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Identification of Multi-Input/Multi-Output Systems

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Abstract

A new method is presented for identifying the controllable and observable part of multi-input/multi-output systems from measurements of the input-output signals contaminated with noises. The pulse transfer function matrix is estimated by using a vector input/output equation of an autoregressive/moving-average type. The minimal realization of the system is given by the Ho-Kalman's algorithm. The Markov parameters are used to determine the degree of the minimal polynomial of the system matrix and the minimal order of the system.

I. Introduction

In recent years, a considerable amount of effort has been made in the field of identifying multi-input/multi-output systems from input/output data. Especially, papers attempting to obtain a minimal order state-space model are of special interest. Gopinath[1] and Budin[2] proposed to use a selector matrix for the realization of systems. Mayne[3], Lobbia and Saridis[4], Chow[5], Guidorzi[6], and Tse and Weinert[7] used canonical forms for the structure determination and parameter estimation. Furuta and Ha[8] proposed a ε-minimal realization. However, it seems that, in these methods, the efficiency of the order determination of the system is greatly degraded because of input/output data contaminated with noises.

In this paper, a new method is presented for obtaining a minimal order state-space model of multi-input/multi-output systems from the noisy input/output data. A pulse transfer function matrix of minimal degree is estimated. Then a minimal order state-space model is given by using the Ho-Kalman's realization algorithm[9]. Special emphasis is laid on using the Markov parameters for the order determination.

The organization of this paper is as follows. In Section II, an input/output relation is derived. Sections III and IV give the identification procedure. Simulation results are presented in Section V.

II. Input/Output Relation

Let us consider an r-input p-output steady-state linear discrete-time system whose controllable and observable part is represented by

$$x_{t+1}=Ax_t+Bu_t \tag{1}$$

$$y_t=Cx_t \tag{2}$$

where x_t is an n-vector and A, B and C are constant matrices of size $n\times n$, $n\times$

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r and $p \times n$, respectively. From the controllability and observability of this system, it follows that

$$\mathrm{rank}[B,\ AB,\ \ldots,\ A^{n-1}B]=n \tag{3}$$
$$\mathrm{rank}[C^T,\ A^TC^T,\ \ldots,\ (A^T)^{n-1}C^T]=n \tag{4}$$

Let the minimal polynomial of the system matrix A be

$$f(z)=z^m+a_1z^{m-1}+\ldots+a_m \tag{5}$$

Then $m \leqq n$ and $\mathrm{rank}[B,\ AB,\ \ldots,\ A^{n-1}B]=\mathrm{rank}[B,\ AB,\ \ldots,\ A^{m-1}B]$. The matrix $[B,\ AB,\ \ldots,\ A^{m-1}B]$ has n rows and rm columns. Thus, from (3), it is necessary that $rm \geqq n$. In the same manner from (4), it follows that $pm \geqq n$. Hence,

$$m \leqq n \leqq \min\ (mp,\ mr) \tag{6}$$

In the case of single-input/single-output systems, we can see that $m=n$ from (6).

From (1), (2) and (5), an input/output equation of an autoregressive/moving-average type can be obtained as follows

$$y_{t+m}+a_1y_{t+m-1}+\ldots+a_my_t=B_1u_{t+m-1}+\ldots+B_mu_t \tag{7}$$

where a_i's ($i=1,\ 2,\ \ldots,\ m$) are scalar values and the coefficients of minimal polynomial of the system and

$$B_j=\sum_{i=0}^{j-1}a_iCA^{j-1-i}B \quad (j=1,\ 2,\ \ldots,\ m) \tag{8}$$

and $a_0=1$. In the input/output equation used by Furuta and Paquet[10] which has the same form as (7), the coefficients for the output are matrices. These coefficients are not unique for the equivalent systems, while the coefficients of (7) are determined uniquely from the input/output deta.

The parameters $\{B_i,\ i=1,\ 2,\ \ldots,\ m\}$ contain the Markov parameters. Then the Ho-Kalman's algorithm can be used to obtain a minimal realization. $\{a_i,\ B_i,\ i=1,\ 2,\ \ldots,\ m\}$ of (7) are also the parameters of the pulse transfer function matrix so that it is possible to use other algorithms for the minimal realization.

