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Fourier Imaging of Rectangular Grating in Partially Coherent Light

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Abstract

Fourier imaging of a rectangu.ly transparent grating under partially coherent illumination was investigated by putting stress on comparison with the sinusoidally transparent grating. It is apparent that the decrease of spatial coherence leads to a distinguishable deformation of Fourier images and also a suppression of modulation depth of the axial intensity in other words, lower spatial frequency components contribute to Fourier imaging under illumination of poor spatial coherence.

I. Introduction

Image formation of a periodic object without essential optical components is known as Fourier imaging or self-imaging. While the Fourier imaging in the case of completely coherent illumination has been studied in considerable details[1-4], partially coherent cases have received but little attention in spite of its importance in applications of optical measurements. Fujiwara[5] treated Fourier imaging of a sinusoidally transparent grating (STG), assuming a spatial stationarity of the coherence function and using paraxial approximation, and further Grousson and Mallick[6] investigated the effect of coherence on the intensity distribution of the grating at closely adjacent fields.

The present paper is concerned with Fourier imaging of a rectangularly transparent grating (RTG) under partially coherent illumination by confining the objective to the problem of comparison of Fourier imaging of RTG with that of STG. The same assumption and approximation as used in the reference[5] are employed.

II. Result and Discussion

Fig. 1 illustrates tho geometry used for analysis of Fourier imaging. We assume that the transparent grating (Ronchi-typed grating) has a period d with a contrast in amplitude C. Let the amplitude transmittance of the grating be represented by

$$\frac{1}{2}\left\{1+\frac{C}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{2n-1}\cos\left[\frac{2\pi(2n-1)\xi}{d}\right]\right\}$$

where the grating is set in the $\xi-\eta$ plane and its rules are in parallel with the η-axis. The incoherent sourc withe a radius a lying in the front focal plane of the collimator radiates quasi-monochromatic light of a mean wavelength λ and

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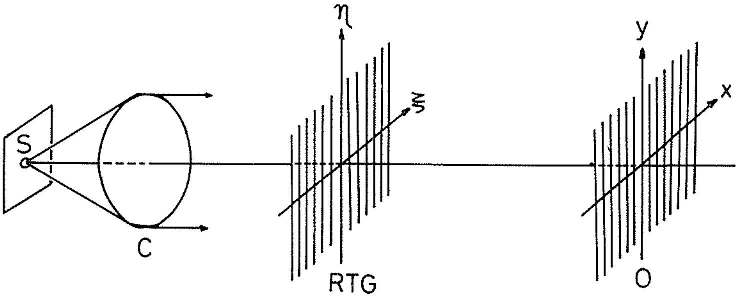

Fig. 1 Geometry used for the analysis of Fourier imaging. S: light source of a radius a, C: collimator with focal length f, RTG: rectangularly transparent grating of a period d, and O: observation plane at a distance from the RTG.

illuminates the grating uniformly. Then, on the grating plane the degree of spatial coherence is given, according to the van Cittert-Zernike theorem[7], by

$$\gamma(l)=\frac{2J_1\left(\frac{2\pi al}{\lambda f}\right)}{\frac{2\pi al}{\lambda f}}$$

where J_1 denotes a Bessel function of the first order, the argument l is a separation of two points over the object plane and f is the focal length of the collimator. Using the generalized formula obtained in the reference[5], we have the intensity distribution at a distance z from the grating;

$$\begin{aligned}I(x)=&1+\frac{1}{2}\sum_{n=1}^{\infty}C_{2n-1}^2+\frac{1}{2}\sum_{n=1}^{\infty}C_{2n-1}^2\gamma[Nd2(2n-1)]\cos\left[\frac{4\pi(2n-1)x}{d}\right]\\&+2\sum_{n=1}^{\infty}C_{2n-1}\cos[\pi N(2n-1)^2]\gamma[Nd(2n-1)]\cos\left[\frac{2\pi(2n-1)x}{d}\right]\\&+\sum_{n>m}^{\infty}\sum_{m=1}^{\infty}C_{2n-1}C_{2m-1}\cos\{\pi N[(2n-1)^2-(2m-1)^2]\}\\&\times\left\{\gamma[Nd2(n+m-1)]\cos\left[\frac{4\pi(n+m-1)x}{d}\right]+\gamma[Nd2(n-m)]\cos\left[\frac{4\pi(n-m)x}{d}\right]\right\}\end{aligned}\tag{1}$$

