depicts the relaying schedule for the proposed multi-relay AFNC. The relaying time schedule itself is a simple extension from the single-relay AFNC, however, it easily verified that the direct application of the relaying protocol for the single-relay model (2) to multi-relay network causes a back stream of the data signal, which we call *data reflection*. To be more precise, with the relaying protocol (2), the transmitted signal $\mathbf{t}_{\mathrm{R}_i}(m-2)$ from relay node R_i in the $(m-2)$-th time slot is included in the signal from R_{i+1} in the $(m-1)$-th time slot, thus, $\mathbf{t}_{\mathrm{R}_i}(m)$ includes $\mathbf{t}_{\mathrm{R}_i}(m-2)$. In this way, the same information is exchanged a number of times not only between local pairs as in this example but also between all the possible pairs of relay nodes in the network. This results in very complex received signal at the edge nodes (S_1 and S_2), therefore, multi-relay AFNC specific relaying protocol is required.

In the proposed method, the center relay node R_L cancels the data reflection from both sides, while relay nodes $\mathrm{R}_2, \cdots, \mathrm{R}_{L-1}$ cancel the reflection only from left hand side and nodes $\mathrm{R}_{L+1}, \cdots, \mathrm{R}_{N-1}$ only from right hand side. As for the edge relay nodes R_1 and R_N, their relaying protocol is the same as the case of single-relay model, because there is no data reflection from S_1 or S_2. In summary, the transmitted signal at the $(m+1)$-th time slot from the relay node R_i is given by

$$\mathbf{t}_{\mathrm{R}_i}(m+1) = \beta_{\mathrm{R}_i} \times$$
$$\begin{cases} \mathbf{r}_{\mathrm{R}_i}(m), \quad (i = 1, N) \\ \mathbf{r}_{\mathrm{R}_i}(m) - (\beta_{\mathrm{R}_{i-1}} h_i^2 + \beta_{\mathrm{R}_{i+1}} h_{i+1}^2)\mathbf{t}_{\mathrm{R}_i}(m-1), \quad (i = L) \\ \mathbf{r}_{\mathrm{R}_i}(m) - \beta_{\mathrm{R}_{i-1}} h_i^2 \mathbf{t}_{\mathrm{R}_i}(m-1), \quad (2 \leq i \leq L-1) \\ \mathbf{r}_{\mathrm{R}_i}(m) - \beta_{\mathrm{R}_{i+1}} h_{i+1}^2 \mathbf{t}_{\mathrm{R}_i}(m-1), \quad (L+1 \leq i \leq N-1), \end{cases} \quad (6)$$

where

$$\beta_{\mathrm{R}_i} = \sqrt{\frac{P}{|h_i|^2 P + |h_{i+1}|^2 P + \sigma_{\mathrm{R}_i}^2}}. \quad (7)$$

Through the procedure at relay nodes above, the received signal at S_1 is given by

$$\mathbf{r}_{\mathrm{S}_1}(m) = \sum_{j=1}^{L} \prod_{k=1}^{j} (\beta_{\mathrm{R}_k} h_k)^2 \beta_{\mathrm{R}_j}^{-1} \mathbf{x}_{\mathrm{S}_1}(m - 2j + 1) + \prod_{j=1}^{N} \beta_{\mathrm{R}_j} \prod_{k=1}^{N+1} h_k \mathbf{x}_{\mathrm{S}_2}(m - 2L + 1) + \mathbf{Z}_1(m), \quad (8)$$

where $\mathbf{Z}_1(m)$ denotes accumulated additive noise component written as

$$\mathbf{Z}_1(m) = \mathbf{z}_{\mathrm{S}_1}(m) + \sum_{j=1}^{N} \prod_{k=1}^{j} (\beta_{\mathrm{R}_k} h_k) \mathbf{z}_{\mathrm{R}_j}(m - j) + \prod_{i=1}^{L-1} (\beta_{\mathrm{R}_i} h_{i+1}) \sum_{j=1}^{L-1} \prod_{k=1}^{j+1} (\beta_{\mathrm{R}_k} h_k) \mathbf{z}_{\mathrm{R}_j}(m - L - j). \quad (9)$$

