北海道大学工学部研究報告
第 136 号（昭和 62 年）

Bulletin of the Faculty of Engineering,
Hokkaido University. No. 136 (1987)

Theoretical Analysis of Feeding Behavior during Solidification of Binary Alloy Ingots

Farooq SIDDIQUI* Tadayoshi TAKAHASHI*
Masayuki KUDOH* and Kenichi OHSASA*

(Received March 31, 1987)

Abstract

The feeding of an alloy ingot consists of a compensation of metal due to volume contraction in liquid and solid states before and after solidification, and is due to the volume contraction during solidification. The measured feeding represents the total amount of feed from the pouring temperature in liquid state to room temperature in solid state, and poses a problem of separation of solidification contraction and thermal contraction. Partitioning became possible by the theoretical analysis of the feeding behavior that utilized the numerical method established by the authors. The analyses were carried out for unidirectionally and cylindrically solidified ingots. In both cases, the comparison of measured and calculated results shows that, the total calculated feed and the actual measured feed shows a good agreement if 0.67 fraction solid is used as the limit to direct feeding during solidification.

1. Introduction

It is important to predict the necessary supply of liquid metal from the feeder head to compensate for the volumetric contractions in a solidifying ingot in order to secure the soundness of the ingot and to cut down on the economical loss. The feeding of an alloy ingot consists of a compensation of metal due to volume contraction in liquid and solid states before and after solidification, and is due to volume contraction during solidification. The former is known as thermal contraction, while the latter is known as solidification contraction. In general, it is difficult to directly measure and compare the amount of feeding. The measured feeding represents the total amount of feed from the pouring temperature in liquid state to room temperature in a solid state[1,2], and it poses the problem of separation of solidification contraction from thermal contraction. Feeding of liquid metal during solidification is limited until the solid entraps the liquid. If the feeding due to the effect of each of the above mentioned factors can be partitioned, then the problem of feeding during solidification can be studied in isolation from other influencing factors. Furthermore it is important to clearly establish the limit of liquid feeding and also to predict the effect of

* Department of Metallurgical Engineering, Faculty of Engineering, Hokkaido University

creep deformation on the feeding behavior.

In this study a method for theoretical analysis of feeding behavior is presented for binary alloy ingots and the calculated results are compared with the measured feeding behavior of Al-3 wt%Si alloy ingots. The measured feeding behavior during unidirectional and cylindrical solidification of these ingots was presented in previous paper[3].

2. Calculation method

The temperature profile in a solidifying ingot is usually spread over a large temperature range. Thus, it is not appropriate to deal with the ingot as a whole. This problem can be overcome by partitioning the ingot into minute volumes, thereby termed as volume elements, in which the temperature difference can be neglected.

It is assumed that the contracted volume of the element per unit time, due to the fall in temperature or change in state of that element, is equivalent to the necessary amount of the liquid metal in the feeder head and can be represented by the following differential equation:

$$-d(F\rho^r)/dt = d(V\rho)/dt \tag{1}$$

where F is the volume of molten metal in the feeder head, t is time, ρ^r and ρ are the densities of metal in the feeder head and volume element and V is the volume of the element. The negative sign indicates a decrease of the metal volume in the feeder head.

The dimensions of each element are dependent on the temperature of the mold, below the limiting fraction solid (to feed the interdendritic spacing), and also on the volumetric contraction of the alloy (beyond the limiting fraction solid). This will be later explained in detail.

The density of the volume element, in the right hand side of equation 1, represents liquid density, when the element is in a liquid state, the density of the liquid-solid coexisting zone, when the element is in the solidification stage, and solid density, when the element is in the solid stage. The term on the left hand side of equation 1 represents the feeding amount of liquid metal from the feeder head, while the right hand side represents the contraction of the element.