where the constant factor is assumed to be unity, N is equal to $\lambda z/d^2$, and $C_{2n-1}=4C(-1)^n/\pi(2n-1)$.

For N taking integers in the coherent limit, Fourier images which are produced at discrete distances, $z=Nd^2/\lambda$, have the same forms as the original object. At discrete distances we have $\cos\{\pi N(2n-1)^2\}=\pm1$ and $\cos\{4\pi N(n^2-m^2-n+m)\}=1$ for arbitrary integers n, m, so that we have the intensity distribution of Fourier images under partially coherent illumination;

$$\begin{aligned}I(x)=&1+\frac{1}{2}\sum_{n=1}^{\infty}C_{2n-1}^2+\frac{1}{2}\sum_{n=1}^{\infty}C_{2n-1}^2\gamma[2Nd(2n-1)]\cos\left[\frac{4\pi(2n-1)x}{d}\right]\\&+2\sum_{n=1}^{\infty}C_{2n-1}\gamma[Nd(2n-1)]\cos\left[\frac{2\pi(2n-1)x}{d}\right]\\&+\sum_{n>m}^{\infty}\sum_{m=1}^{\infty}C_{2n-1}C_{2m-1}\left\{\gamma[2Nd(n+m-1)]\cos\left[\frac{4\pi(n+m-1)x}{d}\right]\right.\\&\left.+\gamma[2Nd(n-m)]\cos\left[\frac{4\pi(n-m)x}{d}\right]\right\}\end{aligned}\tag{2}$$

For further discussion, we chose the same values of parameters as used in the reference[5], that is, $f=500$ mm, $C=0.6$ and $a=0.04$ mm, 0.08 mm and 0.16 mm. In order to illustrate the influence of spatial coherence upon the Fourier image, the numerical results of Eq. (2) are shown in Fig. 2. Since, in the argument of the

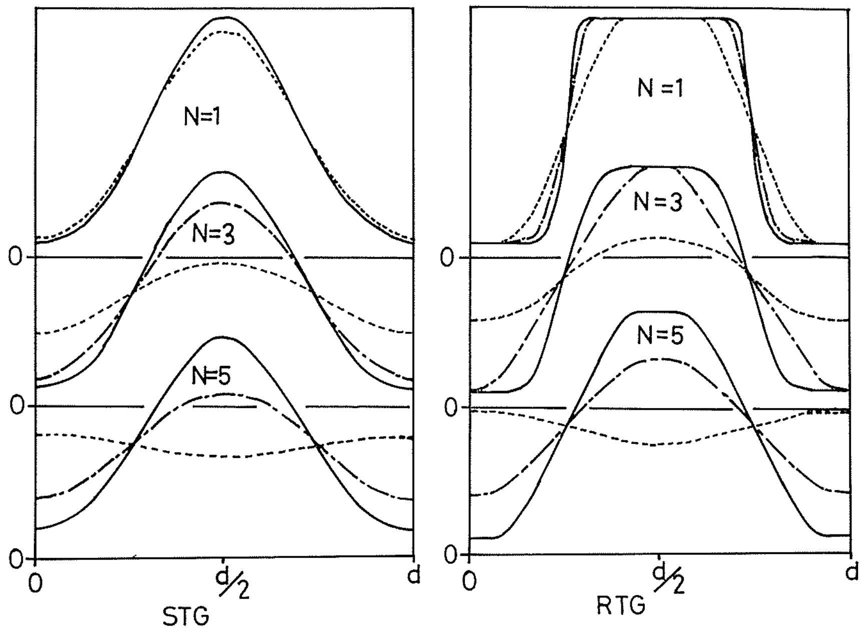

Fig. 2 Intensity distribution of Fourier images of the STG and RTG in the case of odd N. Curves of solid line (——), chain line (-·-·-) and dotted line (······) correspond to the coherence conditions of a=0.04 mm, 0.08 mm and 0.16 mm, respectivery. The notations of these curves are common to figs. 3 and 4.

