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Radiation Effect on the Heat Transfer of a Two-Dimensional Laminar Wall Jet

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Abstract

Simultaneous heat transfer by radiation and convection from a wall surface over which a two-dimensional laminar wall jet of an absorbing and emitting gray gas is issued tangentially, is determined by solving a non-linear energy equation combined with both continuity of mass and momentum equation. The governing equations in the present investigation are obtained by considering a boundary layer type of analysis. The study variable parameters adopted is made to clarify the effects of those including conduction-to-radiation, optical thikness on temperature distributon or heat transfer at the wall surface.

Nomenclature

C_p	specific heat at constant pressure
F	dimensionless stream function
N	conduction-to-radiation parameter
Nu	local Nusselt number defined by equation (32)
Pr	Prandtl number
q	heat flux
T	temperature
u, v	dimensionless velocity components in $x-$ and $y-$ direction, respectively
u', v'	velocity components in $x'-$ $y'-$ direction, respectively
x, y	dimensionless coordinates in parallel and perpendicular to the wall surface, respectively
x', y'	coordinate in parallel and perpendicular to the wall surface, respectively
α	local heat transfer coefficient
η	dimensionless variable defined by equation (17. b)
θ	dimensionless temperature
κ	absorption coeffiicient
λ	thermal conductivity
μ	viscosity
ν	kinematic viscosity
ρ	density
σ	Stefan-Boltzman constant
τ	optical thickness
ϕ	dimensionless constant defined by equation (9. b)
ψ	stream function

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Subscript

c refers to convection

r refers to radiation

t refers to total

w refers to wall

∞ refers to outside thermal boundary layer

1. Introduction

In recent years, considerable attention has been paid to the mass additing of the boundary layer flow, especially in conection with the cooling of turbine blades or skins of high-speed aero-vehicles. Also the problem of mass transfer has been of lasting interest in chemical engineering. This kind of study has been extensively performed hitherto from a practical stand-point of boundary layer control and previous studies generally are mainly focussed on the flow properties or the heat transfer of a non-radiating wall jet issued tangentially over a impermeable plane surface exposed to an otherwise stationary surrounding or uniform approaching flow.

Eckert[1] pointed out that the actual turbine conditions were further complicated by non-uniformity of temperature, three-dimensionality or velocity fluctuation of the approaching stream, and concluded that there was still a considerable need for further research.

In most of the previous studies on free jet and wall jet, generally it has been customary to disregard the possibillity of the flowing gas radiation despite the fact that this causes a temperature variation and velocity distribution in the jet from the primarily predicted ones. Pai[2] examined a tow-dimensional steady free jet mixing problem asserting that the thermal radiation played an important role on heat transfer and reported that the radiation effect could be clarified by averaging the maximum temperature in a thermal mixing region and to increase the extent of the region. Abu-Romia[3] treated a radiating two-dimensional laminar compressible free jet in which the radiation process was described by adopting an optically thin model and found that the specific enthalpy of radiating gas decreased from its value disregarding radiation at any point of the stream if the radiation term in energy equation was assumed to be expressed by a power function of enthalpy. Lately, Golovachev[4] investigated a jet flow of radiating gas and solved a differential approximation taking into account both the transverse and longitudinal radiant energy transfer by a finite difference method. In his paper, Golovachev demonstrated the effects of the longitudinal radiant energy transfer on jet flow of a radiating gas. However, reports of this kind, especially pertaining to a wall jet, have apparently not been published to date.

The present paper deals with the combined momentum and energy transport in a radiating two-dimensional laminar wall jet issued tangentially over an impermeable plane surface exposed to an otherwise stationary surrounding. In this analysis, the governing equations are obtained by considering a boundary layer type of analysis. The radiation part in the energy equation is carefully treated by an application of the technique originally developed by Viskanta[5] and the effects of conduction-to-radiation, optical thickness and other parameters on heat transfer are discussed.

2. Analysis

Now consider a two-dimensional steady laminar wall jet mixing problem in which a gas flow issued tangentially over an impermeable wall surface is mixing with the surrounding stationary gas and restrict to the case when the temperature of jet and of wall are sufficiently high and the fluid can be treated as incompressible. The gas is assumed to be gray and is locally in a thermodynamic equilibrium. Moreover, the specific heat of the gas, the transport coefficient and the absorption coefficient are also each assumed as constant.