Solving the right hand side of equation 1, we get the following differential form:

$$d(V\rho)/dt = V \times d\rho/dt + \rho \times dV/dt. \tag{2}$$

By changing the above equation to the finite difference form, for the time period Δt, between time t and time $t+\Delta t$, it can be written as,

$$d(V\rho)/dt = V_t \times (\rho_{t+\Delta t} - \rho_t)/\Delta t + \rho_{t+\Delta t} \times (V_{t+\Delta t} - V_t)/\Delta t \tag{3}$$

which comes to,

$$d(V\rho)/dt = (V_{t+\Delta t}\rho_{t+\Delta t} - V_t\rho_t)/\Delta t. \tag{4}$$

Similarly, rearranging the left hand side of equation 1, for Δt, we get,

$$-d(F\rho^r)/dt = -(F_{t+\Delta t}\rho^r_{t+\Delta t} - F_t\rho^r_t)/\Delta t \tag{5}$$

Now, if we consider ΔF as the difference between the initial and final volumes of the liquid metal in the feeder head in time Δt, i. e. $F_{t+\Delta t} = F_t - \Delta F$, then equation 5 becomes,

$$-d(F\rho^r)/dt = [\Delta F\rho^r_{t+\Delta t} - F_t(\rho^r_{t+\Delta t} - \rho^r_t)]/\Delta t \tag{6}$$

Now, by equating equations 4 and 6 and rearranging for ΔF we get,

$$\Delta F/\Delta t=(V_{t+\Delta t}\rho_{t+\Delta t}-V_t\rho_t)/(\Delta t\rho^r_{t+\Delta t})+F_t(\rho^r_{t+\Delta t}-\rho^r_t)/(\Delta t\rho^r_{t+\Delta t}) \quad (7)$$

The first term on the right hand side of equation 7 shows the contracted volume of the element for time interval Δt and the second term indicates the contracted volume of the liquid in the feeder head due to the fall in temperature of the melt. Feed volume, to compensate the volumetric contraction in a solidifying ingot, for the unit time interval, can be calculated by summating the first term of the right hand side of equation 7 for total number of elements, n, and adding the second term of the right hand side.

Thus, the total feed volume compensated from the feeder head between time t and time $t+\Delta t$, can be expressed as,

$$\Delta F/\Delta t=[\sum_{i=1}^{n}(V_{t+\Delta t}\rho_{t+\Delta t}-V_t\rho_t)/(\Delta t\rho^r_{t+\Delta t})]+F_t(\rho^r_{t+\Delta t}-\rho^r_t)/(\Delta t\rho^r_{t+\Delta t}) \quad (8)$$

For calculating the total volumetric contraction, F_v from the initial time $t=0$ to time t_l, we can sum up the feeding for each time interval. This can be expressed as,

$$F_v=\sum_{t=0}^{t_l}\Delta F \quad (9)$$

If the mold of the feeder head is assumed to be cylindrical, then the total volumetric change F_v, can be converted into the fall in the level of the melt in the feeder head, by the following relationship:

$$F_h=F_v/(\pi r_r^2) \quad (10)$$

where F_h is the shift in metal depth for the corresponding feed volume in the feeder head of the radius r_r.

3. Experimental model for analysis

An experimental model was necessary to compare the calculated and actually measured feeding behavior of the ingots. The unidirectionally and cylindrically solidified cylindrical ingots, for which the feeding behavior was measured and presented in our previous paper[3], were used as models for the analysis.

The unidirectionally solidified ingot was 60 mm in diameter and 85 mm in height. The ingot had a feeder head of 20 mm in diameter at its top. The molds for the ingot and the feeder head were made of graphite. The ingot was partitioned at 85 levels along its height and each level was again partitioned into six radial elements, as shown in Figure 1, for the purpose of the analysis of the feeding behavior of the ingot.

The cylindrically solidified ingot was 80 mm in diameter and 85 mm in height, The ingot had a feeder head of 25 mm in diameter at its top. The mold for the ingot was made of a steel pipe, and the mold base was made of isolite and the mold for the feeder head was made of graphite. The ingot was theoretically partitioned in 85 levels along its height and each level was again partitioned into 40 radial elements, as shown in Figure 2.

