A Quadratic Phase Distribution of Local Oscillator Beams and Directional Characteristics in Optical Heterodyne Detection[*]

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Contents

Abstract

The effect of quadratic phase distribution of local oscillator beams on directional characteristics in optical heterodyne detection of uniform plane signal waves was discussed. The derivation was based on Corcoran and Sakuraba's analysis for a one-dimensional photocathode.

Results of the analysis show that the allowed angular tolerance on the directivity factors is increased by increasing the amount β of quadratic phase distributions.

1. Introduction

Directional characteristics of optical heterodyne detection were first described by Stroke[1] and have been examined by various authors from different points of view[2-10]. Recently, the wavefront curvature effect on detected power output was given by Sakuraba[11].

The purpose of this paper is to deduce the directivity factors in optical heterodyne detection when the local oscillator wave is assumed to have quadratic phase distribution.

2. Quadratic Phase Distribution of Local Oscillator Beams

A local oscillator wave of complex scalar amplitude A_1 and optical frequency ω_1 emanating from point P_{01} (x_{01}, y_{01}) and a signal wave of complex scalar amplitude A_2 and optical frequency ω_2 emanating from point P_{02} (x_{02}, y_{02}) are superimposed at the square-law detector which is centered at $x=y=0$ in the $z=0$ plane. It is assumed that the wavefronts are not too strongly divergent or convergent on the active length of the detector and $1/r$ dependence of amplitudes was neglected by the above approximation. It is also assumed that the two beams have the same optical modes and they are identically polarized. The difference-frequency photocurrent is then given by

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$$i(t)=e^{-j(\omega_1-\omega_2)t}\int_A \eta(P)E_1(P)E_2^*(P)dA \tag{1}$$

where

$$E_1(P)=A_1 \exp jk_1r_1 \tag{2}$$

$$E_2(P)=A_2 \exp jk_2r_2 \tag{3}$$

and k_1 and k_2 are the phase constants of the local oscillator and the signal waves, respectively, $\eta(P)$ is the effective quantum efficiency for light striking the point P and $E_n(P)$ is the field intensity at a point P on the photosurface. For simplicity, the photosurface is assumed to be a strip of width d in the x-direction and uniform in the z-direction. It is assumed that an equivalent point of origin of the nth wave is on the line at the angle θ_n. The distance from a point P_{on} to a point P on the detector is

$$r_n=\{(x-x_{on})^2+y_{on}^2\}^{1/2}=(r_n'-2x_{on}x+y_{on}^2)^{1/2}$$

where

$$r_n'^2=x_{on}^2+y_{on}^2 \ .$$

Since the linear dimension of the detector is small compared to r_n', r_n can be expanded by means of the binominal theorem. The result is

$$r_n \approx r_n'-x\sin\theta_n+\frac{x^2}{2r_n'}\cos\theta_n+\ldots \tag{4}$$

where

$$\theta_n=\tan^{-1}(x_{on}/y_{on}) \ . \tag{5}$$

Through use of Eqs. (2) and (5) the detector output current can be written as

$$i(t)=e^{-j(\omega_1-\omega_2)t}e^{j(k_1r_1'-k_2r_2')}A_1A_2^*\int_{-d/2}^{+d/2}\eta(x)e^{jf(x)}dx \tag{6}$$

where

$$f(x)=(-k_1\sin\theta_1+k_2\sin\theta_2)x+\left(\frac{k_1\cos^2\theta_1}{2r_1'}-\frac{k_2\cos^2\theta_2}{2r_2'}\right)x^2+\ldots \tag{7}$$