Although such an approach mentioned above is probably an over-simplification to the actual wall jet flow problems, it may be assumed that the present model serves as a convenient stepping stone in the course of analysing more realistic problems. The dynamic problem is solved without any consideration of the interaction with temperature fields, and thus the obtained velocity distributiou is used for solving the energy equation involving the effects of radiation transfer.

The starting system of equations are integro-differential equations in which the energy equation contains the divergence of radiant flux. By Glauert[6], the wall jet can be treated categorically within a framework of the boundary layer theory. According to Sparrow and Cess[7], the boundary layer equations in accordance with the above assumptions are

$$\frac{\partial u'}{\partial x'}+\frac{\partial v'}{\partial y'}=0 \tag{1}$$

$$\rho\left(u'\frac{\partial u'}{\partial x'}+v'\frac{\partial u'}{\partial y'}\right)=\mu\frac{\partial^2 u'}{\partial y'^2} \tag{2}$$

$$\rho C_p\left(u'\frac{\partial T}{\partial x'}+v'\frac{\partial T}{\partial y'}\right)=\lambda\frac{\partial^2 T}{\partial y'^2}-\mathrm{div}\, q_r \tag{3}$$

where the viscous dissipation is considered to be negligibly small. q_r denotes the vector of radiation flux and the radiation term Eq. (3) is frequently approximated in the following one dimensiomal propagation form, in case of non-scattering gray gas and black wall surface

$$q_r=2\sigma T_w^4E_3(\tau)+2\sigma\int_0^\tau T^4(x',t)E_2(\tau-t)dt-2\sigma\int_\tau^\infty T^4(x',t)E_2(\tau-t)dt \tag{4}$$

and correspondingly

$$-\mathrm{div}\, q_r=\kappa[2\sigma T_w^4E_2(\tau)+2\sigma\int_0^\infty T^4(x',t)E_1(|\tau-t|)dt-4\sigma T^4(x',\tau)] \tag{5}$$

where the magnitude of the optical thickness τ is defined as

$$\tau=\int_0^{y'}\kappa dy' \tag{6}$$

and $E_n(t)$ is a exponential integral function defined by

$$E_n(t)=\int_0^1\mu'^{(n-2)}e^{-t/\mu'}d\mu' \tag{7}$$

The boundary conditions are taken as

$$u'=v'=0,\qquad T=T_w \text{ at } y'=0 \tag{8. a}$$

$$u'\to 0,\qquad T\to T_\infty \text{ as } y'\to\infty \tag{8. b}$$

The isothermal wall temperature is denoted by T_w and the surrounding fluid temperature by T_∞.

If one introduces the non-dimensional variables defined by

$$x=x'/L,\qquad y=y'/L,\qquad u=u'L/\nu,\qquad v=v'L/\nu \tag{9. a}$$

$$\theta=(T-T_\infty)/(T_w-T_\infty), \qquad \phi=T/T_\infty \tag{9. b}$$

where L is a characteristic length, Eqs. (1), (2) and (3) become

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{10}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2} \tag{11}$$

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2}+\frac{1}{\phi_w-1}\frac{(\kappa L)^2}{2PrN}[\phi_w^4E_2(\tau)+$$
$$+\int_0^\infty\{1+\theta(x,t)(\phi_w-1)\}^4E_1(|\tau-t|)dt-2\{1+\theta(x,\tau)(\phi_w-1)\}^4] \tag{12}$$

where

$$N=\frac{\lambda\kappa}{4\sigma T^3} \tag{13}$$

is a scale to evaluate the relative effectiveness of conduction versus radiation, while Pr denotes the Prandtl number of a gas. For $N=\infty$, heat within the medium is transfered only by conduction and convection, while $N\rightarrow 0$ corresponds to the case in which radiation plays an important role on heat transfer. The boundary conditions are

$$u=v=0, \qquad \theta=1 \text{ at } y=0 \tag{14. a}$$

$$u\rightarrow 0, \qquad \theta\rightarrow 0 \text{ as } y\rightarrow\infty \tag{14. b}$$