The thermal distribution in the ingot was measured by inserting 17 thermocouples in the unidirectionally solidified ingots and 18 in the cylindrically solidified ingots. The temperatures of the volume elements, for the analysis of the feeding behavior, were determined from the thermal distribution diagram obtained by the thermocouples.

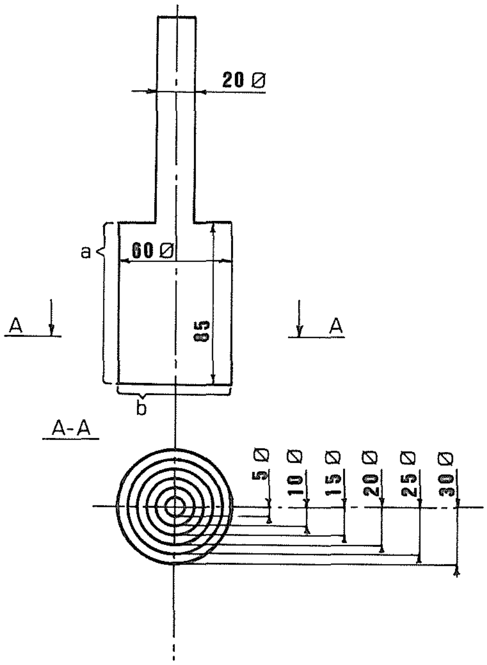

Fig. 1 Volume partition scheme of the unidirectionally solidified ingot for computer simulation of feed behavior during solidification. (unit-mm)
a ; 85 horizontal partitions of 1 mm height each along the height of the ingot,
b ; 6 radial partitions for each horizontal partition level,
A-A is the cross section of the ingot.

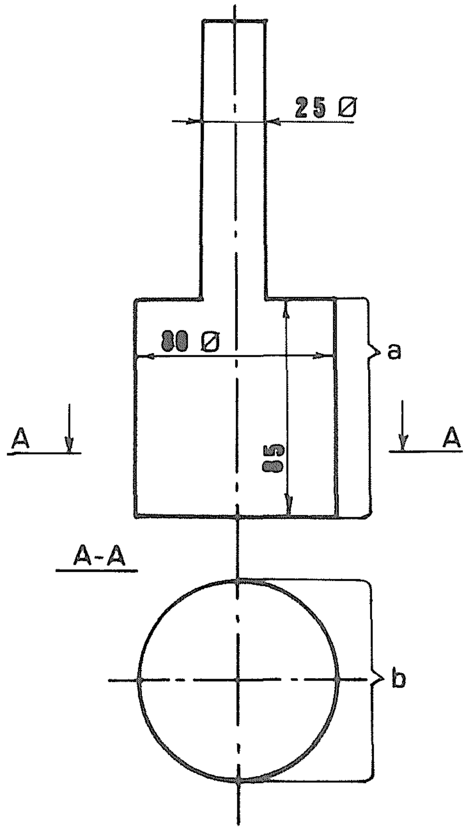

Fig. 2 Volume partition scheme of the cylindrically solidified ingot for computer simulation of feed behavior during solidification. (unit-mm)
a ; 85 horizontal partitions of 1 mm height each along the height of the ingot,
b ; 40 radial partitions for each horiznontal partition level,
A-A is the cross section of the ingot.

4. Dimensions of the element

The ingot, during the liquid state, takes the shape of the mold, which by itself, expands and contracts with the changes in temperature. After the initiation of solidification, the ingot holds the shape of the mold until the direct feeding stops from the metal in the feeder head. A solid skeleton is formed, during this period, and develops to such an extent that further direct feeding becomes impossible. However, after crossing the limit of direct feeding, the solid skeleton, which entraps the liquid, shrinks radially and an air gap is formed between the mold and the solidifying ingot. These changes have a significant influence upon the correct prediction of feeding behavior. To cover these aspecs which affect the feeding behavior, a concept of variable element is developed. The dimensions of the element are adjustable according to the respective conditions of the element.

