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Free Vibration Analysis of Cylindrical Shells with Spherical Cap by the Collocation Method

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Abstract

The collocation method is presented for determining the vibration characteristics of shells involved in revolution which are composed of combinations of cylindrical and spherical shells. The results obtained from the present treatment are compared with the existing ones for shells of this type. The comparison shows that the method yields very good results. Finally, several numerical examples are given to examine the effects of dimensions of the shell on the dynamic behavior system.

1. INTRODUCTION

The vibration problems of cylindrical shells closed by spherical shells are important in the practice of engineering. Such structures are used in many industries.

For such shells which have a discontinuity in geometry, most of the reported studies[1)~3)] have been attempted using numerical methods, such as the finite difference method, and the finite element method. However, to obtain accurate estimates for the natural frequencies and the corresponding mode shapes (expecially near the points of discontinuity), these methods usually require a large number of discretization points or a large number of elements and result in a correspondingly large set of algebraic equations. Therefore, numerical difficulties may arise in the calculation of the natural frequencies and the mode shapes.

In the present study, the formulation of free vibrations of cylindrical shells with spherical caps is presented using the collocation method. The collocation points are taken at the zeroes Mth shifted Legendre polynomials $P_M^*(x)$ defined on the domain $0 \leq x \leq 1$. These zeroes are distributed closer to $x=0$, 1. Therefore, the location of such zeroes may be utilized to avoid the numerical difficulties when shells which have a discontinuity in geometry must be dealt with.

The main objectives of the present paper are : 1) to examine the applicability of the collocation method to free vibration analysis of cylindrical shells with spherical caps, and 2) to determine the dynamic characteristics of shells of this type.

2. MATHEMATICAL FORMULATION

A cylindrical shell with a spherical shell is shown in Fig. 1. In the figure, a, h_c, and L_c are the radius, thickness, and height of the cylindrical shell, respectively, and R, h_s, L_s, and

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Φ are the radius, thickness, meridional length, and half opening angle of the spherical shell, respectively.

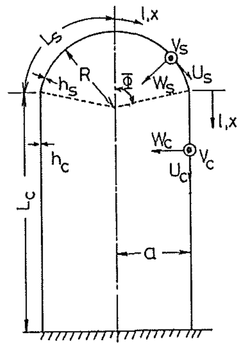

Fig. 1 Cylindrical shell with spherical cap.

The analysis is based on Novozhilov's thin shell theory[4].

2. 1 Governing Differential Equations

The governing differential equations for a shell of revolution can be formulated in terms of the displacements by substituting the stress resultant-displacement relations into the equations of equilibrium.

Assuming that the cylindrical and spherical shells are undergoing free vibration with a frequency ω, then the meridional, circumferential, and radial displacements, u, v, and w are described as

$$\begin{Bmatrix} u_j \\ v_j \\ w_j \end{Bmatrix} = \sum_{n=0}^{\infty} \begin{Bmatrix} h_j U_j(x) \cos n\theta \\ h_j V_j(x) \sin n\theta \\ h_j W_j(x) \cos n\theta \end{Bmatrix} e^{i\omega t}, \quad (j=c,\ s) \tag{1}$$

in which U, V, and W are nondimensional displacement functions, n is the harmonic number, and θ is the circumferential angle. Moreover, in Eq. (1) $j=c$ refers to the quantities of the cylindrical shell, and $j=s$ to the quantities of the spherical shell.

