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On the Dynamic Response of a Vibro-Impact System to Random Force

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Abstract

A method of calculation was established for studying the dynamic response of a mechanical system with a play to arbitrary exciting forces. A computer simulation was carried out on the stationary response of a single degree-of-freedom vibro-impact system to ergodic stationary random input and a harmonic force superimposed by random input.

The following conclusions were obtained from the study. When the stationary random force acts on the system with a play, the power spectral density of the dynamic response is large in the vicinity of the resonance frequency of the vibro-impact system. The colliding velocities and the time intervals between two adjacent collisions become smaller, with the decrease of the play. When a harmonic force superimposed by random force acts on the system, the stationary fundamental and super/sub-impact vibrations are caused at some probabilities in the system. The contour maps made on these probabilities show that the stationary impact vibration of each type is caused at high probability in the system causing a corresponding impact vibration at high stability under the action of a harmonic force.

1. Introduction

In general, machines and mechanical structures assembled and constructed of many parts have some play (or clearance), even if the play may be small. These plays generate complicated impact vibrations in the mechanical systems and render the accuracy or performance to become inferior and also decreases the lifetime of machines. However, there are some machines such as pile drivers, vibrating screens, forging machines and impact dampers that positively utilize the energy of impact vibrations. Hence these problems have been dealt with for various purposes in various papers[1)~8)], however, the majority of which have involved only the impact vibration caused by harmonic or periodic forces.

In this paser, the dynamic property of a vibro-impact system with a play was studied by simulating the stationary response of a single degree-of-freedom system to stationary random input and harmonic input superimposed by random noise on a digital computer. Here the deformation of bodies produced by the collision and the colliding time intervals were all neglected, because they are usually small. It was also assumed that there is a proportional relation defined by the restitution coefficient

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between the velocities of the body before and after collisions.

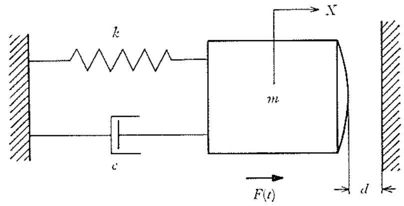

Fig. 1. A vibro-impact system

2. Fundamental equations

As shown in Fig. 1, a mass m supported by a linear spring k and a viscous damper c is set in a position of statical equilibrium, maintaining a distance d (a play) from the surface of a wall whose mass is infinitely large. When an exciting force $F(t)$ acts on the mass, the motion of equation of the mass is written as

$$m\ddot{X}+c\dot{X}+kX = F(t) \tag{1}$$

where X denotes the displacement of the mass measured from the statical position. When the mechanical system has a negative play, X is measured from the statical position of a free mass without a wall.

Eq. (1) can be written as

$$\ddot{x}+2\zeta\dot{x}+x = f(\tau) \tag{2}$$

using the dimensionless displacement $x=X/X_{st}(X_{st}=F_0/k)$, dimensionless force $f(\tau)=F(t)/F_0(F_0=\text{reference force})$, damping ratio $\zeta=c/2\sqrt{mk}\,(\zeta<1)$ and dimensionless time $\tau=\omega_0 t(\omega_0=\sqrt{k/m})$. The symbol $\cdot$ used in Eq. (2) denotes the derivative with respect to τ.

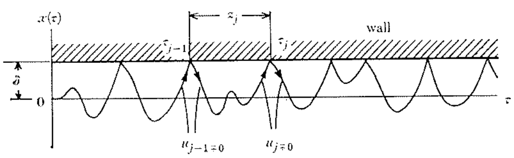

Fig. 2. Impact vibration of a mass

When the play is small or the exciting force attains a certain level, some impact vibrations occur in the mechanical system as shown in Fig. 2. If the mass which collides at time τ_{j-1} is simultaneously rebounded from the wall surface at a velocity U_{j-1+0}, the displacement after the collision is expressed by

$$\begin{aligned} x_j(\tau) = \frac{1}{\sqrt{1-\zeta^2}}\, e^{-\zeta(\tau-\tau_{j-1})} \Big[& \Big\{\delta-\eta(\tau_{j-1})\Big\} \\ & \times\Big\{\zeta \sin\sqrt{1-\zeta^2}\,(\tau-\tau_{j-1})+\sqrt{1-\zeta^2}\cos\sqrt{1-\zeta^2}\,(\tau-\tau_{j-1})\Big\} \\ & +\Big\{u_{j-1+0}-\dot{\eta}(\tau_{j-1})\Big\}\sin\sqrt{1-\zeta^2}\,(\tau-\tau_{j-1})\Big]+\eta(\tau) \\ & (\tau_{j-1}<\tau<\tau_j) \end{aligned} \tag{3}$$

where $\eta(\tau)$ is written as

$$\eta(\tau) = \int_{-\infty}^{\tau} f(\tau')\, g(\tau-\tau')\, d\tau' \tag{4}$$

using the unit impulse response of the system: $q(\tau)=(1-\zeta^2)^{-1/2} e^{-\zeta\tau} \sin\sqrt{1-\zeta^2}\,\tau$. In Eq. (3), $\delta(=d/X_{st})$ expresses dimensionless play and $u_{j-1}(=U_{j-1}/\omega_0 X_{st})$ denotes dimensionless velocity. When the mass collides again at τ_j, the displacement is