It is assumed that an equivalent point of origin of the local oscillator wave is on the line perpendicular to the detector and passes through the detector center. When the quadratic terms in Eq. (7) cannot be neglected and the higher order terms are neglected, the local oscillator wave has an exponential factor exp $j\beta x^2$, where $\beta=k_1/2r_1'$. This means a quadratic phase distribution of the local oscillator beam. Experiments with uncollimated laser beams can easily be performed in this region. Namely, when the detector surface is in the near-field of laser, or when a lens is used to direct the radiation on to it, the detector surface is illuminated by the local oscillator wave with quadratic phase distribution. Also, this corresponds to the effective quantum efficiency $\eta(x)=\exp j\beta x^2$ or a curved photosurface proportional to the value of x^2. When the signal wave is a plane wave, the quadratic and higher order terms can be neglected in Eq. (7). The expression for the output can, therefore, be written as

$$P=\frac{1}{2}R_{eq}|A_1A_2^*|^2\left|\int_{-d/2}^{+d/2}\eta(x)\exp\{j(k_2x\sin\theta_2+\beta x^2)\}dx\right|^2 \tag{8}$$

where R_{eq} is the equivalent resistance of the detector[2,12]. The similarity between the phase distribution effects in optical heterodyne detection and phase error effects of microwave antenna can be seen by an inspection of Eq. (8). In fact, if $\eta(x)$ is assumed to be the amplitude of the aperture field, Eq. (8) shows the effect of quadratic phase errors on the power pattern of the antenna system[13 14].

3. Effects of the Quadratic Phase Distribution on Directional Characteristics

When the effective quantum efficiency is uniform over the photosurface, $\eta(x)$

$=1$, the power output is given by

$$P=P_0|D(\theta)|^2 \tag{9}$$

$$P_0=(1/2)R_{eq}|A_1A_2^*|^2d^2 \tag{10}$$

$$|D(\theta)|^2=(1/2\delta)[\{C(a_1)+C(a_2)\}^2+\{S(a_1)+S(a_2)\}^2] \tag{11}$$

$$a_1=\beta\left(\frac{d}{2}+\frac{k_2\sin\theta_2}{2\beta}\right)^2, \qquad a_2=\beta\left(\frac{d}{2}-\frac{k_2\sin\theta_2}{2\beta}\right)^2 \tag{12}$$

$$\beta\equiv\delta d^2/\pi \tag{13}$$

where the notation of Fresnel integrals is used as

$$C(x)=\sqrt{\frac{2}{\pi}}\int_0^{\sqrt{x}}\cos t^2dt, \qquad S(x)=\sqrt{\frac{2}{\pi}}\int_0^{\sqrt{x}}\sin t^2td\ . \tag{14}$$

Plots of Eq. (11) with βd^2 as the parameter are presented in Fig. 1. The directivity factor of uniform plane waves, namely $\beta d^2=0$, is also shown in Fig. 1. As the amount β of the quadratic phase distribution of local oscillator beam increases, the value $|D(\theta)|$ of the main lobe becomes lower and side lobes become larger and finally the critical angular selectivity decreases.

In the case where $\eta(x)=\cos(\pi x/d)$, the detected power output is shown by

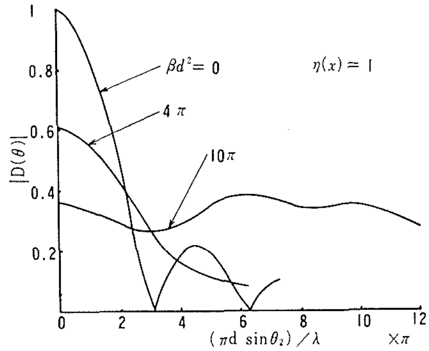

Fig. 1 The calculated value of $|D(\theta)|$ vs. $(\pi d\sin\theta_2)/\lambda$ with βd^2 as the parameter in the case where $\eta(x)=1$.