As in the single-relay case, assuming that S_1 knows $\prod_{k=1}^{j} (\beta_{\mathrm{R}_k} h_k)^2 \beta_{\mathrm{R}_j}^{-1}$ $(j = 1 \cdots L)$, S_1 can obtain the estimate of the transmitted signal from S_2 after the cancellation of the first term of the right hand side in (8) as

$$\hat{\mathbf{x}}_{\mathrm{S}_2}(m - 2L + 1) = \mathbf{x}_{\mathrm{S}_2}(m - 2L + 1) + \frac{\prod_{k=1}^{N+1} h_k^*}{\prod_{j=1}^{N} \beta_{\mathrm{R}_j} \prod_{k=1}^{N+1} |h_k|^2} \mathbf{Z}_1(m). \quad (10)$$

In a similar manner, S_2 can obtain the estimate of $\mathbf{x}_{\mathrm{S}_1}(m)$.

IV. Proposed Interference-Aware Two-Relay AFNC

The method presented in the previous section has assumed that a transmitted signal is received only at adjacent nodes, but in practice, the signal is also received at some distant nodes as interference. Here, we consider a two-relay network model, which is the minimum configuration where the interference is

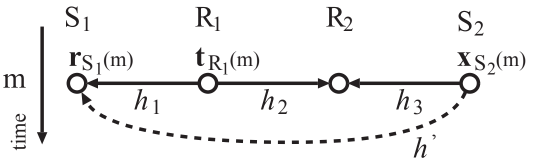

Fig. 3. Two-relay AFNC with interference

included in the received signal, and propose AFNC methods, which can cope with the interference.

Fig. 3 shows the two-relay network model with interference, where $h^{'}$ denotes the direct channel between S_1 and S_2, which causes the interference. The received signal at S_1 in the m-th time-slot can be written as

$$\begin{aligned}\mathbf{r}_{\mathrm{S}_1}(m) &= h_1\mathbf{t}_{\mathrm{R}_1}(m) + h^{'}\mathbf{x}_{\mathrm{S}_2}(m) + \mathbf{z}_{\mathrm{S}_1}(m)\\ &= \beta_{\mathrm{R}_1}\beta_{\mathrm{R}_2}h_1h_2h_3\mathbf{x}_{\mathrm{S}_2}(m-2) + \beta_{\mathrm{R}_1}\beta_{\mathrm{R}_2}h_1h_2\mathbf{z}_{\mathrm{R}_2}(m-2)\\ &\quad + \beta_{\mathrm{R}_1}h_1\mathbf{z}_{\mathrm{R}_1}(m-1) + h^{'}\mathbf{x}_{\mathrm{S}_2}(m) + \mathbf{z}_{\mathrm{S}_1}(m)\\ &= h_{all}\mathbf{x}_{\mathrm{S}_2}(m-2) + h^{'}\mathbf{x}_{\mathrm{S}_2}(m) + \mathbf{z}(m),\end{aligned} \tag{11}$$

where $\mathbf{z}(m) = \beta_{\mathrm{R}_1}\beta_{\mathrm{R}_2}h_1h_2\mathbf{z}_{\mathrm{R}_2}(m-2) + \beta_1h_1\mathbf{z}_{\mathrm{R}_1}(m-1) + \mathbf{z}_{\mathrm{S}_1}(m)$ and $h_{all} = \beta_{\mathrm{R}_1}\beta_{\mathrm{R}_2}h_1h_2h_3$. From (11), we can see that the problem of estimating $\mathbf{x}_{\mathrm{S}_2}(m-2)$ from $\mathbf{r}_{\mathrm{S}_1}(m)$ can be considered as an equalization problem. Therefore, we propose to apply the technique of a block transmission using CP to the problem. Assuming that one redundant signal vector is inserted as a block wise CP for evey M transmitted signal vectors ($\mathbf{x}_{\mathrm{S}_2}(m-2) = \mathbf{x}_{\mathrm{S}_2}(m+2(M-1))$), the received signal vector block $\mathbf{R}(m) = \left[\mathbf{r}^{\mathrm{T}}_{\mathrm{S}_1}(m), \mathbf{r}^{\mathrm{T}}_{\mathrm{S}_1}(m+2), \cdots, \mathbf{r}^{\mathrm{T}}_{\mathrm{S}_1}(m+2(M-1))\right]^{\mathrm{T}}$ can be written as