By using Eq. (1) a system of ordinary differential equations can be obtained for each harmonic n. These equations are

$$\left.\begin{aligned} & a_{j,1} U_j'' + a_{j,2} U_j' + a_{j,3} U_j + a_{j,4} V_j' + a_{j,5} V_j + a_{j,6} W_j''' + a_{j,7} W_j'' + \\ & \qquad a_{j,8} W_j' + a_{j,9} W_j - \lambda_j U_j = 0, \\ & a_{j,10} U_j' + a_{j,11} U_j + a_{j,12} V_j'' + a_{j,13} V_j' + a_{j,14} V_j + a_{j,15} W_j'' + \\ & \qquad a_{j,16} W_j' + a_{j,17} W_j - \lambda_j V_j = 0, \\ & a_{j,18} U_j''' + a_{j,19} U_j'' + a_{j,20} U_j' + a_{j,21} U_j + a_{j,22} V_j'' + a_{j,23} V_j' + \\ & a_{j,24} V_j + a_{j,25} W_j'''' + a_{j,26} W_j''' + a_{j,27} W_j'' + a_{j,28} W_j' + a_{j,29} W_j - \\ & \qquad \lambda_j W_j = 0, \quad (j=c,\ s) \end{aligned}\right\} \tag{2}$$

in which the subscripts $j=c$ and $j=s$ denote the quantities with respect to the cylindrical and spherical shells, respectively. Moreover, $(\ )' = d(\ )/dx$, $x = l/L_j$, l is the meridional length measured from the apex of the spherical shell or the junction of the cylindrical shell and the spherical shell, λ_c $(=\rho a^2 (1-\nu^2)\ \omega^2/E)$ and λ_s $(=\rho R^2 (1-\nu^2)\ \omega^2/E)$ are the frequency parameters, ρ is the mass density, ν is Poisson's ratio, and E is Young's modulus.

The coefficients $a_{j,i}$ $(i=1,\ldots, 29)$ are functions of the shell geometry and material characteristics, and the harmonic number. Although detailed expressions for the coefficients have been omitted here for brevity, by considering the geometric properties of a surface of revolution the coefficients of the cylindrical and spherical shells may be obtained from those of a hyperboloidal shell given in Ref. 5.

2. 2 Boundary and Compatibility Conditions

The force quantities ($[n]$, $[m]$, $[s]$, and $[t]$) which appear in the boundary conditions now will be expanded into Fourier series, where $[n]$ and $[m]$ are the meridional force and moment, respectively, and $[s]$ and $[t]$ are the effective shear forces. These are shown in the following forms :

$$\left.\begin{aligned} &[n]_j=\sum_{n=0}^{\infty}[N]_j\cos n\theta,\quad [m]_j=\sum_{n=0}^{\infty}[M]_j\cos n\theta,\\ &[s]_j=\sum_{n=0}^{\infty}[S]_j\sin n\theta,\quad [t]_j=\sum_{n=0}^{\infty}[T]_j\cos n\theta, \end{aligned}\quad (j=c,\ s)\right\} \tag{3}$$

in which the quantities $[N]_j$, $[M]_j$, $[S]_j$, and $[T]_j$ may be expressed in terms of the displacements as

$$\left.\begin{aligned} &[N]_j=K_{j,1}(b_{j,1}U_j'+b_{j,2}U_j+b_{j,3}V_j+b_{j,4}W_j),\\ &[M]_j=K_{j,2}(b_{j,5}U_j'+b_{j,6}U_j+b_{j,7}V_j+b_{j,8}W_j''+b_{j,9}W_j'+b_{j,10}W_j),\\ &[S]_j=K_{j,3}(b_{j,11}U_j+b_{j,12}V_j'+b_{j,13}V_j+b_{j,14}W_j'+b_{j,15}W_j),\\ &[T]_j=K_{j,4}(b_{j,16}U_j''+b_{j,17}U_j'+b_{j,18}U_j+b_{j,19}V_j'+b_{j,20}V_j+b_{j,21}W_j'''+\\ &\qquad b_{j,22}W_j''+b_{j,23}W_j'+b_{j,24}W_j),\quad (j=c,\ s). \end{aligned}\right\} \tag{4}$$