The continuity equation of Eq. (10) is automatically satisfied if the stream function ψ is introduced by the usual definition as

$$u=\frac{\partial\psi}{\partial y}, \qquad v=-\frac{\partial\psi}{\partial x} \tag{15}$$

Here one introduces the new variables

$$\xi=\frac{2(\kappa L)^2}{PrN}x^{3/2}, \qquad \eta=\frac{1}{2}yx^{-3/4} \tag{16}$$

where a new independent variable ξ characterizes the optical thickness. Using ξ and η, the stream function and temperature are expressed in the following forms.

$$\psi=2x^{1/4}F(\eta) \tag{17}$$

$$\theta=\theta(\xi,\eta) \tag{18}$$

As for Eqs. (16), (17) and (18), the momentum and energy equations can be rewritten in terms of the new variables as follows

$$\frac{d^3F}{d\eta^3}+F\frac{d^2F}{d\eta^2}+2\left(\frac{dF}{d\eta}\right)^2=0 \tag{19}$$

$$\frac{1}{Pr}\frac{\partial^2\theta}{\partial\eta^2}+F\frac{\partial\theta}{\partial\eta}-6\xi\frac{dF}{d\eta}\frac{\partial\theta}{\partial\xi}+\frac{\xi}{\phi_w-1}[\phi_w^4E_2(\tau)+$$
$$+\int_0^\infty\{1+\theta(x,t)(\phi_w-1)\}^4E_1(|\tau-t|)dt-2\{1+\theta(x,t)(\phi_w-1)\}^4]=0 \tag{20}$$

where η can be expressible in terms of ξ and τ as

$$\eta=\frac{\tau}{(2PrN)^{1/2}} \tag{21}$$

It should be noted that Eq. (19) coincides well with a dynamic governing equation for a usual laminar wall jet issued tangentially over an impermeable wall surface, the solution of which has already been plotted by Glauert[6]. The boundary conditions described as Eqs. (14. a) and (14. b) are transformed to

$$F=\frac{dF}{d\eta}=0, \qquad \theta=1 \text{ at } \eta=0 \tag{22. a}$$

$$\frac{dF}{d\eta}=0, \qquad \theta=0 \text{ at } \eta=\infty \tag{22.b}$$

The new energy equation (20) constitutes a partial non-linear, integro-differential equation and involves partial derivatives pertaining to the variable ξ, so an additional boundary condition is needed. It is chosen as

$$\theta=\theta_0 \text{ for } \xi=0 \tag{23}$$

where θ_0 represents the solution for a non-radiative case.

3. Method of solution

To determine the temperature distribution in the medium concerned, the partial integro-differential equation (20) must be solved with appropriate boundary conditions. In the present analysis, the partial integro-differential equation is transformed into an ordinary integro-differential equation in terms of η by replacing the following derivative $(\partial\theta/\partial\xi)$ with its finite difference as

$$\left(\frac{\partial\theta}{\partial\xi}\right)=\frac{\theta_i-\theta_{i-1}}{\xi_i-\xi_{i-1}} \tag{24}$$

The resulting equation becomes

$$\frac{1}{Pr}\theta_i''+F\theta_i-6\xi_i F'\frac{\theta_i-\theta_{i-1}}{\theta_i-\theta_{i-1}}+\frac{\xi_i}{\phi_w-1}[\phi_w^4 E_2(\zeta\eta)+$$

$$+\zeta\int_0^\infty \{1+\theta_{i-1}(\phi_w-1)\}^4 E_1(\zeta|\eta-s|)ds-2\{1+\theta_{i-1}(\phi_w-1)\}^4]=0 \tag{25}$$

where the variable ζ does not actually constitute an independent one, because ζ is connected with ξ and N through a relation defined as

$$\zeta=\sqrt{2PrN\xi} \tag{26}$$

and the primes denote differentiation with respect to η.