$$\mathbf{R}(m) = \mathbf{H}_{\mathrm{C}}\mathbf{X}(m) + \mathbf{Z}(m), \tag{12}$$

where $\mathbf{X}(m) = \left[\mathbf{x}^{\mathrm{T}}_{\mathrm{S}_2}(m), \mathbf{x}^{\mathrm{T}}_{\mathrm{S}_2}(m+2), \cdots, \mathbf{x}^{\mathrm{T}}_{\mathrm{S}_2}(m+2(M-1))\right]^{\mathrm{T}}$ and $\mathbf{Z}(m) = \left[\mathbf{z}^{\mathrm{T}}(m), \mathbf{z}^{\mathrm{T}}(m+2), \cdots, \mathbf{z}^{\mathrm{T}}(m+2(M-1))\right]^{\mathrm{T}}$. The channel matrix $\mathbf{H}_{\mathrm{C}}$ is a block circulant matrix of size $KM \times KM$ given by

$$\mathbf{H}_{\mathrm{C}} = \begin{bmatrix} \mathbf{H}^{'} & & & \mathbf{H}_{\mathrm{all}} \\ \mathbf{H}_{\mathrm{all}} & \ddots & & \\ & \ddots & \ddots & \\ & & \mathbf{H}_{\mathrm{all}} & \mathbf{H}^{'} \end{bmatrix}, \tag{13}$$

where $\mathbf{H}_{\mathrm{all}}$ and $\mathbf{H}^{'}$ are diagonal matrices of size $K \times K$ defined as $\mathbf{H}_{\mathrm{all}} = \mathrm{diag}[h_{\mathrm{all}}, \cdots, h_{\mathrm{all}}]$ and $\mathbf{H}^{'} = \mathrm{diag}[h^{'}, \cdots, h^{'}]$, respectively. The matrix $\mathbf{H}_{\mathrm{C}}$ is a block circulant matrix composed by diagonal matrices, therefore, $\mathbf{H}_{\mathrm{C}}$ itself is a circulant matrix. Thus, $\mathbf{H}_{\mathrm{C}}$ can be diagonalized by using KM-point DFT matrix $\mathbf{D}_{KM}$ as

$$\mathbf{H}_{\mathrm{C}} = \mathbf{D}^{\mathrm{H}}_{KM}\mathbf{\Lambda}_{\mathrm{H}_{\mathrm{C}}}\mathbf{D}_{KM}, \tag{14}$$

TABLE I
SYSTEM PARAMETERS

Mod./Demod. Scheme	BPSK / Coherent detection
Packet Length K	32 symbols
Number of Relay Nodes N	2
Transmission Power P	1
Channel Model	Flat Fading Channel
Channel Estimation	Ideal
Number of Blocks M	4, 16
Path Loss Exponent q	1.5, 2, 3

where $\mathbf{\Lambda}_{\mathrm{H}_{\mathrm{C}}}$ is a diagonal matrix whose diagonal elements $\lambda_{\mathrm{H}_{\mathrm{C}}}[0], \cdots, \lambda_{\mathrm{H}_{\mathrm{C}}}[KM-1]$ are given by

$$\begin{bmatrix} \lambda_{\mathrm{H}_{\mathrm{C}}}[0] \\ \vdots \\ \lambda_{\mathrm{H}_{\mathrm{C}}}[KM-1] \end{bmatrix} = \mathbf{D}_{KM} \begin{bmatrix} h^{'} \\ \mathbf{0}_{(K-1)\times 1} \\ h_{\mathrm{all}} \\ \mathbf{0}_{(KM-K-1)\times 1} \end{bmatrix}. \tag{15}$$