III. Identification Procedure

The equation (7) can be rewritten as follows,

$$Y_t=[Y_{t-1},\ \ldots,\ Y_{t-m},\ U_m]\alpha \tag{9}$$

where

$$Y_{t-i}=\begin{pmatrix} y_{t-N+1-i} \\ \vdots \\ y_{t-i} \end{pmatrix}, \quad (i=0,\ 1,\ \ldots,\ m) \tag{10}$$

$$U_m=\begin{pmatrix} \bar{u}_{t-N}, & \ldots, & \bar{u}_{t-m-N+1} \\ \vdots & & \vdots \\ \bar{u}_{t-1}, & \ldots, & \bar{u}_{t-m} \end{pmatrix} \tag{11}$$

$$\bar{u}_t=\begin{pmatrix} u_t^T & 0 & \ldots & 0 \\ 0 & u_t^T & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & u_t^T \end{pmatrix} \tag{12}$$

$$\alpha=[-a_1,\ \ldots,\ -a_m,\ b_{11}^T,\ \ldots,\ b_{1p}^T,\ \ldots,\ b_{m1}^T,\ \ldots,\ b_{mp}^T]^T \tag{13}$$

$$B_j=\begin{pmatrix} b_{j1}^T \\ \vdots \\ b_{jp}^T \end{pmatrix}, \quad (j=1,\ 2,\ \dots,\ m) \tag{14}$$

To be able to solve (9), N should be the integer which satisfies

$$Np \geqq m+mpr \tag{15}$$

since the number of unknowns is $m+mpr$ and the number of equations is Np.

If the inputs are sufficiently random, then U_m in (9) has always a full rank and the vectors Y_{t-1}, Y_{t-2}, ..., Y_{t-m} are linearly independent of (7). Hence the matrix $[Y_{t-1}, \dots, Y_{t-m}, U_m]$ has a full rank.

As m is unknown a priori, we must find it from the input/output data. Let an assumed m be $\hat{m}$. If $\hat{m} \leqq m$, then the vectors Y_{t-1}, Y_{t-2}, ..., $Y_{t-\hat{m}}$ are linearly independent so that the matrix $[Y_{t-1}, \dots, Y_{t-\hat{m}}, U_{\hat{m}}]$ has a full rank. If $\hat{m}>m$, then the vectors Y_{t-m-1}, Y_{t-m-2}, ..., $Y_{t-\hat{m}}$ are linearly dependent upon the vectors Y_{t-1}, Y_{t-2}, ..., Y_{t-m} so that the matrix $[Y_{t-1}, \dots, Y_{t-\hat{m}}, U_{\hat{m}}]$ has a rank $m+\hat{m}pr$. Hence we have

$$\operatorname{rank}[Y_{t-1}, \dots, Y_{t-\hat{m}}, U_{\hat{m}}]=\begin{cases} \hat{m}+\hat{m}pr : \hat{m} \leqq m \\ m+\hat{m}pr : \hat{m}>m \end{cases} \tag{16}$$

If m is found, the unknown parameters can be obtained as follows

$$\alpha=[D^TD]^{-1}D^TY_t \tag{17}$$

where $D=[Y_{t-1}, \dots, Y_{t-m}, U_m]$.

In this way we can obtain the parameters $\{a_1, \dots, a_m, B_1, \dots, B_m\}$ of the pulse transfer function matrix from the input/output data, and we can calculate the Markov parameters $\{CB, CAB, \dots\}$ by using (8).

Hence by using the Ho-Kalman's minimal realization algorithm, we can obtain the matrices $\{A, B, C\}$ of the minimal order state-variable model as follows.