In Eqs. (3) and (4), the subscripts $j=c$ and $j=s$ refer to the quantities of the cylindrical and spherical shells, respectively, and the coefficients $b_{j,i}$ $(i=1,\dots,24)$ can be obtained from those of Ref. 5 similarly to the coefficients $a_{j,i}$ in Eq. (2). Moreover, $K_{j,i}$ $(i=1,\dots,4)$ are as follows : for the cylindrical shell

$$K_{c,1}=\frac{Eh_c^2}{(1-\nu^2)a},\quad K_{c,2}=K_{c,1}\frac{h_c^2}{12a},\quad K_{c,3}=2(1-\nu)K_{c,1},\quad K_{c,4}=K_{c,1}\frac{h_c^3}{12a^2} \tag{5}$$

and for the spherical shell

$$K_{s,1}=\frac{Eh_s^2}{(1-\nu^2)R},\quad K_{s,2}=K_{s,1}\frac{h_s^2}{12R},\quad K_{s,3}=2(1-\nu)K_{s,1},\quad K_{s,4}=K_{s,1}\frac{h_s^3}{12R^2} \tag{6}$$

The conditions to be applied at the apex of the spherical shell are as follows :

$$\left.\begin{aligned} &U_s=V_s=W_s'=W_s'''-U_s''=0 \text{ for } n=0,\\ &U_s+V_s=W_s=U_s'=[M]_s=0 \text{ for } n=1,\\ &U_s=V_s=W_s=W_s'=0 \text{ for } n\geq 2. \end{aligned}\right\} \tag{7}$$

The compatibility conditions at the joint between the cylindrical shell and spherical shell are :

$$\left.\begin{aligned} &h_cU_c=h_sU_s\sin\Phi+h_sW_s\cos\Phi,\quad h_cW_c=h_sW_s\sin\Phi-h_sU_s\cos\Phi,\\ &h_cV_c=h_sV_s,\quad \frac{h_c}{L_c}W_c'=\frac{h_s}{R}U_s+\frac{h_s}{L_s}W_s'\\ &[N]_c=[N]_s\sin\Phi+[T]_s\cos\Phi,\quad [M]_c=[M]_s,\\ &[T]_c=[T]_s\sin\Phi-[N]_s\cos\Phi,\quad [S]_c=[S]_s. \end{aligned}\right\} \tag{8}$$

For the boundary conditions at the base of the cylindrical shell, we will prescribe the appropriate four of the quantities U_c, V_c, W_c, W_c', $[N]_c$, $[M]_c$, $[S]_c$, and $[T]_c$. Thus, the boundary conditions to be considered are :

$$\left.\begin{array}{l} U_c=0 \text{ or } [N]_c=0, \quad V_c=0 \text{ or } [S]_c=0, \\ W_c'=0 \text{ or } [M]_c=0, \quad W_c=0 \text{ or } [T]_c=0. \end{array}\right\} \tag{9}$$

3. METHOD OF SOLUTION

3.1 The Collocation Method

Without going into details, let us consider the application of the collocation method to specific differential equations. For this purpose, let us assume that there is the problem given by a second ordinary differential equation

$$L(u)=0\,;\ 0<x<1, \tag{10}$$

and two boundary conditions

$$S_j(u)=0\,;\ j=1,2, \tag{11}$$

in which L and S are a linear differential operaters.

We, for example, seek an approximate solution of the form

$$u=\sum_{i=1}^{M+2} a_{i-1}x^{i-1} \tag{12}$$

in which a_{i-1} ($i=1,\ldots, M+2$) are undetermined coefficients. Substituting Eq. (12) into Eq. (10), the residual $R(x)$ becomes

$$R(x)=\sum_{i=1}^{M+2} a_{i-1}L(x^{i-1}). \tag{13}$$

In the collocation method the residual is eliminated by setting $R(x)=0$ at M specified collocation points. Then, with two boundary conditions, $M+2$ linear equations are obtained for the $M+2$ unknowns.

The roots of the orthogonal polynomial are frequently used as collocation points with considerable success. In this paper, the collocation points are selected to be zeroes of the Mth shifted Legendre polynomial $P_M^*(x)$ defined on $0\leq x\leq 1$. These M points are called the interior collocation points.