$$\delta = \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta z_j} \Big[\Big\{ \delta - \eta(\tau_{j-1}) \Big\} \Big\{ \zeta \sin \sqrt{1-\zeta^2}\, z_j + \sqrt{1-\zeta^2} \cos \sqrt{1-\zeta^2}\, z_j \Big\}$$
$$+ \Big\{ u_{j-1+0} - \dot{\eta}(\tau_{j-1}) \Big\} \sin \sqrt{1-\zeta^2}\, z_j \Big] + \eta(\tau_j) \tag{5}$$

and the colliding velocity is given by

$$u_{j-0} = \frac{-1}{\sqrt{1-\zeta^2}} e^{-\zeta z_j} \Big[\Big\{ \delta - \eta(\tau_{j-1}) \Big\} \sin \sqrt{1-\zeta^2}\, z_j$$
$$+ \Big\{ u_{j-1+0} - \dot{\eta}(\tau_{j-1}) \Big\} \Big\{ \zeta \sin \sqrt{1-\zeta^2}\, z_j - \sqrt{1-\zeta^2} \cos \sqrt{1-\zeta^2}\, z_j \Big\} \Big] + \dot{\eta}(\tau_j) \tag{6}$$

where $z_j = \tau_j - \tau_{j-1}$ is the time interval between two adjacent collisions. Considering that τ_{j-1} and u_{j-1+0} are already known, the colliding time τ_j is determined by obtaining the smallest positive root satisfying Eq. (5). Although Eq. (5) is a transcendental equation with respect to z_j, it is not difficult to calculate the value of z_j numerically by Newton's method. The colliding velocity u_{j-0} at τ_j is calculated by Eq. (6) and the velocity after the collision u_{j+0} is determined by the relation :

$$u_{j+0} = -\varepsilon u_{j-0} \qquad (0<\varepsilon<1) \tag{7}$$

where ε is the restitution coefficient between the mass and wall. Thus the motion of the mass colliding repeatedly can be determined numerically.

3. The stationary response of a vibro-impact system to random force

The method of calculation mentioned above makes a computer simulation possible on the dynamic response of the vibro-impact system to arbitrary exciting forces. Here the stationary response to an ergodic stationary random input is illustrated. The random input was made by a noise generator of the computer HIDAS 2000 and the motion of the mass was simulated on the digital computer FACOM 230-60.

Fig. 3 shows an example of (a) the stationary random input and (b) the dynamic

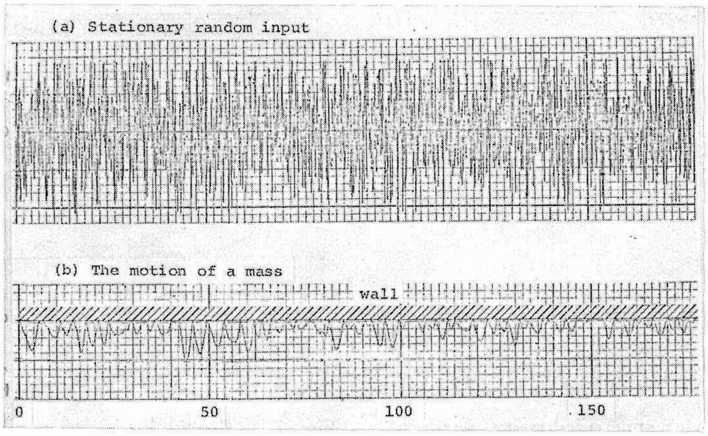

Fig. 3. An example of random input and the motion of a mass ($\zeta=0.05$, $\varepsilon=0.5$, $\delta=0$)

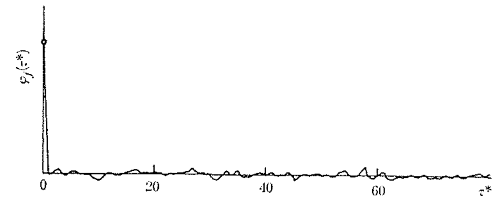

Fig. 4. Autocorrelation function of random input

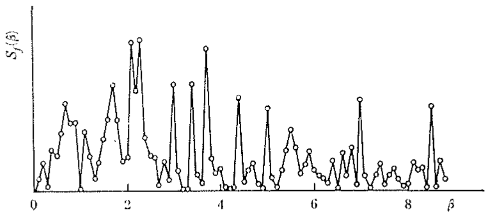

Fig. 5. Power spectral density of random input

response of the mass to this random input. It may be seen in this example that the motion of the mass which was at rest at $\tau=0$ attained a statistically stationary vibration rapidly after several collisions. Figs. 4 and 5 present the autocorrelation function

$$\varphi_f(\tau^*) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} f(\tau)\, f(\tau+\tau^*)\, d\tau \qquad (8)$$

and the power spectral density

$$S_f(\beta) = \int_{-\infty}^{\infty} \varphi_f(\tau^*)\, e^{-j\beta\tau^*}\, d\tau^* \qquad (\beta=\omega/\omega_0) \qquad (9)$$

of the input respectively. The correlation of the input is considerably small and the power is distributed over a fairly wide range of frequency, although the input is not a completely white noise.