Thus, the estimate of $\mathbf{X}(m)$ is obtained at the output of the frequency domain equalizer as

$$\hat{\mathbf{X}}(m) = \mathbf{D}^{\mathrm{H}}_{KM}\mathbf{\Gamma}_{\mathrm{H}_{\mathrm{C}}}\mathbf{D}_{KM}\mathrm{R}(m), \tag{16}$$

where $\mathbf{\Gamma}_{\mathrm{H}_{\mathrm{C}}}$ is a diagonal matrix of the one tap equalizer. For the case of zero-forcing (ZF) equalization, the (i,i) element of $\mathbf{\Gamma}_{\mathrm{H}_{\mathrm{C}}}$ is given by

$$\left\{\mathbf{\Gamma}^{(\mathrm{ZF})}_{\mathrm{H}_{\mathrm{C}}}\right\}_{(i,i)} = \frac{1}{\lambda_{\mathrm{H}_{\mathrm{C}}}[i]}, \tag{17}$$

while, for the case of the minimum mean-square-error (MMSE) criterion, the (i,i) element is given by

$$\left\{\mathbf{\Gamma}^{(\mathrm{MMSE})}_{\mathrm{H}_{\mathrm{C}}}\right\}_{(i,i)} = \frac{\lambda^{*}_{\mathrm{H}_{\mathrm{C}}}[i]}{|\lambda_{\mathrm{H}_{\mathrm{C}}}[i]|^2 + 1/\gamma_{\mathrm{S}_1}}, \tag{18}$$

where

$$\gamma_{\mathrm{S}_1} = \frac{KM\left\{\prod_{j=1}^{2}\beta^2_{\mathrm{R}_j}\prod_{k=1}^{3}|h_k|^2 + |h^{'}|^2\right\}}{E[\mathbf{Z}(m)\mathbf{Z}^{\mathrm{H}}(m)]}. \tag{19}$$

V. NUMERICAL RESULTS

We have conducted computer simulations to evaluate the performance of the proposed methods. System parameters are summarized in Table I. The equi-spaced chain topology with the number of relay nodes $N = 2$ is assumed for the network model. Since the distance between S_1 and S_2 is three times as far as that between adjacent nodes, the interference power is assumed to be $1/3^q$, where q denotes the path loss exponent, while the received signal power from the adjacent node is set to be 1. As a performance measure, we have employed throughput $R^{'}$, which is defined as

$$R^{'} = R\left(1 - \frac{P_{e,\mathrm{S}_1} + P_{e,\mathrm{S}_2}}{2}\right), \tag{20}$$

where P_{e,S_1} and P_{e,S_2} denote packet error rate at S_1 and S_2, respectively. R [bit/sec] denotes the transmission rate and is normalized to 1. Note that the unit of the transmission power is omitted in Table I, since the actual transmit power varies depending on the transmission rate.

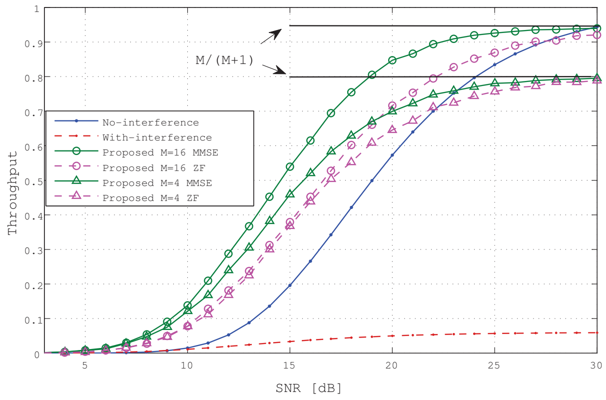

Fig. 4. Throughput performance ($M = 4$, 16, $q = 2$)