(1) Dimensions of the element below the direct feeding limit

An air gap is formed when the element neighboring the inner walls of the mold crosses the limit of direct feeding. Therefore, before the inner wall temperature of the mold at the horizontal level reaches the temperature corresponding to the limiting fraction solid, the volume of that element is calculated as follows:

$$V = V_r \times [1 + (T_o - 20) \times \alpha_0 \times 3] \quad (11)$$

where V_r is the volume of the element at room temperature, T_o is the temperature of the mold wall at the same height of the mold as the element and α_0 is the linear expansion coefficient from the room terperature to the temperature T_o for the material of the mold.

(2) Dimensions of the element above the limiting fraction solid

After the outermost shell of the ingot crosses the liniting fraction solid, an air gap is formed between the mold and the solidifying ingot and there is no feeding to compensate this gap. This can be easily taken into account, by changing the volume of the elements at the same horizontal level, accordingly.

After the formation of the air gap the solidifying outer shell contracts inwards. Therefore the radii of the respective elements, at a certain horizontal level, which have crossed the limiting fraction of the solid, do not depend on the inner radius of the mold, but it changes proportionally to the volumetric contraction of the alloy.

By knowing the thermal distribution in the ingot, we can calculate the dimensions of the element and predict its volume at time $t+\Delta t$. Let us consider the schematic diagram in Figure 3. It represents a radial volume element which has passed the limiting fraction solid. The inner and outer radii and the height of the element, at time t, are shown as Ri_t, Ro_t and H_t respectively. The difference between Ri_t and Ro_t, for all the elements at a certain horizontal level, is taken as constant at the beginning of each cycle of calculation for the subsequent time intervals. Above the limiting fration solid direct feeding stops and the remnant liquid is entrapped by the growing solid skeleton. With a further fall in temperature this radial element shrinks because of the solidification of the entrapped liquid which in turn forces the contraction of the surrounding solid. This is termed as creep deformation. At time $t+\Delta t$, the previous volume of the element V_t has contracted in radial proportionality to new dimensions of the inner and outer radii and the contracted height, denoted as $Ri_{t+\Delta t}$ and $Ro_{t+\Delta t}$ and height H^*. This new volume, V^* still does not represent the volume of the element, at time $t+\Delta t$, but represents the proportionately contracted volume of the element, from time t to time $t+\Delta t$. Volumes V_t and V^* represent the same

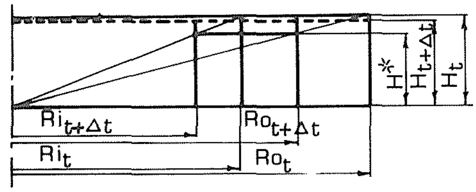

Fig. 3 Schematic illustration of shrinkage of a radial volume element after crossing the feeding limit in the solidification zone.
Ri, Ro and H are the inner and outer radii and the height of an element respectively. H^* is the contracted height of the element after shrinkage. Suffix t and $t+\Delta t$ indicate time.

enclosed mass of the alloy.

The inner and outer radii, $Ri_{t+\Delta t}$, $Ro_{t+\Delta t}$, of the volume element, at time $t+\Delta t$, are the same as those of the contracted volume V^*, because the radial contraction is not fed by the liquid from the feeder head due to the formation of the air gap. The height of the volume element depends upon the thermal conditions of the mold. Thus, by knowing the temperature of the mold wall, the element height $H_{t+\Delta t}$ can be determined.

The following relationships can be deduced from the proportional geometry of the radial elements:

$$Ro_t/H_t = Ro_{t+\Delta t}/H^* = Y \tag{12}$$

$$Ri_t/H_t = Ri_{t+\Delta t}/H^* = Z \tag{13}$$

$$H_t/H^* = Ro_t/Ro_{t+\Delta t} \tag{14}$$

$$1-(Z^2/Y^2) = (Ro_t^2 - Ri_t^2)/Ro_t^2 \tag{15}$$

Y and Z are constants of proportionality.