Fig. 6 shows the power spectral density of the displacement $a(\tau)$ of the mass

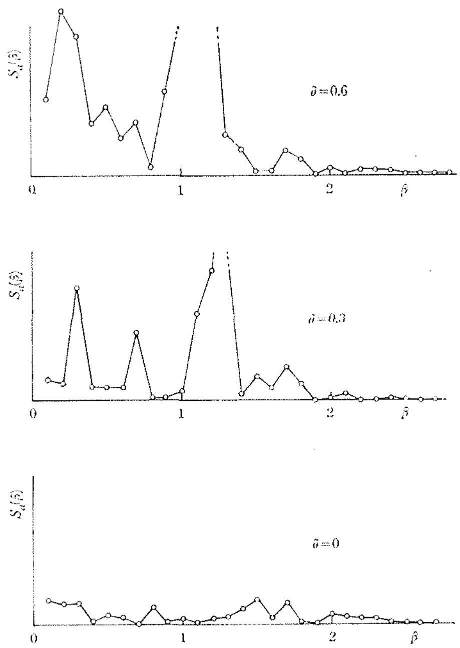

Fig. 6. Power spectral density of the displacement of a mass ($\zeta=0.05$, $\varepsilon=0.5$)

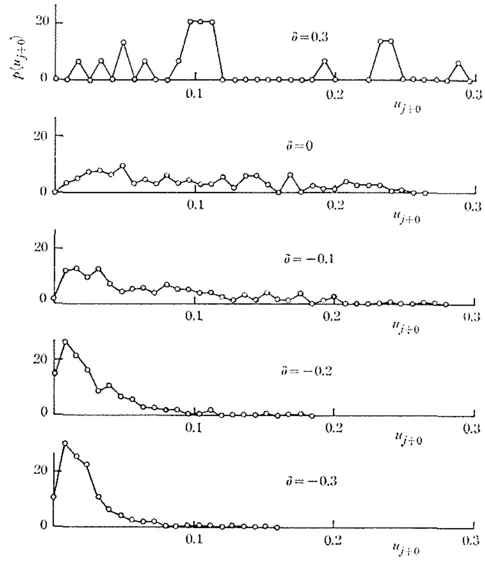

Fig. 7. Probability density of the velocity of a mass after collision ($\zeta=0.05$, $\varepsilon=0.5$)

measured from the wall surface. The spectral density is large in the vicinity of the resonance frequency of the vibro-impact system; however, it becomes small at a rapid rate in the range of a higher frequency. With the decrease of the play, the frequency having the largest spectral density increases and the level of the density becomes small. No remarkable peak values of the spectral density can be seen in the case of $\delta=0$.

Fig. 7 presents the probability density of the colliding velocities, from which it may be seen that the velocities become small with the decrease of the play and that this tendency is more remarkable in the system with a negative play. As seen in Fig. 8, the time intervals between two collisions show the same tendency with that of the colliding velocities. Fig. 9 shows the average values of the colliding velocities and the time intervals. With the increase of the play, these values also increase; however, when the play exceeds a critical value, no collisions occur and thus the colliding velocities become zero.

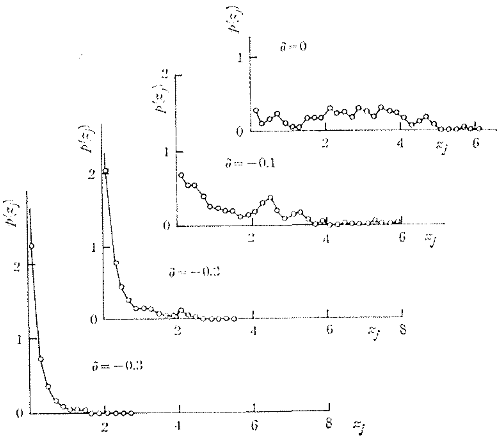

Fig. 8. Probability of the time intervals between adjacent collisions ($\zeta=0.05$, $\varepsilon=0.5$)

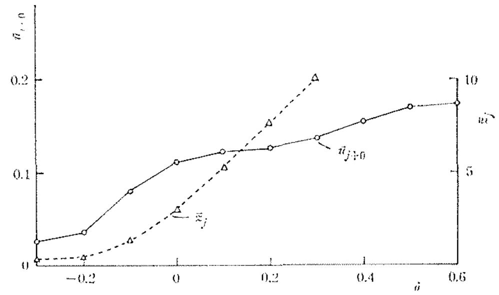

Fig. 9. Average values of the colliding velocities and the time intervals between adjacent collisions ($\zeta=0.05$, $\varepsilon=0.5$)