3.2 Application to Cylindrical Shell with Spherical Cap

The solutions to Eq. (2) are taken as the following forms :

$$U_j=\sum_{i=1}^{M_j+2} e_{j,i-1}x^{i-1},\quad V_j=\sum_{i=1}^{M_j+2} f_{j,i-1}x^{i-1},\quad W_j=\sum_{i=1}^{M_j+4} g_{j,i-1}x^{i-1},\qquad (j=c,s) \tag{14}$$

in which $e_{j,i-1}$, $f_{j,i-1}$, and $g_{j,i-1}$ are unknown coefficients, and M_c and M_s are the number of interior collocation points in the cylindrical and spherical shells, respectively.

It seems convenient to write the eigenvalue equation in terms of the solution of the interior collocation points. To this end, two matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are used to approximate the first, second, third, and fourth derivatives of Eq. (14). The expression for the solution W, for example, is given by

$$\boldsymbol{W}_j^{(i)}=\boldsymbol{A}_j^{(i)}\boldsymbol{W}_j+\boldsymbol{B}_j^{(i)}\boldsymbol{g}_j\qquad (i=1,\ldots,4\,;\ j=c,s), \tag{15}$$

in which the superscript (i) denotes the ith derivative,

$$\left.\begin{array}{l} \boldsymbol{W}_j^{(i)T}=\{W_j^{(i)}(x_1), W_j^{(i)}(x_2),\ldots, W_j^{(i)}(x_{M_j+1}), W_j^{(i)}(x_{M_j+2})\}, \\ \boldsymbol{W}_j^{T}=\{W_j(x_1), W_j(x_2),\ldots, W_j(x_{M_j+1}), W_j(x_{M_j+2})\}, \end{array}\right\} \tag{16}$$

and $\boldsymbol{g}^T=\{g_{j,M_j+3},\ g_{j,M_j+4}\}$. Moreover, x_i $(i=2,\ldots,M_j+1)$ are the interior collocation points, $x_1=0$, $x_{M_j+2}=1$, and $\boldsymbol{A}_j^{(i)}$ and $\boldsymbol{B}_j^{(i)}$ are the $(M_j+2)\times(M_j+2)$ and $(M_j+2)\times 2$ matrices, respectively.

The derivatives of U and V may, on the other hand, be written as

$$\boldsymbol{U}_j^{(i)}=\boldsymbol{A}_j^{(i)}\boldsymbol{U}_j,\quad \boldsymbol{V}_j^{(i)}=\boldsymbol{A}_j^{(i)}\boldsymbol{V}_j, \tag{17}$$

in which $\boldsymbol{U}_j^{(i)}$, $\boldsymbol{U}_j$, etc. are given by an expressions similar to Eq. (16).

There are 3 (M_c+M_s) +16 unknowns in Eq. (14). However, we obtain 3 (M_c+M_s) equations from Eq. (1). In addition to this, we have 16 boundary conditions.

Equation (14) should satisfy the equations of motion (Eq. (1)), at the interior collocation points. Therefore, we have the following equation :

$$\boldsymbol{\alpha}_1\boldsymbol{\delta}_1+\boldsymbol{\alpha}_2\boldsymbol{\delta}_2-\omega^2\boldsymbol{\delta}_1=\boldsymbol{0}, \tag{18}$$

in which $\boldsymbol{\delta}_1^T=\{U_s(x_2),\ldots,U_s(x_{M_s+1}),\ V_s(x_2),\ldots,V_s(x_{M_s+1}),\ W_s(x_2),\ldots,W_s(x_{M_s+1}),$
$U_c(x_2),\ldots,U_c(x_{M_c+1}),\ V_c(x_2),\ldots,V_c(x_{M_c+1}),\ W_c(x_2),\ldots,W_c(x_{M_c+1})\}$,

and $\boldsymbol{\delta}_2^T=\{U_s(x_1),\ U_s(x_{M_s+2}),\ V_s(x_1),\ V_s(x_{M_s+2}),\ W_s(x_1),\ W_s(x_{M_s+2}),\ g_{s,M_s+3},\ g_{s,M_s+4},$
$U_c(x_1),\ U_c(x_{M_c+2}),\ V_c(x_1),\ V_c(x_{M_c+2}),\ W_c(x_1),\ W_c(x_{M_c+2}),\ g_{c,M_c+3},\ g_{c,M_c+4}\}$.