Equation (25) is integrated numerically and directionally by utilizing the Runge-Kutta-Gill method at each nodal point ξ_i after starting the calculation at $\xi=0.0$. It should be noted that in this integration the boundary condition $\eta=\infty$ must be replaced by an appropriate constant value which is taken as $\eta=50\sim100$ in the present calculation. Also the exponential integral functions $E_n(t)$ are approximated according to Sparrow and Cess[7].

4. Heat transfer

From equation (4), the surface radiation fiux q_{rw} is obtained as the value at $\tau=0$, as follows

$$q_{rw}=2\sigma T_w^4 E_3(0)-2\sigma\int_0^\infty T^4 E_2(t)dt \tag{27}$$

The convective heat flux from the surface q_{rw} is given by

$$q_{cw}=-\lambda\left(\frac{\partial T}{\partial y'}\right)y'=0 \tag{28}$$

and the total net heat flux at the wall surface q_{tw} by

$$q_{tw}=q_{cw}+q_{rw} \tag{29}$$

If the local Nusselt number is permitteb to be befined conventionally

$$Nu=\frac{\alpha x'}{\lambda}=\frac{q_{tw}x'}{\lambda(T_w-T)} \tag{30}$$

where α is the local heat transfer coefficient. Substituting equation (29) into equation (30), the local Nusselt number Nu, after some arranging, is given as

$$\frac{Nu}{\left(\frac{1}{2}x^{1/4}\right)}=-\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0}+\frac{\sqrt{\xi}}{\phi_w-1}\sqrt{Pr/2N}\,[\phi_w^4E_3(0)-$$
$$-2\zeta\int_0^\infty\{1+\theta(\phi_w-1)\}^4E_2(\zeta s)ds] \tag{31}$$

For a large value of N or for $\sqrt{\xi/N}\langle\langle 1$, the radiation terms in Eq. (31) is negligible and the resulting expression coincides fully with the previous result for the Nusselt number of a tow-dimensional laminal wall jet[8].

5. Result and discussions

To understand the effects of thermal radiation on temperature distribution in a wall jet of radiating gas and its convective heat transfer, a number of solutions are obtained for various combinations of parameters N, ϕ_w or ξ. The dimensionless N indicates a relative role of energy transport by conduction to that by radiation. For $N=\infty$, the energy transport is by convection, for $N=0$, the energy transport is only by radiation. ϕ_w is a ratio of the temperature of a wall to that of the surrounding gas. It follows that ξ is a scale of optical thickness.

The dependabillity of temperature on dimensionless parameter N is shown in Fig. 1 for particular values of ϕ_w, ξ and Pr. The temperature profile for $N=\infty$ is very close that for $N=1.0$, and therefore each separate curve could not be shown. Temperature distribution for the case in which a great part of the energy transport is by radiation, $N=10^{-4}$, is also included in this figure for comparison.

The influences of the ratio of ambient gas temperature T_∞ to wall temperature T_w are indicated in Fig. 2 for particular values of N, ξ and Pr. $\phi_w \to 1$,