4. Periodic impact vibration caused by a harmonic force and the stability of motion

The stationary impact vibration caused by a harmonic force is studied, prior to the dynamic response to a harmonic force ($\sin\beta_f\tau$) superimposed by random force is discussed. When the mass collides m times ($m=1, 2, \cdots$) with the wall in the time interval $z=2n\pi/\beta_f$ ($n=1, 2, \cdots$) and an impact vibration occurs periodically at a regular time interval z, the motion is called the (m/n) super/sub-impact vibration. Here n is the number of cycles of the harmonic force acting during a period z. Applying Eqs. (5)~(7) to each collision caused during z, the following 2m equations are derived

$$\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta z_j}\Big[\Big\{\delta-q\sin\beta_f\Big(\tau_0+\sum_{p=1}^{j-1}z_p\Big)\Big\}\Big\{\zeta\sin\sqrt{1-\zeta^2}\,z_j+\sqrt{1-\zeta^2}\cos\sqrt{1-\zeta^2}\,z_j\Big\}$$
$$-\Big\{\varepsilon u_{j-1-0}+\beta_f q\cos\beta_f\Big(\tau_0+\sum_{p=1}^{j-1}z_p\Big)\Big\}\sin\sqrt{1-\zeta^2}\,z_j\Big]$$
$$=\delta-q\sin\beta_f\Big(\tau_0+\sum_{p=1}^{j}z_p\Big) \tag{10}$$

$$\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta z_j}\left[-\left\{\delta-q\sin\beta_f\left(\tau_0+\sum_{p=1}^{j-1}z_p\right)\right\}\sin\sqrt{1-\zeta^2}\,z_j\right.$$
$$\left.-\left\{\varepsilon u_{j-1-0}+\beta_f q\cos\beta_f\left(\tau_0+\sum_{p=1}^{j-1}z_p\right)\right\}\left\{\sqrt{1-\zeta^2}\cos\sqrt{1-\zeta^2}\,z_j-\zeta\sin\sqrt{1-\zeta^2}\,z_j\right\}\right]$$
$$= u_{j-0}-\beta_f q\cos\beta_f\left(\tau_0+\sum_{p=1}^{j}z_p\right) \qquad (j=1,\,2,\,\cdots,\,m) \tag{11}$$

where $q=1/\sqrt{(1-\beta_f^2)^2+(2\zeta\beta_f)^2}$. Eqs. (10) and (11) contain also the 2m unknown quantities:

$$\tau_0,\ z_1,\ z_2,\ \cdots,\ z_{m-1},\qquad \left(\sum_{j=1}^{m}z_j=2n\,\pi/\beta_f\right);\ u_{-0},\ u_{1-0},\ \cdots,\ u_{m-1-0} \tag{12}$$

When the positive values of these quantities satisfying Eqs. (10) and (11) are obtained, the (m/n) impact vibration is determined. For the impact vibration to be caused for practical purposes, in addition to Eqs. (10) and (11), the conditions demand that the mass should not pass through the wall surface. Thus

$$\xi_j(\tau)\geq\delta \qquad (j=1,\,2,\,\cdots,\,m) \tag{13}$$

must be satisfied.

When the small disturbances $\Delta\tau_{j-1}^{(q)}$ and $\Delta u_{j-1-0}^{(q)}$ are caused on the $j-1$th colliding time $\tau_{j-1}^{(q)}$ and the velocity $u_{j-1-0}^{(q)}$ of the qth periodic motion, the small disturbances

$$\begin{pmatrix}\Delta\tau_j\\ \Delta u_{j-1}\end{pmatrix}^{(q)} = \boldsymbol{T}_j^{(q)}(T_{ik}^{(j)})\begin{pmatrix}\Delta\tau_{j-1}\\ \Delta u_{j-1-0}\end{pmatrix}^{(q)} \tag{14}$$

are caused on the jth collision. $\boldsymbol{T}_j^{(q)}$ is a 2×2 matrix whose elements $T_{ik}^{(j)}$ are

$$\left.\begin{aligned}
T_{11}^{(j)} &= -\frac{1}{\sqrt{1-\zeta^2}}\frac{e^{-\zeta z_j}}{u_{j-0}}\Big\{\theta_{j-1}\sin\sqrt{1-\zeta^2}\,z_j\\
&\qquad+\varepsilon u_{j-1-0}(\sqrt{1-\zeta^2}\cos\sqrt{1-\zeta^2}\,z_j-\zeta\sin\sqrt{1-\zeta^2}\,z_j)\Big\}\\
T_{12}^{(j)} &= \frac{1}{\sqrt{1-\zeta^2}}\frac{e^{-\zeta z_j}}{u_{j-0}}\,\varepsilon\sin\sqrt{1-\zeta^2}\,z_j\\
T_{21}^{(j)} &= \frac{1}{\sqrt{1-\zeta^2}}\frac{e^{-\zeta z_j}}{u_{j-0}}\Big[\theta_{j-1}\Big\{\theta_j\sin\sqrt{1-\zeta^2}\,z_j\\
&\qquad+u_{j-0}(\sqrt{1-\zeta^2}\cos\sqrt{1-\zeta^2}\,z_j+\zeta\sin\sqrt{1-\zeta^2}\,z_j)\Big\}\\
&\qquad+\varepsilon u_{j-1-0}\Big\{\theta_j(\sqrt{1-\zeta^2}\cos\sqrt{1-\zeta_j}\,z_j-\zeta\sin\sqrt{1-\zeta^2}\,z_j)\\
&\qquad\qquad-u_{j-0}\sin\sqrt{1-\zeta^2}\,z_j\Big\}\Big]\\
T_{22}^{(j)} &= -\frac{1}{\sqrt{1-\zeta^2}}\frac{e^{-\zeta z_j}}{u_{j-0}}\,\varepsilon\Big\{\theta_j\sin\sqrt{1-\zeta^2}\,z_j\\
&\qquad+u_{j-0}(\sqrt{1-\zeta^2}\cos\sqrt{1-\zeta^2}\,z_j+\zeta\sin\sqrt{1-\zeta^2}\,z_j)\Big\}
\end{aligned}\right\} \tag{15}$$