$\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$ are the $3(M_c+M_s)\times 3(M_c+M_s)$ and $3(M_c+M_s)\times 16$ matrices which are composed of the elements of $\boldsymbol{A}_j^{(i)}$ and $\boldsymbol{B}_j^{(i)}$, and these matrices are expressed in partitioned matrix forms :

$$\boldsymbol{\alpha}_1=\left[\begin{array}{c|c} \underset{3M_s\times 3M_s}{\boldsymbol{\alpha}_s^1} & \boldsymbol{0} \\ \hline \boldsymbol{0} & \underset{3M_c\times 3M_c}{\boldsymbol{\alpha}_c^1} \end{array}\right],\quad \boldsymbol{\alpha}_2=\left[\begin{array}{c|c} \underset{3M_s\times 8}{\boldsymbol{\alpha}_s^2} & \boldsymbol{0} \\ \hline \boldsymbol{0} & \underset{3M_c\times 8}{\boldsymbol{\alpha}_c^2} \end{array}\right], \tag{19}$$

in which $\boldsymbol{0}$ is the null matrix, and the submatrices $\boldsymbol{\alpha}_s^i$ and $\boldsymbol{\alpha}_c^i$ $(i=1,2)$ are built from the equations of motion of the cylindrical and spherical shells, respectively.

The remaining 16 equations are obtained from the boundary conditions. These may be written as an expression similar to Eq. (18)

$$\boldsymbol{\beta}_1\boldsymbol{\delta}_1+\boldsymbol{\beta}_2\boldsymbol{\delta}_2=\boldsymbol{0}, \tag{20}$$

in which $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$ are the 16 x 3 (M_c+M_s) and 16 x 16 matrices, and these matrices are made up from appropriate submatrices as follows :

$$\boldsymbol{\beta}_1=\left[\begin{array}{c|c} \underset{4\times 3M_s}{\boldsymbol{\beta}_1^1} & \boldsymbol{0} \\ \hline \multicolumn{2}{c}{\underset{8\times 3(M_s+M_c)}{\boldsymbol{\beta}_1^2}} \\ \hline \boldsymbol{0} & \underset{4\times 3M_c}{\boldsymbol{\beta}_1^3} \end{array}\right],\quad \boldsymbol{\beta}_2=\left[\begin{array}{c|c} \underset{4\times 8}{\boldsymbol{\beta}_2^1} & \boldsymbol{0} \\ \hline \multicolumn{2}{c}{\underset{8\times 16}{\boldsymbol{\beta}_2^2}} \\ \hline \boldsymbol{0} & \underset{4\times 8}{\boldsymbol{\beta}_2^3} \end{array}\right], \tag{21}$$

in which the submatrices $\boldsymbol{\beta}_i^1$, $\boldsymbol{\beta}_i^2$, and $\boldsymbol{\beta}_i^3$ $(i=1,2)$ are built from Eqs. (7), (8), and (9), respectively.

Combination of Eqs. (18) and (20) leads to an 3 (M_c+M_s) eigenvalue problem

$$(\boldsymbol{\alpha}_1-\boldsymbol{\alpha}_2\boldsymbol{\beta}_2^{-1}\boldsymbol{\beta}_1)\boldsymbol{\delta}_1=\omega^2\boldsymbol{\delta}_1. \tag{22}$$

The solution of Eq. (22) yields the estimates for the 3 (M_c+M_s) natural frequencies and the corresponding eigenmodes.