[Ho-Kalman's algorithm]

i). Find k which satisfies rank $\Gamma_k=$rank $\Gamma_{k+1}=\dots=n$ where

$$\Gamma_k=\begin{pmatrix} CB & CAB & \dots & CA^{k-1}B \\ CAB & CA^2B & \dots & CA^kB \\ \vdots & \vdots & & \vdots \\ CA^{k-1}B & CA^kB & \dots & CA^{2k-2}B \end{pmatrix} \tag{18}$$

ii). Find two nonsingular matrices P and Q of size $pk \times pk$ and $rk \times rk$ satisfying

$$P\Gamma_kQ=\begin{pmatrix} I_n & 0_{n,rk-n} \\ 0_{pk-n,n} & 0_{pk-n,rk-n} \end{pmatrix} \tag{19}$$

where $0_{l,q}$ is a zero matrix of a size $l \times q$.

iii). A minimal realization $\{A, B. C\}$ is given by

$$A=[I_n,\ 0_{n,pr-n}](P\Gamma_k'Q)\begin{pmatrix} I_n \\ 0_{rk-n,n} \end{pmatrix} \tag{20}$$

$$B=[I_n,\ 0_{pk-n}](P\Gamma_k)\begin{pmatrix} I_r \\ 0_{rk-r,r} \end{pmatrix} \tag{21}$$

$$C=[I_p,\ 0_{p,pk-p}](\Gamma_kQ)\begin{pmatrix} I_n \\ 0_{rk-n,n} \end{pmatrix} \tag{22}$$

where

$$\Gamma'_k=\begin{pmatrix} CAB & \dots & CA^kB \\ \vdots & & \vdots \\ CA^kB & \dots & CA^{2k-1}B \end{pmatrix} \tag{23}$$

Noting (5), wc can let $k=m$ in (18) when we construct Γ_k. The degree m is determined from the input/output data by using (16). For using this Ho-Kalman's algorithm, we need only $\{CB, CAB, \dots, CA^{2m-1}B\}$ as Markov parameters. The equation (19) can be solved by Jordan's method or elementary operations of matrix.

IV. Identification Procedure for Noisy Data

Let the input/output data be contaminated with noises as follows

$$\begin{cases} \tilde{u}_t=u_t+w_t \\ \tilde{y}_t=y_t+v_t \end{cases} \tag{24}$$

Then the equation (9) will be

$$Z_t=H_t\alpha+E_t \tag{25}$$

where

$$Z_t=\begin{pmatrix} \tilde{y}_{t-N+1} \\ \vdots \\ \tilde{y}_t \end{pmatrix} \tag{26}$$

$$H_t=\begin{pmatrix} \tilde{y}_{t-N}, & \dots, & \tilde{y}_{t-m-N+1}, & \tilde{\bar{u}}_{t-N}, & \dots, & \tilde{\bar{u}}_{t-m-N+1} \\ \vdots & & \vdots & \vdots & & \vdots \\ \tilde{y}_{t-1}, & \dots, & \tilde{y}_{t-m}, & \tilde{\bar{u}}_{t-1}, & \dots, & \tilde{\bar{u}}_{t-m} \end{pmatrix} \tag{27}$$

$$E_t=\begin{pmatrix} e_{t-N+1} \\ \vdots \\ e_t \end{pmatrix} \tag{28}$$

$$e_t=v_t+a_1v_{t-1}+\dots+a_mv_{t-m}-B_1w_{t-1}-\dots-B_mw_{t-m} \tag{30}$$

The least squares estimate of α is given by

$$\hat{\alpha}=[H_t^TH_t]^{-1}H_t^TZ_t \tag{31}$$

However it is well known that this estimate containts a bias since $E[H_t^TE_t]\neq 0$. To avoid this, we can use maximum likelihood or the instrumental variable method.

Noting that the matrix H_t has a full rank almost always when the inputs are sufficiently random, it is impossible to determine m by using (16).

However we can use the property in that the Markov parameters are unique among equivalent systems. Namely,

$$C_1A_1^kB_1=C_2A_2^kB_2, \quad (k=0, 1, 2, \dots) \tag{32}$$

where the system $\{A_1, B_1, C_1\}$ of order n_1 is equivalent to the one $\{A_2, B_2, C_2\}$ of order n_2. From (32) we can see that for some assumed $\hat{m}$'s, estimated Markov parameters have curves such as a or b in Figure 1. From this property, the degree of the minimal polynomical of the system matrix can be found. If m is determined, we can obtain an estimate of the pulse transfer function matrix of minimal degree and also the Markov parameters.