where

$$\theta_j = \delta-\sin\left\{\beta_f\left(\tau_0+\sum_{p=1}^{j}z_p+\tan^{-1}\frac{2\zeta\beta_f}{1-\beta_f^2}\right)\right\} \tag{16}$$

After m collisions are caused during a period of vibration, the resulting disturbances will be

$$\begin{pmatrix} \Delta\tau_m \\ \Delta u_{m-0} \end{pmatrix}^{(q)} = \boldsymbol{T}^{(q)} \begin{pmatrix} \Delta\tau_0 \\ \Delta u_{-0} \end{pmatrix}^{(q)} \tag{17}$$

$$\boldsymbol{T}^{(q)} = \boldsymbol{T}_m^{(q)} \boldsymbol{T}_{m-1}^{(q)} \cdots \boldsymbol{T}_1^{(q)} \tag{18}$$

Taking the small disturbances as $\Delta\tau_j^{(q)} = \bar{T}\lambda^{q+j}$, $\Delta u_{j-0}^{(q)} = \bar{U}\lambda^{q+j}$ and eliminating $\bar{T}$ and $\bar{U}$ from the equations obtained by substituting $\Delta\tau_j^{(q)}$ and $\Delta u_{j-1}^{(q)}$ in Eq. (17), the characteristic equation on the stability of the periodic motion is written in the form

$$\begin{vmatrix} T_{11}-\lambda^m & T_{12} \\ T_{21} & T_{22}-\lambda^m \end{vmatrix} = 0 \tag{19}$$

where T_{ij} are the elements of the matrix $\boldsymbol{T}$. Expansion of Eq. (19) gives the quadratic equation with respect to λ^m

$$\lambda^{2m} - b_m e^{-\zeta z} \varepsilon^m \lambda^m + e^{-2\zeta z} \varepsilon^{2m} = 0 \tag{20}$$

where

$$\left.\begin{aligned} b_1 &= \frac{1}{\sqrt{1-\zeta^2}}\left\{2\sqrt{1-\zeta^2}\cos\sqrt{1-\zeta^2}\,z + \left(1+\frac{1}{\varepsilon}\right)\frac{\theta_0}{u_{-0}}\sin\sqrt{1-\zeta^2}\,z\right\} \\ b_2 &= \frac{1}{\sqrt{1-\zeta^2}}\left\{2\sqrt{1-\zeta^2}\cos\sqrt{1-\zeta^2}\,z + \left(1+\frac{1}{\varepsilon}\right)\left(\frac{\theta_0}{u_{-0}}+\frac{\theta_1}{u_{1-0}}\right)\sin\sqrt{1-\zeta^2}\,z \right. \\ &\quad \left. + \frac{1}{\sqrt{1-\zeta^2}}\left(1+\frac{1}{\varepsilon}\right)^2 \frac{\theta_0}{u_{-0}}\frac{\theta_1}{u_{1-0}}\sin\sqrt{1-\zeta^2}\,z_1 \sin\sqrt{1-\zeta^2}\,z_2\right\} \\ &\cdots\cdots\cdots\cdots\cdots\cdots \end{aligned}\right\} \tag{21}$$

To have the small disturbances decrease gradually with every collision, the value of λ must be

$$|\lambda| < 1 \tag{22}$$

and in this case the motion of the vibro-impact system is dynamically stable. When the values of $|\lambda|$ are small, the disturbances decrease at a rapid rate. In contrast, when the values of $|\lambda|$ approach unity, the disturbances do not easily die out. For $|\lambda|<1$, the condition

$$|b_m|\varepsilon^m e^{-\zeta z} < 1 + (\varepsilon^m e^{-\zeta z})^2 \tag{23}$$

must be satisfied.