4. NUMERICAL EXAMPLES

4. 1 Convergence and Comparative Study

Consider the example of a cylindrical shell with a hemispherical cap. The properties of the shell are : $h_c = h_s$, $h_c/a = 0.02$, $L_c/a = 1$, and $\nu = 0.2$. The shell is considered to have fully-clamped conditions ($U_c = V_c = W_c = W'_c = 0$) at the base of the cylinder. The same shell has also been analyzed by several investigators.

In the present study, both the cylindrical and spherical shells employed the same number of collocation points M (i. e. $M = M_c = M_s$), and the values of M were varied from $M = 9$ to $M = 12$. For $M = 9$ and 11, for example, location of the interior collocation points are shown in Fig. 2. As can be seen, the collocation points are distributed in the neighborhood of the apex, the base and the junction of the cylinder and the cap.

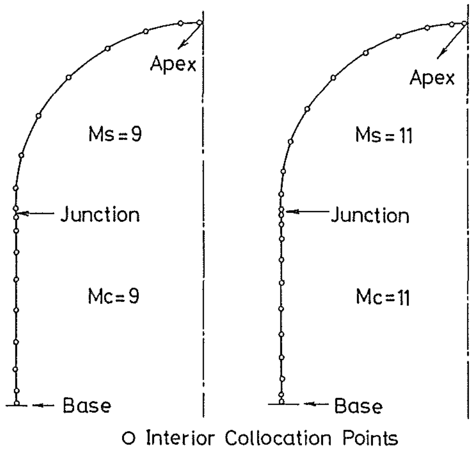

Fig. 2 Location of interior collocation points for $M = 9$ and 11 ($M = M_c = M_s$).

The results obtained by the present method for the first mode ($m = 1$) are summarized in Table 1, in which the computed values of the natural frequencies $a\omega\sqrt{\rho/E}$ are tabulated and compared with those obtained by the finite element method (**F.E.M**), and the finite difference method (**F.D.M**). The total number of mesh points for both the finite element and finite difference methods was 124. It may be observed from Table 1, that even with a small number of collocation points, the results compare very favorably with the results by other methods.

The numerical convergence of the present approach for higher modes is shown in Table 2. As can be seen, the natural frequencies converge rapidly to steady values. The analysis with the largest collocation points of $M = 12$ will be assumed to be 'exact' for comparison purposes. As it may be observed in Table 2, even with $M = 9$, the errors were within acceptable limits. For example, the errors in the fifth mode frequency are within 1 %.

4. 2 Parametric Studies

Numerical calculations were carried out to investigate the vibration characteristics of cylindrical shells with a spherical cap. The results for the natural frequencies and mode shapes are presented in Figs. 3~6 and table 3.

The results shown and tabulated are meant to illustrate the following aspects : 1) the influence of Poisson's ratio ν and half opening angle Φ, 2) the effect of boundary conditions at the base of the cylinder, and 3) influence of thickness to radius ratio h_c/a and length to radius ratio L_c/a. The boundary conditions considered are :

$$\left.\begin{aligned} C_1 &: U_c = V_c = W_c = W'_c = 0, \\ C_2 &: U_c = [S]_c = W_c = W'_c = 0. \end{aligned}\right\} \qquad (23)$$

Table 1 Natural frequencies ($a\omega\sqrt{\rho/E}$) for the axial mode $m=1$.

Circumferential mode n	F.D.M[2)]	F.E.M[2)]	This study $M=9$	$M=10$	$M=11$	$M=12$
0	0.64682	0.64682	0.64664	0.64664	0.64664	0.64664
1	0.29641	0.29650	0.29629	0.29629	0.29629	0.29629
2	0.50963	0.50910	0.50910	0.50910	0.50910	0.50910
3	0.41061	0.41061	0.41062	0.41062	0.41062	0.41062
4	0.34369	0.34369	0.34374	0.34374	0.34374	0.34374

Table 2 Natural frequencies ($a\omega\sqrt{\rho/E}$) for the higher modes.