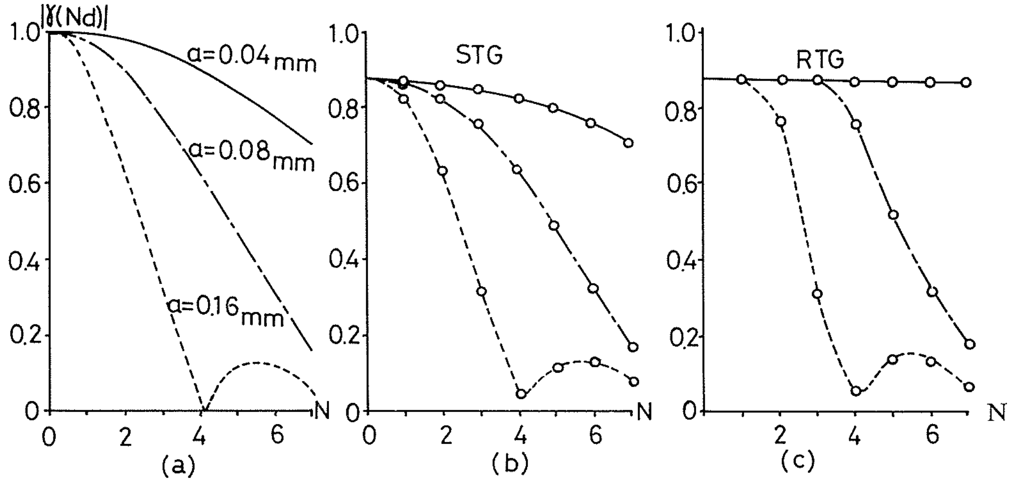

Fig. 3 Strong dependence of the degree of spatial coherence on the contrast of Fourier images of the STG and RTG: (a) the degree of spatial coherence, (b) the contrast of Fourier images of the STG and (c) the contrast of Fourier images of the RTG.

degree of spatial coherence, the increase in the order of Fourier series and N causes the degree of spatial coherence to decrease, the contribution of high orders of the diffraction to the intensity distribution of Fourier images will be destroyed. Thus, for the case of partially coherent illumination, as is shown in this figure, the intensity distribution of Fourier images deforms and becomes similar to that of STG as the integer N becomes large. At the same time Gibbs phenomenon which may occur in the Fourier images of RTG corresponding to the edges of the rule is suppressed. This phenomenon is more remarkable in the case of coherent illumination and a smaller N. There is still an important fact: when the degree of spatial coherence changes the signs, the Fourier images shift laterally by an amount of half the period $d/2$. We should pay attention to this shifting property to apply Fourier images to optical measurements.

The contrast of the intensity distribution of Fourier images of RTG is shown in Fig. 3. It is apparent that the contrast storongly depends upon the degree of spatial coherence. For example, the contrast is almost zero for $a=0.16$ mm and $N=4$ becausc $\gamma(Nd)$ nearly equals zero in this case. A comparison between the contrast of Fourier images of RTG and STG indicates that the contrast of the former decreases more slowly according as N increases and is somewhat higher than the contrast of the latter.