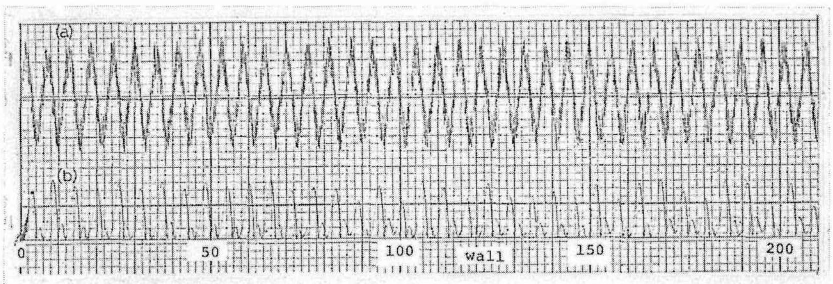

Fig. 10. An example of a harmonic force superimposed by random input and the motion of a mass ($\zeta=0.05$, $\varepsilon=0.5$, $\delta=0$; $f(\tau)=\sin(1.1\,\tau)+r(\tau)$, $rms_r=0.29$)

5. The stationary response to a harmonic force superimposed by random force

Fig. 10 shows an example of the dynamic behavior of the mass to a harmonic input (sin $\beta_f \tau$) superimposed by random noise ($rms_f = 0.29$). The power spectral density of the input is concentrated in the vicinity of the exciting frequency $\beta_f = 1.1$ of the harmonic force as seen in Fig. 11. However, as shown in Fig. 12, the power of the displacement $a(\tau)$ of the mass is distributed over a wider range than that of the input and has a tendency to move to a lower frequency with the decrease of the play.

Fig. 13 show the probability density of the ratio of the time interval between adjacent collisions z_j to the period z. With the increase of the play, a large probability density appears at $z_j/z = 1$. This means that the fundamental (1/1) impact vibration is easily caused in the vibro-impact system with a large play. When the play is small, two symmetric curves with respect to $z_j/z = 1/2$ (chain line in Fig. 13) appear in the range of $z_j/z < 1$. This suggests the existence of the (2/1) super-impact vibration which is caused when the mass collides twice during the action of a one cycle harmonic force. In contrast, the impact vibrations appearing at $z_j/z = 2, 3, \cdots$, although they are not seen in Fig. 13, mean the (1/2, 1/3, $\cdots$) sub-impact vibrations caused in the system whose mass collides one time during the action of two, three, $\cdots$ cycle harmonic force.

As shown in Fig. 14, the colliding velocities are distributed in slightly narrow range for the system with a large play in which the fundamental impact vibration are apt to occur. In contrast, the distribution of the velocities becomes wider in the system with a small play in which the sub-impact vibration occurs.

The curves on the $1/\beta - \delta$ planes shown in Fig. 15 present the contour lines on the probability at which the impact vibrations of three types mentioned above are caused in the vibro-impact system under the action of a harmonic force superimposed by random force. The numbers written on each curve express the value

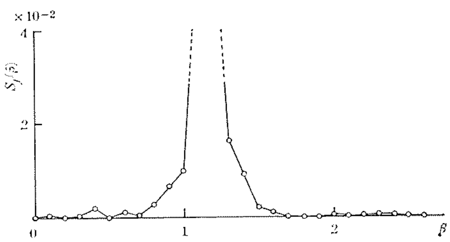

Fig. 11. Power spectral density of a harmonic force superimposed by random force ($f(\tau) = \sin. (1.1\,\tau) + r(\tau)$, $rms_r = 0.29$)

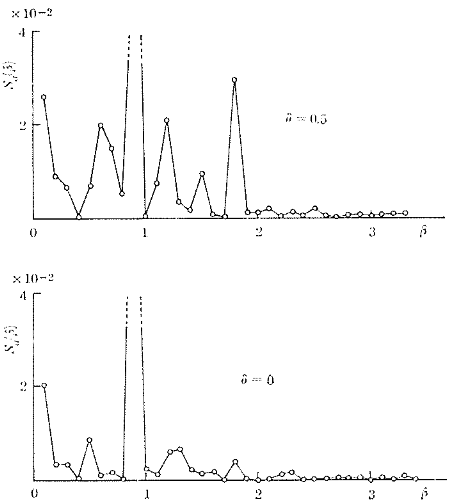

Fig. 12. Power spectral density of the displacement of a mass ($\zeta = 0.05$, $\varepsilon = 0.5$; $f(\tau) = \sin(\tau/1.1) + r(\tau)$, $rms_r = 0.29$)

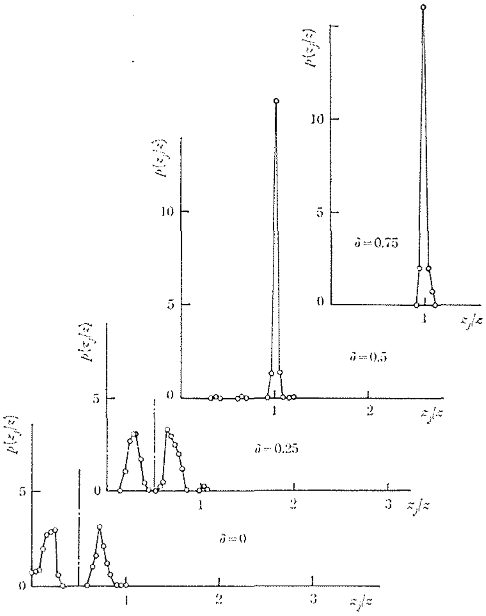

Fig. 13. Probability density of the time intervals between adjacent collisions ($\zeta=0.05$, $\varepsilon=0.5$; $f(\tau)=\sin(\tau/1.1)+r(\tau)$, $rms_r=0.29$)