Circumferential mode n	Mode number m	This study $M=9$	$M=10$	$M=11$	$M=12$
0	2	0.92289	0.92289	0.92289	0.92289
	3	0.98076	0.98077	0.98077	0.98077
	4	1.00581	1.00591	1.00590	1.00591
	5	1.02976	1.02997	1.02996	1.02996
1	2	0.78722	0.78722	0.78722	0.78721
	3	0.86635	0.86636	0.86635	0.86636
	4	0.96626	0.96626	0.96626	0.96625
	5	0.98172	0.98172	0.98172	0.98172
2	2	0.84153	0.84153	0.84153	0.84152
	3	0.91468	0.91467	0.91468	0.91462
	4	0.97238	0.97238	0.97238	0.97240
	5	1.00953	1.00966	1.00966	1.00968
3	2	0.77107	0.77107	0.77107	0.77107
	3	0.94299	0.94299	0.94299	0.94299
	4	0.97666	0.97666	0.97666	0.97666
	5	1.01899	1.01890	1.01891	1.01891

Table 3 Effect of Poisson's ratio on frequency parameter ($a\omega\sqrt{\rho/E}$).

Circumferential mode n	Mode number $m=1$ $\nu=0.15$	$\nu=0.3$	Mode number $m=2$ $\nu=0.15$	$\nu=0.3$
0	0.4969	0.4897	0.8706	0.8457
1	0.1782	0.1721	0.5247	0.5178
2	0.2891	0.2820	0.5927	0.5852
3	0.1995	0.1961	0.4507	0.4460
4	0.1477	0.1465	0.3480	0.3449
5	0.1268	0.1276	0.2813	0.2801

The calculations were performed with $M_c = M_s = 11$.

Table 3 shows the effect of Poisson's ratio on the frequency parameter. The calculation was carried out for $h_c = h_s$, $h_c/a = 0.01$, $L_c/a = 2$, $\Phi = 90°$, and the boundary condition C_1. As shown in Table 3, Poisson's ratio has little effect on the frequency parameter.

The variation of the natural frequency $a\omega\sqrt{\rho/E}$ with the half opening angle Φ is shown in Fig. 3 for $h_c = h_s$, $h_c/a = 0.01$, $L_c/a = 2$, $\nu = 0.3$, and the boundary condition C_1. The values of n were varied from 0 to 10. The results indicate that

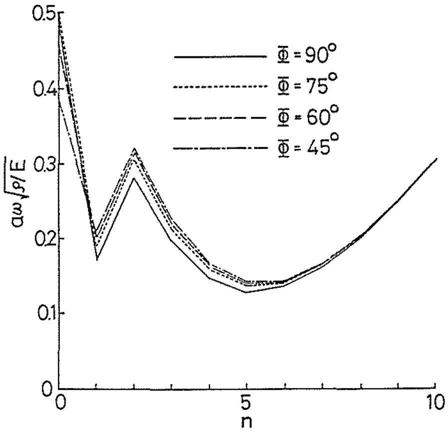

Fig. 3 Effect of half opening angles on fundamental frequencies ($h_c = h_s$, $L_c/a = 2$, $h_c/a = 0.01$, $\nu = 0.3$).

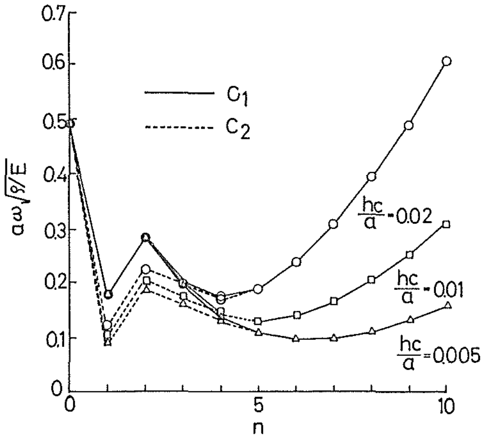

Fig. 4 Effect of length to radius ratios (L_c/a) on fundamental frequencies ($h_c = h_s$, $\Phi = 90°$, $h_c/a = 0.01$, $\nu = 0.3$).