By substituting the value of H^* from equation 12 into equation 13 we get,

$$Ri_{t+\Delta t} = Ro_{t+\Delta t} \times (Z/Y) \tag{16}$$

The mass enclosed by V_t and that of the contracted volume V^*, is the same. Thus, the following relationship between the two volumes can be deduced,

$$V^* = \rho_t \times V_t / \rho_{t+\Delta t} \tag{17}$$

Furthermore, the two volumes can also be presented through their dimensions as follows,

$$V_t = \pi(Ro_t^2 - Ri_t^2)H_t\ ;\ V^* = \pi(Ro_{t+\Delta t}^2 - Ri_{t+\Delta t}^2)H^* \tag{18}$$

By substituting the values of V_t and V^* from equation 18, into equation 17 we get,

$$\rho_{t+\Delta t}/\rho_t = (Ro_t^2 - Ri_t^2)H_t/(Ro_{t+\Delta t}^2 - Ri_{t+\Delta t}^2)H^* \tag{19}$$

By substituting the values of $Ri_{t+\Delta t}$ from equation 16 and H_t/H^* from equation 14, into equation 19, we get,

$$\rho_{t+\Delta t}/\rho_t = (Ro_t^2 - Ri_t^2)Ro_t/[1-(Z^2/Y^2)]Ro_{t+\Delta t}^3 \tag{20}$$

By substituting the $1-(Z^2/Y^2)$ from equation 15 into equation 20 and rearranging for $Ro_{t+\Delta t}$ we get,

$$Ro_{t+\Delta t} = Ro_t \times (\rho_t/\rho_{t+\Delta t})^{1/3} \tag{21}$$

$$\text{Similarly } Ri_{t+\Delta t} = Ri_t \times (\rho_t/\rho_{t+\Delta t})^{1/3} \tag{22}$$

Now, as explained before, the height of the new volume element $H_{t+\Delta t}$, depends on the linear expansion of the mold material, at its temperature T_o, thus, it can be determined as follows,

$$H_{t+\Delta t} = H_o \times [1+(T_o-20)\times \alpha_o] \tag{23}$$

where H_o is the initial height of volume element at room temperature.

The volume at time $t+\Delta t$, denoted by $V_{t+\Delta t}$, can be given as,

$$V_{t+\Delta t} = \pi(Ro_{t+\Delta t}^2 - Ri_{t+\Delta t}^2) \times H_{t+\Delta t}$$

By using the values from equation 21 and 22 and 23, and by eliminating the unknown values, the volume of the element at time $t+\Delta t$ can be calculated as,

$$V_{t+\Delta t} = \pi(\rho_t/\rho_{t+\Delta t})^{2/3} \times (Ro_t^2 - Ri_t^2) \times H_{t+\Delta t} \tag{24}$$

(3) Dimensions of the inner elements after formation of air gap

The formation of an air gap between the solidifying ingot and the mold occurs when the radial elements, adjacent to the inner walls of the mold, cross the limiting fraction solid. In cylindrical solidification the solidification front proceeds from circumference to the center of the ingot. Therefore, at a certain horizontal level, the metal, in the inner elements which are below the limiting fraction solid, is squeezed due to the inward shrinkage of the solidifying elements, which crossed the limiting fraction solid.

This situation is illustrated in Figure 4. We assume that the temperatures of the two inner elements are higher than that corresponding to the limiting fraction solid, while that of the outer elements is lower. This indicates that direct feeding has stopped for the outer element, while the inner two elements still have access to liquid feeding.

Though the alloy in the inner elements does not contract radially, but the adjacent outer element does, so that the subsequent radii of the inner elements also decreases and are calculated as follows:

$$Ro_{t+\Delta t} = R_s \times m/N \text{ and } Ri_{t+\Delta t} = R_s \times (m-1)/N \tag{25}$$

where R_s is the inner radius of the innermost element which has crossed the limiting fraction solid temperature, N is the total number of elements below the limiting fraction solid temperature, at a given height and m is the serial number of the elements under consideration as counted from the innermost element.