It must be noted that estimated Markov parameters contain estimation errors so that we can not find the minimal order by using (19) since Γ_k has always a full rank. However we know that the minimal order exists in the interval between m and rm or pm as indicated in Section II. Hence we may find the minimal order as follows. For each integer in this interval, we can obtain realizations by solving (19) to (23) and also their Markov parameters (or transfer function matrices). If some realization has the nearest Markov parameters (or transfer function matrix) to the estimated ones, then we consider the order of this realization as the true minimal order since there exists only one solution to this problem.

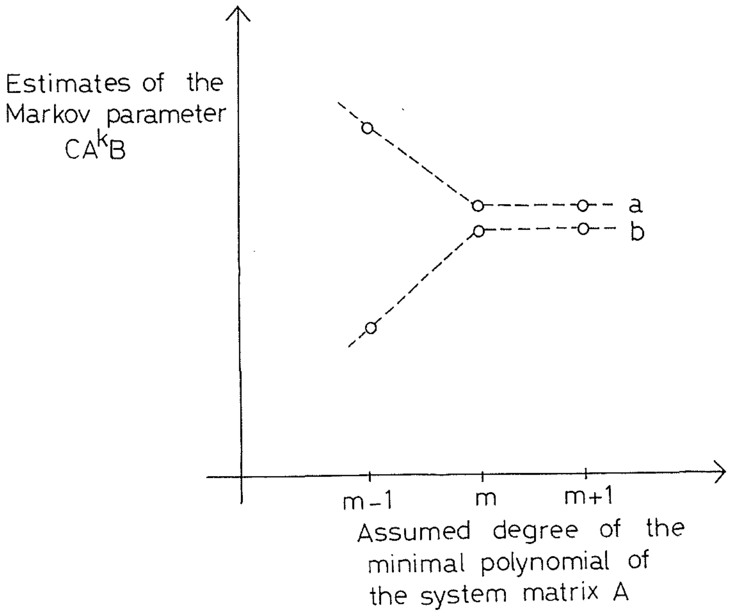

Fig. 1 Behavior of estimates of the Markov parameter

V. Simulation Results

1). Noiseless case

The first example is a system with the following transfer function matrix.

$$G(z)=\frac{\begin{pmatrix} z-0.9 & 1 \\ z^2-0.9z & z \end{pmatrix}}{z^3-0.9z^2+0.23z-0.015} \tag{33}$$

The input data were zero mean Gaussian random numbers with covariance 1. The degree of the minimal polynomial of the system matrix was assumed to be 2, 3, 4 and 5. In order to determine m, the determinants of the matrices in (16) were calculated. When the matrix was not square, the determinant of the product of this matrix and its transposed matrix was calculated. The results are indicated in Table 1.

From these results, we found that $m=3$. The estimated pulse transfer function matrix had the same values as (33). Then we calculated the Markov parameters and the following minimal realization was given.