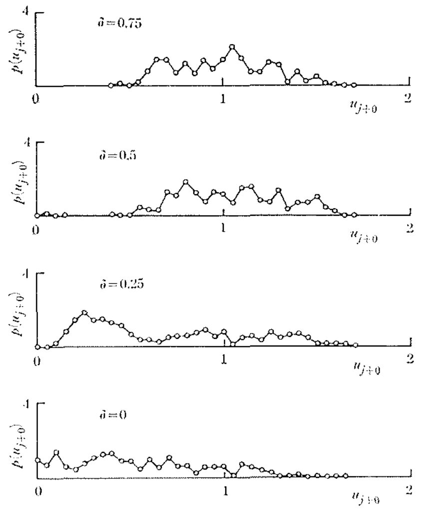

Fig. 14. Probability density of the velocities of a mass after collision ($\zeta=0.05$, $\varepsilon=0.5$; $f(\tau)=\sin(\tau/1.1)+r(\tau)$, $rms_r=0.29$)

of the probability as a percentage. By superimposing the three contour maps shown in Fig. 15, it is found that the dense contour lines overlap in small domains where the impact vibrations of two types occur simultaneously.

The curves shown in Fig. 16 present the $1/\beta$–δ domain where the periodic impact vibrations are caused by a harmonic force alone ($\sin\beta_f\tau$). The stable periodic impact vibration can be caused only in the system having the play and the frequency within the bold lines. In the domain outside the bold lines, no periodic impact vibrations can be caused or the vibrations are unstable. In fact, there are no solutions satisfying Eqs. (10), (11) and (13) in the domains D, V, X and the conditions on the stability (Eq. (22)) are not satisfied in the domain U. The thin lines drawn inside the domains causing the stable vibration show the contour lines on the stability of the motion. The numbers written on each line indicate the degree of stability defined by

$$S = 100(1-\lambda) \quad (\%) \tag{24}$$

When comparing Figs. 15 and 16, it is found that the domains of the corresponding impact vibration are similar to each other and hence the stationary impact vibration is caused at a high probability in the system causing the periodic vibration at high stability under the action of a harmonic forces.

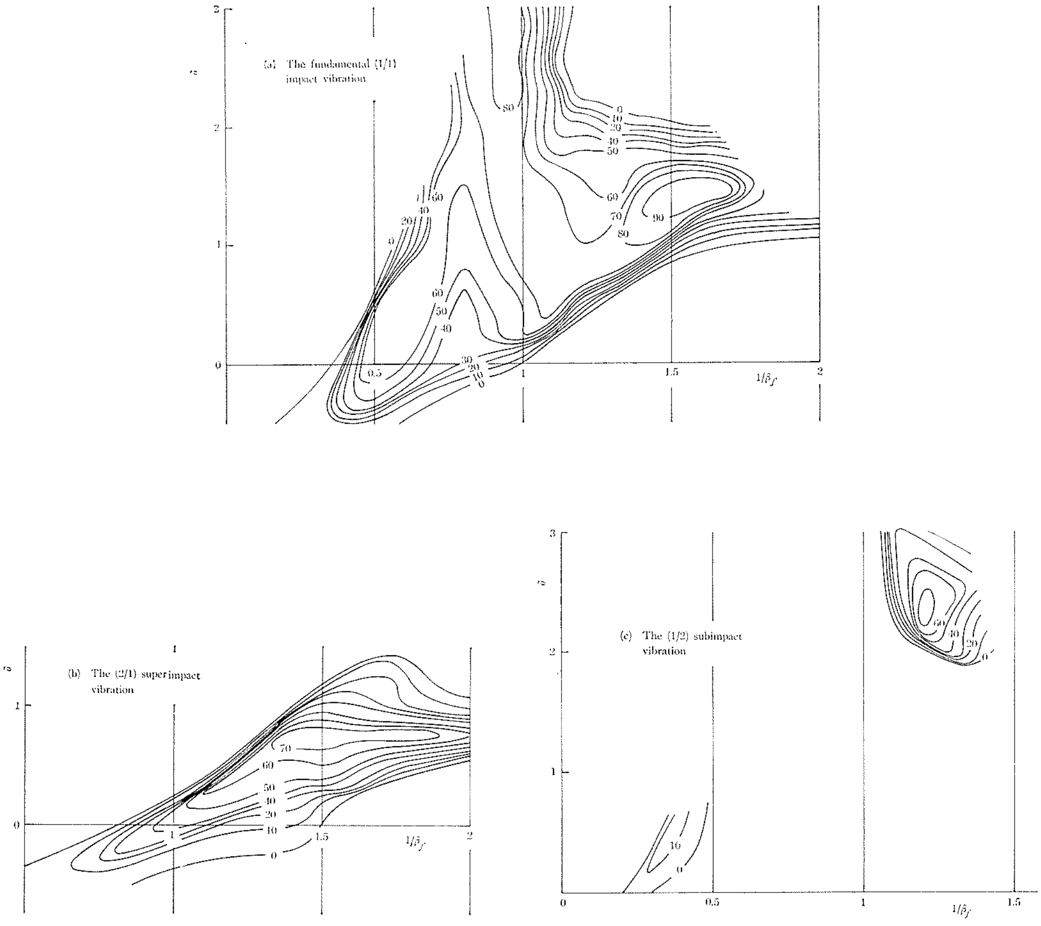

Fig. 15. Contour maps on the probability of the impact vibrations ($\xi=0.05$, $\varepsilon=0.5$; $f(\tau)=\sin\beta_f\tau+r(\tau)$, $rms_r=0.29$)