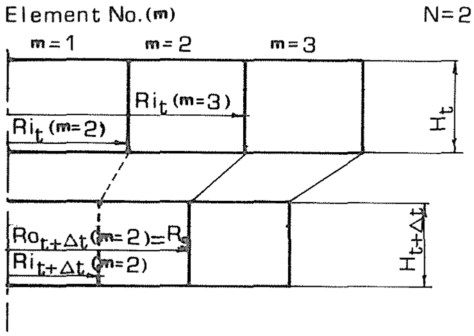

Fig. 4 Schematic illustration of change in dimensions of inner volume elements after the outer element crosses the feeding limit in the solidification zone.
m is the serial number of the radial elements. N is the number of the elements which have still not crossed the limiting fraction solid. Ri, Ro and H are the inner and outer radii and the height of an element respectively. Suffix t and $t+\Delta t$ indicate time.

5. Results and discussion

5. 1 Unidirectionally solidified ingot

The results of the calculated feed, for a unidirectionally solidified ingot are given in Figure 5. The calculated curves show the partitioned feeding of the solidifying ingot expressed as the fall of the metal level in the feeder head, in relation to passage of time. Furthermore the figure also shows the comparison of the total feeding curve with the actual measured feeding curve for that ingot. The figure shows the partitioned feeding expressed as a fall of the feeder head level, in relation to the passage of time.

Curve L shows the feeding from the feeder head, arising from the thermal contraction of the alloy in the liquid state, and from the pouring temperature to the liquidus temperature. Feed due to contraction in the liquid state becomes faster, with the beginning of cooling by water shower, from the base of the ingot, and the feed gradually decreases as the volume of liquid metal above the liquidus temperature decreases. The slope of the curve eventually

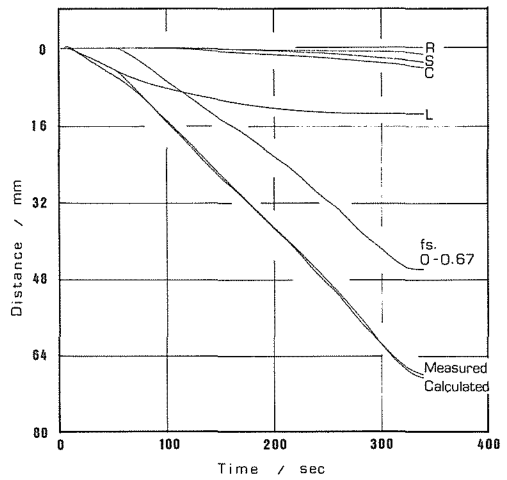

Fig. 5 Calculated partitions of the fall in feeder head level and comparison of total calculated fall and measured fall in the feeder head level due to feeding during unidirectional solidification of Al-3 wt%Si alloy ingot having an initial cooling rate of 3.0℃/sec.
L, fs 0-0.67, C and S indicate the fall in the feeder head level due to, the thermal contraction in liquid state, the solidification contraction before crossing the limiting fraction solid, the solidification contraction after crossing the limiting fraction solid and the thermal contraction in solid state respectively of the alloy in the ingot. R shows the fall due to the contraction of the alloy in the feeder head.

becomes zero which shows that no more feeding occurs due to the liquid state contraction.

Curve, denoted as " fs. 0-0.67 ", indicates the fall in the feeder head level due to the direct feeding of the alloy to the interdendritic spacing during the liquid-solid transformation. The fraction solid of 0.67 represents the limit of flow into interdendritic channels resulting from the growth of dendrites which entrap the remnant liquid. Therefore the fraction solid of 0.67 also represents the limit of direct feeding from the bulk liquid. This limit to direct feeding is discussed in detail in our previous study[3].

Curve C indicates the feeding occurring due to creep deformation of the solid, because of the solidification of the entrapped liquid above the limiting fraction solid. This is an indirect feeding, as direct contact with the liquid metal is broken off, but still it results in a fall of the metal level in the feeder head, because of the volume change due to creep deformation. At this stage an air gap is formed between the ingot and the mold. This air gap is not fed from the feeder head, so the feeding from the feeder head becomes comparatively low.

Curve S shows similar feeding due to thermal contraction, with the fall in temperature of the completely solidified alloy. This is also comparatively small because the contraction along the horizontal axis of the ingot is also not fed from the feeder head, similar to the case of creep deformation.