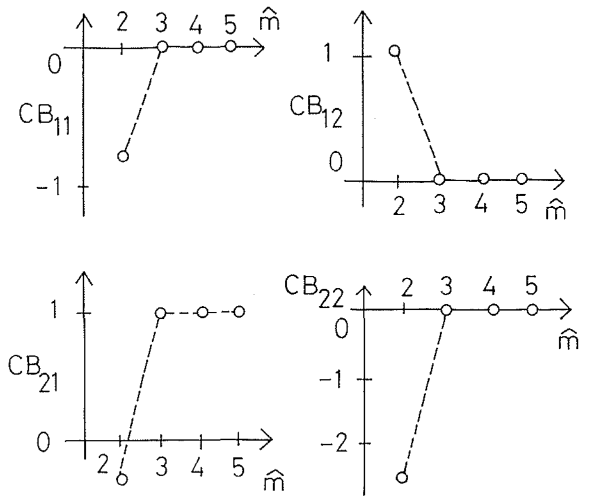

Fig. 2 Behavior of estimates of the Markov parameter

Table 1 Determination of m

$\hat{m}$	2	3	4	5
determinant of D (or D^TD)	2.28	4.28×10^8	0	0

Table 2 Determination of n

$\hat{n}$	3	4	5	6
CA^2B_{11}	0.00889	0.0961	0.132	88.95

$$A=\begin{pmatrix}0 & 1 & 0\\ -0.23 & 0.9 & 0.015\\ 1 & 0 & 0\end{pmatrix},\quad B=\begin{pmatrix}1 & 0\\ 0 & 1\\ 0 & 0\end{pmatrix},\quad C=\begin{pmatrix}0 & 0 & 1\\ 1 & 0 & 0\end{pmatrix} \tag{34}$$

2). Noisy case

The second example is a system with the following transfer function matrix.

$$G(z)=\frac{\begin{pmatrix}z-2.5 & 1\\ z^2-2.5z & z\end{pmatrix}}{z^3-2.5z^2+2.11z-0.595} \tag{35}$$

The inputs were zero mean Gaussian random numbers with covariance 1 and the outputs were contaminated with the measurement noises which were zero mean Gaussian random numbers with covariance 0.01.

For $m=2$, 3, 4, and 5, the pulse transfer function matrices were estimated by the recursive least squares method and the Markov parameters were calculated. The effects of the bias were ignored. Figure 2 illustrates estimates of a Markov parameter CB. (the same curves are obtained for the other around $m=3$).

Therefore, $m=3$. The estimated pulse transfer function matrix was

$$G(z)=\frac{\begin{pmatrix}0.0028z^2+0.999z-2.4998 & 0.0052z^2-0.0019z+0.9997\\ 1.0028z^2-2.4997z-0.0047 & -0.0014z^2-1.004z-0.0014\end{pmatrix}}{z^3-2.4995z^2+2.109z-0.5945} \tag{36}$$

The Markov parameters were

$$CB=\begin{pmatrix}0.00278 & 0.0052\\ 1.0028 & -0.00139\end{pmatrix},\quad CAB=\begin{pmatrix}1.006 & 0.0111\\ 0.00683 & 1.001\end{pmatrix},\quad CA^2B=\begin{pmatrix}0.00889 & 1.017\\ -2.103 & 2.503\end{pmatrix}$$
$$CA^3B=\begin{pmatrix}-2.098 & 2.52\\ -4.673 & 4.115\end{pmatrix},\quad CA^4B=\begin{pmatrix}-4.664 & 4.162\\ -7.243 & 5.676\end{pmatrix},\quad CA^5B=\begin{pmatrix}-7.228 & 5.693\\ -9.497 & 6.934\end{pmatrix} \tag{37}$$

Next we must find the minimal order. Noting that $m=3$, it exists in the interval between 3 and 6. For $n=3$, 4, 5 and 6, we calculated realizations and their Markov parameters. Table 2 indicates the calculated values of 1-1 element of a Markov parameter CA^2B for each n's.

The realization of order 3 had the nearest value to (37). Hence the minimal order was considered to be 3. The minimal realization was given a follows.

$$A=\begin{pmatrix} 1.3113 & 0.2648 & -0.07331 \\ -1.1847 & 1.1876 & -2.390 \\ -2.0610 & 1.5756 & -0.000682 \end{pmatrix}, \quad B=\begin{pmatrix} 0.2903 & -0.3456 \\ 1.0 & -0.4421 \\ 0.0 & 0.008166 \end{pmatrix}$$
$$C=\begin{pmatrix} 0.00889 & 0.000198 & 1.0234 \\ -2.1025 & 1.6131 & -0.0199 \end{pmatrix} \tag{38}$$

IV. Conclusion

A new identification procedure has been developed for multi-input/multi-output systems with an unknown order. Simulation results have been presented for this method. Even if the input/output data are contaminated with noises, the minimal order of the system is determined by using the Markov parameters. Moreover the estimated pulse transfer function matrix is of a minimal degree.

It is desirable that the state-space model is in the canonical form of Luenberger. This will be treated in a future publication.

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