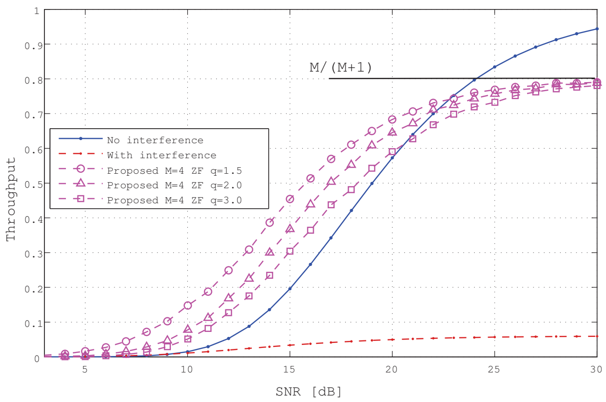

Fig. 5. Throughput performance with ZF criterion ($M = 4$, $q = 1.5$, 2, 3)

Fig. 4 depicts the throughput performance of the proposed method presented in Sec. III and IV with ZF and MMSE equalization in interference environment, where the number of blocks M is set to be 4 and 16, and the path loss exponent $q = 2$. For comparison purpose, the throughput performance of the proposed AFNC without the FDE technique is also shown in the same figure for both the case with and without the interference. From the figure, we can see that the proposed methods can improve throughput performance quite a lot compared with the method without the FDE technique, although the throughput is saturated at $M/(M + 1)$ because of the requirement of one redundant signal block for the cyclic prefix. It should be noted that the performance of the proposed method with interference is better than that of the method without the FDE technique even in the absence of interference.

Figs. 5 and 6 show the throughput performance of the proposed method for different values of q with ZF and MMSE criteria, respectively. It can be seen that the throughput performance of the proposed method gets better as the value of q decreases, and hence as the interference power increases. This means that the proposed method can successfully utilize the interference to obtain the diversity gain. Although the interference power is quite small for the case of $q = 3$, significant performance gain can be obtained even for this

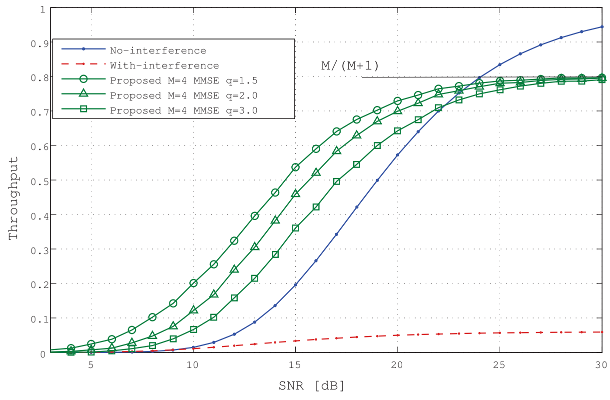

Fig. 6. Throughput performance with MMSE criterion ($M = 4$, $q = 1.5$, 2, 3)

case. It is also interesting that there is not so much difference in performance between the ZF and the MMSE criteria.

VI. Conclusion

In this paper, we have extended the conventional single-relay AFNC to multi-relay network with a chain topology. We have pointed out that the phenomenon of the data reflection happens if the conventional relaying protocol is directly applied to the multi-relay network. The proposed relaying algorithm can compensate the data reflection by using the locally available information. Moreover, we have proposed an AFNC scheme for two-relay network with inter-hop interference, where the fundamental idea is the effective utilization of interference by the FDE technique. Computer simulation results show that, the proposed method can even outperform the conventional method without interference in wide range of the SNR. Furthermore, it can achieve better performance as the interference signal power increases.

Future work will include extension of the proposed method to the multi-relay network with more than two-relay nodes and to frequency selective fading environments.

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