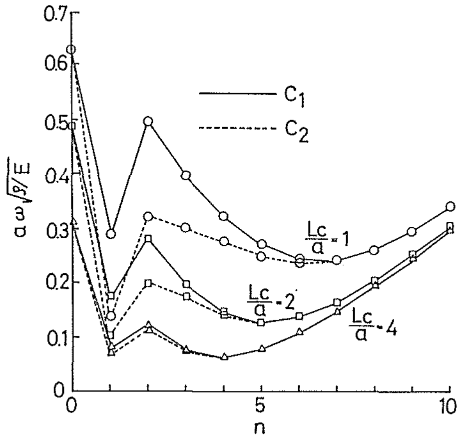

Fig. 5 Effect of thickness to radius ratios (h_c/a) on fundamental frequencies ($h_c = h_s$, $\Phi = 90°$, $L_c/a = 2$, $\nu = 0.3$).

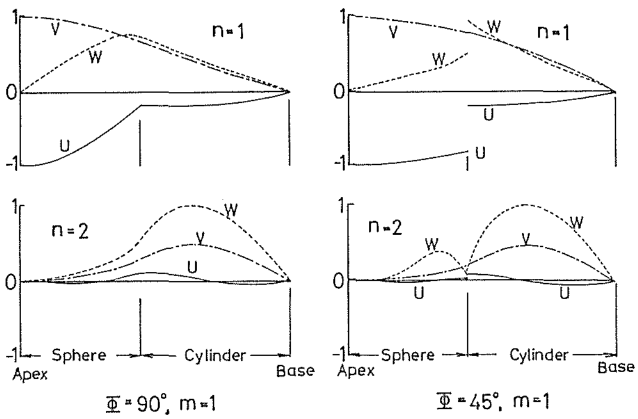

Fig. 6 Mode shapes of cylindrical-spherical shell for half opening angle $\Phi = 45°$ and $90°$ ($h_c = h_s$, $L_c/a = 2$, $h_c/a = 0.01$, $\nu = 0.3$).

changes in the half opening angle have little influence on the frequency.

In Figs. 4 and 5, the frequency parameters of cylindrical-hemispherical shells with the boundary conditions C_1 and C_2 are given for different ratios of L_c/a and h_c/a. As can be seen, the effect of the boundary conditions is greater for low values of n, L_c/a, and h_c/a. Moreover, the results in Figs. 4 and 5 indicate that for high values of n, the influence of the boundary conditions is limited regardless of the L_c/a and h_c/a. Furthermore, for the boundary condition C_2 the minimum natural frequency occurs at $n=1$. For the boundary condition C_1, on the other hand, the minimum natural frequency occurs at values of n larger than 1.

Figure 6 shows mode shapes for mode 1 and $n=1$, 2. The figure indicates that as n increases, the motion in the cylinder is predominant. Moreover, it is also indicated that the presence of the discontinuity point results in rapid changes in the mode shape in the neighbourhood of this point.

5. SUMMARY AND CONCLUSIONS

The collocation method was presented for the free vibration analysis of shells which are composed of combinations of cylindrical and spherical shells.

Numerical results involving comparisons with other numerical solutions indicate the advantages of the present approach. One of the main advantages of the present method as compared to other approximate methods is that a relatively high degree of accuracy is obtained even with a reasonably small degrees-of-freedom. In addition, this method appears to be easy to formulate and simple to use when shells having the discontinuity point must be considered.

The method was used to determine the dynamic characteristics of cylindrical shells with spherical caps and the frequencies were obtained for different shell parameters. A series of charts which would be readily applicable for the determination of the fundamental natural frequencies was developed.

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