$$P=P_0|D(\theta)|^2 \tag{15}$$

$$|D(\theta)|^2=(1/8\delta)\{C_{12}^2+C_{34}^2+S_{12}^2+S_{34}^2+2(C_{12}C_{34}+S_{12}S_{34})(\cos\gamma_1\cos\gamma_2+\sin\gamma_1\sin\gamma_2)+2(S_{12}C_{34}+S_{34}C_{12})(\sin\gamma_1\cos\gamma_2-\cos\gamma_1\sin\gamma_2)\} \tag{16}$$

where

$$\left.\begin{array}{ll} C_{12}=C(b_1)+C(b_2), \quad C_{34}=C(b_3)+C(b_4), & S_{12}=S(b_1)+S(b_2), \quad S_{34}=S(b_3)+S(b_4) \\ \gamma_1=\{k_2\sin\theta_2+(\pi/d)\}^2/4\beta, & \gamma_2=\{k_2\sin\theta_2-(\pi/d)\}^2/4\beta \\ b_1=\left\{\frac{d}{2}+\frac{k_2\sin\theta_2+(\pi/d)}{2\beta}\right\}^2, & b_2=\left\{\frac{d}{2}+\frac{k_2\sin\theta_2-(\pi/d)}{2\beta}\right\}^2 \\ b_3=\left\{\frac{d}{2}-\frac{k_2\sin\theta_2+(\pi/d)}{2\beta}\right\}^2, & b_4=\left\{\frac{d}{2}-\frac{k_2\sin\theta_2-(\pi/d)}{2\beta}\right\}^2 \end{array}\right\} \tag{17}$$

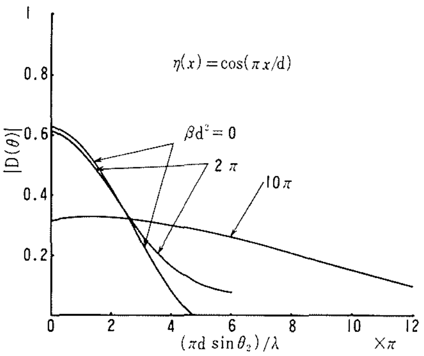

Fig. 2 The calculated value of $|D(\theta)|$ vs. $(\pi d\sin\theta_2)/\lambda$ with βd^2 as the parameter in the case where $\eta(x)=\cos(\pi x/d)$.

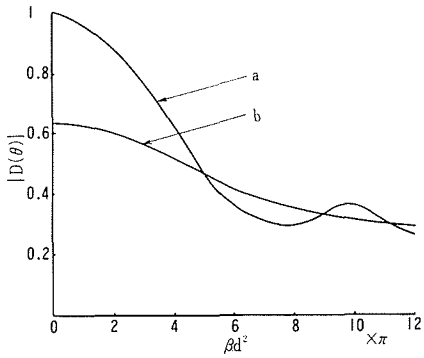

Fig. 3 Reduction in power output as the amount β of quadratic phase distribution of local oscillator wave increases: a) $\eta(x)=1$, b) $\eta(x)=\cos(\pi x/d)$. It was assumed that $\theta_1=\theta_2=0$.

Plots of Eq. (16) with βd^2 as the parameter are presented in Fig. 2. The directional pattern of cosine distribution of effective quantum efficiency is less sensitive to the angle than that of the uniform distribution of effective quantum distribution, as was expected. Values of $|D(\theta)|$ as function βd^2 for $\eta(x)=1$ and cos $(\pi x/d)$ are shown in Fig. 3. A study of these figures shows that the allowed angular tolerance on the directivity is increased but the detected power output is decreased by the technique which increases the amount β of quadratic phase distribution. The calculated data on the effects of the quadratic distribution in local oscillator beams is summarized in Table 1.

Table 1 Comparison of Performance Figure

		$\beta=0$	$\beta d^2=10\pi$
$\eta(x)=1$	Detected Power Output at $\theta_2=0$	1.0	0.35
	Full Width of Main Lobe	1.0	>3.0
$\eta(x)=\cos(\pi x/d)$	Detected Power Output at $\theta_2=0$	0.64	0.3
	Full Width of Main Lobe	~ 1.5	~ 3.0

4. Conclusions

The effect of quadratic phase distribution of lccal oscillator beame on directional characteristics in optical heterodyne detection of uniform plane signal waves were investigated. We can draw an interesting conclusion in which the allowed angular tolerance on the directivity is increased by a technique which increases the amount β of the quadratic phase distributions.

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