We examined the axial intensity distribution along the directoin of $x=0$ and $x=d/2$ which is represented by

$$I=1+\frac{1}{2}\sum_{n=1}^{\infty}C_{2n-1}^{2}+\frac{1}{2}\sum_{n=1}^{\infty}C_{2n-1}^{2}\gamma[2Nd(2n-1)]\pm 2\sum_{n=1}^{\infty}C_{2n-1}\cos[\pi N(2n-1)^{2}]\gamma[Nd(2n-1)]$$
$$+\sum_{n>m}^{\infty}\sum_{m=1}^{\infty}C_{2n-1}\cos[4\pi N(n^{2}-m^{2}-n+m)]\{\gamma[2Nd(n-m-1)]+\gamma[2Nd(n-m)]\} \quad (3)$$

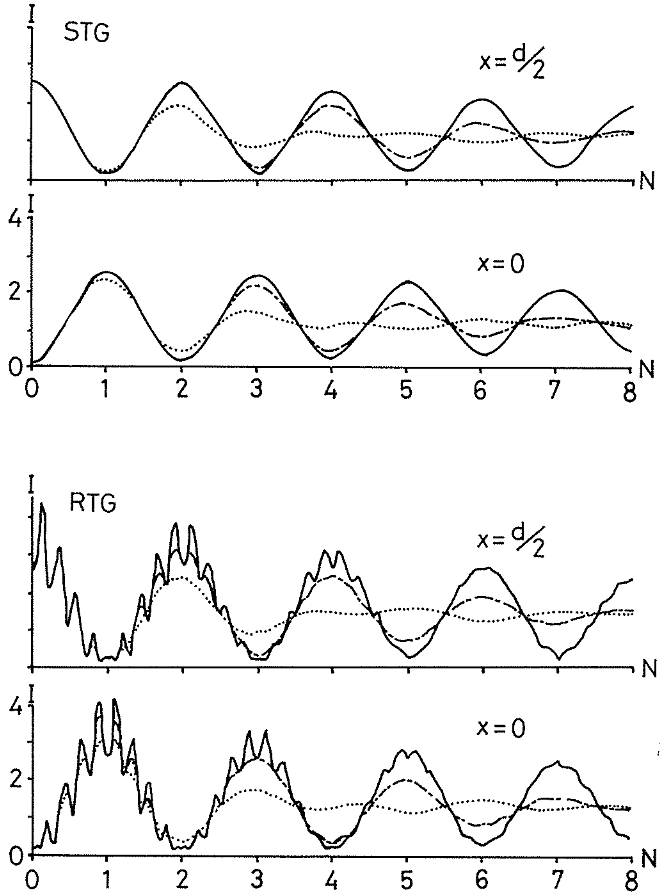

Fig. 4 Axial intensity distribution along the direction of $x=d/2$ and $x=0$.

The numerical result of Eq. (3) is shown in Fig. 4. As the degree of spatial coherence decreases, the slowly damped oscillation becomes remarkable and the positions where the axial intensity takes extreme values shift from the positions where the coherent Fourier images are constructed. Fig. 4 indicates that, for small N, the axial intensity is modulated with a period nearly equal to $1/4$ while it does not appear in the STG. This modulation phenomenon results from the change of signs of terms containing high orders of Fourier series in Eq. (3). However, since the higher orders of Fourier series and large N reduce the degree of spatial coherence which is a coefficient of thc cosine terms, the modulation will be suppressed.

III. Conclusion

We have investigated the effects of spatial coherence on the Fourier imaging of RTG. It is concluded that the decrease of spatial coherence leads to a distinguishable deformation of Fourier images and suppresses the rapid oscillation of the axial intensity, in other words, lower spatial-frequency components contribute mainly to the formation of the Fourier image as the degree of spatial coherence decreases. Hence, we should pay close attention to the examination of the spatial coherence of light in the experiment of Fourier imaging. The RTG rather than the STG is often used in the field of Fourier imaging because of the facility of construction, but the analysis of Fourier-imaging characteristics of the RTG under partially coherent illumination is more complex than that of STG. The present study facilitates the treatment which will serve for investigating the Fourier imaging, especially, that of RTG.

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