6. Conclusions

A computer simulation was carried out on the dynamic response of a single degree-of-freedom system with a play to random input and a harmonic input superimposed by random noise. From the results, the following conclusions were obtained.

(1) The power spectral density of the dynamic response to stationary random force is large in the vicinity of the resonance frequency of the vibro-impact system when a play is large. With the decrease of the play, the level of the density become small.

(2) The colliding velocities become small with the decrease of the play. This tendency is remarkable in the system with a negative play.

(3) The time intervals between adjacent collisions show the same tendency with that of the colliding velocities.

(4) When a harmonic force superimposed by random force acts on a vibro-

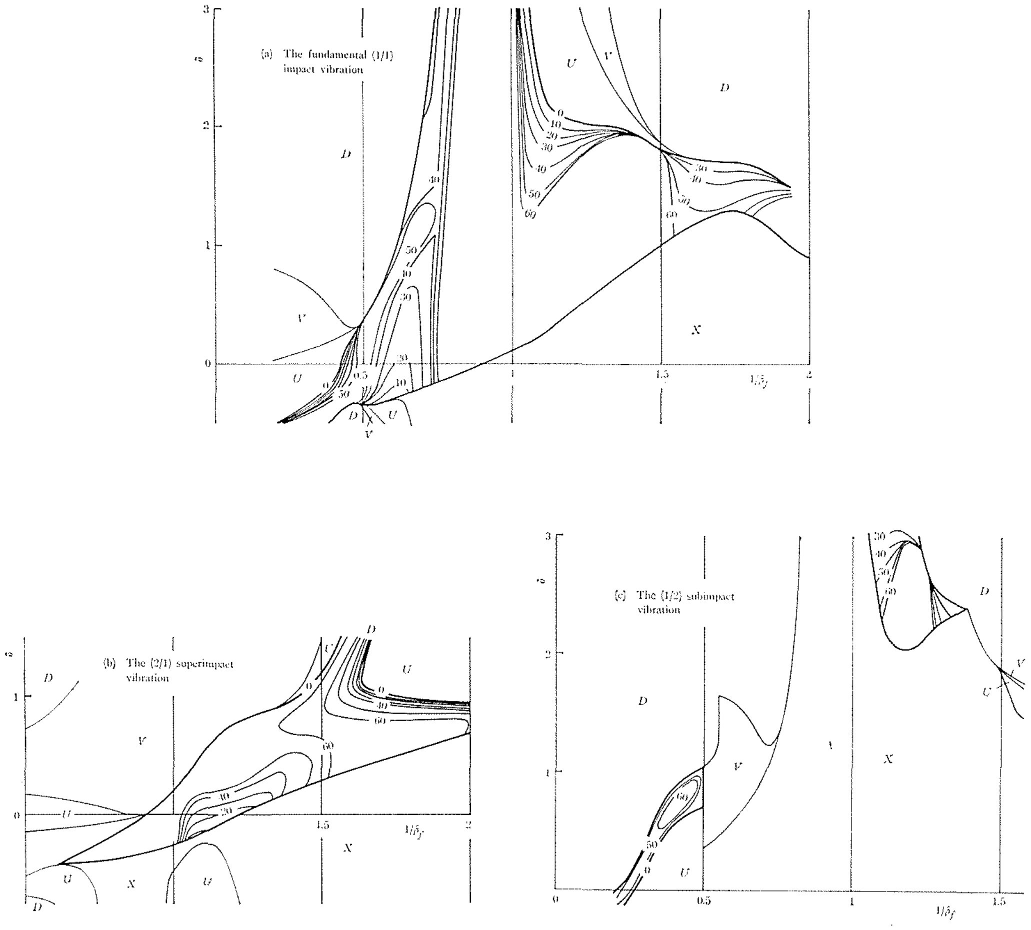

Fig. 16. Parameter domains causing the stable impact vibrations under the action of a harmonic force ($\zeta=0.05$, $\varepsilon=0.5$; $f(\tau)=\sin\beta_f\tau$)

impact system, the fundamental impact vibration is caused in the system with a large play. With the decrease of the play, the super-impact vibration appears and the distribution of the colliding velocities becomes considerably wide.

(5) It is found from the contour maps on the probability at which the periodic motions occur that the stationary impact vibrations are caused at high probability in the system causing periodic vibrations at high stability under the action of a harmonic force.

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