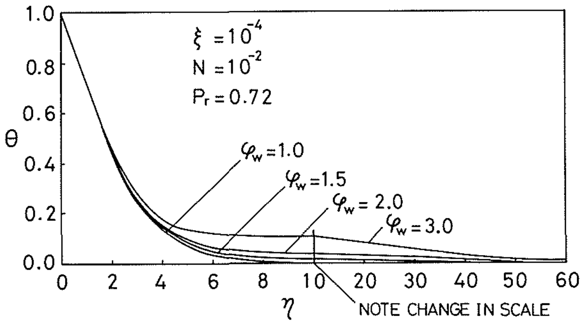

Fig. 1 Effect of parameter N on temperature distribution

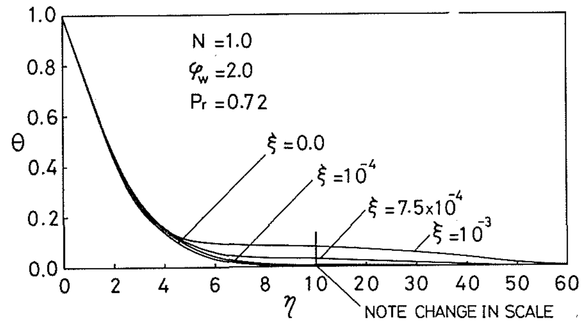

Fig. 2 Effect of temperature ratio ϕ_w on temperature distribution

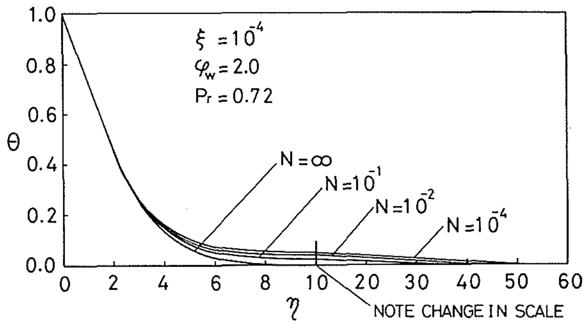

Fig. 3 Effect of parameter ξ on temperature distribution

Table 1. a Numerical results for temperature profile ($\xi=10^{-4}$, $\phi_w=2$ and $Pr=0.72$)

η	$(T-T_\infty)/(T_w-T_\infty)$ $N=\infty$	$N=0.1$	$N=0.01$	$N=0.0001$
0.0	1.0000	1.0000	1.0000	1.0000
0.5	0.8570	0.8572	0.8573	0.8574
1.0	0.7157	0.7162	0.7165	0.7167
2.0	0.4553	0.4571	0.4579	0.4586
4.0	0.1342	0.1437	0.1484	0.1523
6.0	0.0331	0.0528	0.0636	0.0725
8.0	0.0079	0.0325	0.0463	0.0577
10.0	0.0019	0.0271	0.0415	0.0535
20.0	0.0000	0.0186	0.0302	0.0399
50.0	0.0000	0.0005	0.0008	0.0011

Table 1. b Numerical results for temperature profile ($\xi=10^{-4}$, $N=10^{-2}$ and $Pr=0.72$)

η	$(T-T_\infty)/(T_w-T_\infty)$ $\phi_w=1.9$	$\phi_w=1.5$	$\phi_w=2.0$	$\phi_w=3.0$
0.0	1.0000	1.0000	1.0000	1.0000
0.5	0.8570	0.8571	0.8573	0.8582
1.0	0.7157	0.7159	0.7165	0.7184
2.0	0.4553	0.4563	0.4579	0.4632
4.0	0.1342	0.1399	0.1484	0.1751
6.0	0.0331	0.0452	0.0636	0.1209
8.0	0.0079	0.0232	0.0463	0.1184
10.0	0.0019	0.0176	0.0415	0.116
20.0	0.0000	0.0119	0.0302	0.0869
50.0	0.0000	0.0004	0.0008	0.0023

the temperature profile approaches that for pure convection, $N=\infty$

Figure 3 shows the temperature profiles at several axial positions for particular values of N, ϕ_w and Pr. The temperature profile of $\xi=0$ characterizes the non-radiative case.

In table 1 we present the numerical values of temperature as a function of η at $\xi=10^{-4}$ for different values of the parameters N and ϕ_w. The case for

Table 2 Numerical results for local Nusselt number ($Pr=0.72$)

	N	ϕ_w	ξ	$Nu/(\frac{1}{2}\times^{1/4})$
Effect of				
N	0.0001	2.0	10^{-4}	5.0162
	0.01			0.7163
	0.1			0.4160
	∞			0.2862
Effect of				
ϕ_w	0.01	1.5	10^{-4}	0.5791
		2.0		0.7105
		3.0		1.3165
		4.5		3.1562
Effect of				
ξ	1.0	2.0	10^{-4}	0.3367
			10^{-3}	0.6498

$N=\infty$ represents a non-radiative one. An examination of this table suggests that the thickness of thermal boundary layer increases with the decreasing ϕ_w.

Table 2 illusrates the effects of parameters N, ϕ_w and ξ on the local Nusselt number for the conditions specified table. The ratio $Nu(x^{1/4}/2)$ increases with the decreasing N or increasing ϕ_w or increasing ξ. For large values of N or for $\phi_w \rightarrow 1$ this ratio is the same as that for the non-radiative case.

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