Curve R indicates the fall in feeder head level, due to the thermal contraction of the liquid in the feeder head. As the volume of liquid in the feeder head is small, compared to that of the ingot, the feeding is also relatively smaller. Furthermore, the alloy in the feeder head is kept in a liquid state, by heating it with an electrical furnace, thus, it does not solidify, although the liquid in the mold just below the feeder head solidifies.

Curves, for direct feeding in the solidification zone, feeding arising from creep deformation and feeding due to solid contraction starts when the first volume element enters the respective state.

Total calculated feeding from the feeder head can be ascertained, by adding feeding due to liquid and solid contraction, creep deformation and direct feeding to the solidification zone. The calculated feeding is shown by the curve marked as " calculated ". The actual measured feeding curve of the ingot, obtained from the same experiment, is marked as " measured ". The calculated and the measured feeding curves are in good agreement with each other. This is also true for all other experiments carried out by the authors[4]. The calculated feeding curve is somewhat lower than the measured one, because the calculations are carried out for ideal density, whereas experimentally obtained ingots contains some microporosity. The calculated and the measured curves show a good agreement only when 0.67 fraction solid is used, as the limiting fraction solid, to feeding. If the limiting fraction solid is taken below this value, then the calculated feeding curve does not meet the measured one. In the opposite case, if the limiting fraction solid is taken as more than 0.67 fraction solid, then the calculated curve shows higher feeding than that of the actual measured feeding curve.

Comparing the partitioned feeding for different stages from complete liquid to complete solid, it becomes clear that direct feeding in the solidification zone, i. e. in the range of 0 to 0.67 fraction solid, represents the biggest component of total feed to the ingot and any feeding deficiency in this region can be determinant to the soundness of the ingot.

5. 2 Cylindrically solidified ingot

The results of the calculated feed, for a cylindrically solidified ingot are given in Figure 6. The calculated curves show partitioned feeding of the solidifying ingot expressed as the fall of the metal level in the feeder head, in relation to passage of time. Furthemore the figure also shows the comparison of the total feeding curve with the actual measured feeding curve for that ingot. The figure shows the partitioned feeding expressed as a fall of the feeder head level, in relation to the passage of time.

Curves L, " f_s 0-0.67", C, S, R, " calculated " and " measured " show the feeding due to liquid contraction, direct feeding during liquid-solid transformation,

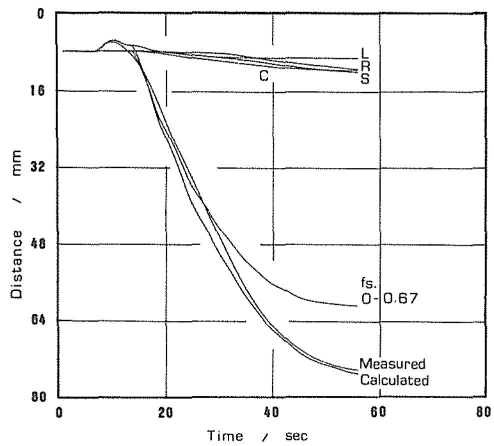

Fig. 6 Calculated partitions of the fall in feeder head level and comparison of total calculated fall and measured fall in the feeder head level due to feeding during cylindrical solidification of Al -3 wt%Si alloy ingot having an initial cooling rate of 15.0℃/sec.
L, fs 0-0.67, C and S indicate the fall in the feeder head level due to, the thermal contraction in liquid state, the solidification contraction before crossing the limiting fraction solid, the solidification contraction after crossing the limiting fraction solid and the thermal contraction in solid state respectively of the alloy in the ingot. R shows the fall due to the contraction of the alloy in the feeder head.

feeding due to creep deformation, feeding due to solid contraction, total calculated feeding and measured feeding, respectively.

The fall in the feeder head level due to liquid contraction is very small. This is due to the contraction of the mold, which in these cases is made of steel. As the mold is sprayed with water, when it is at 800℃, it immediately contracts. The contraction of liquid metal is comparatively smaller than that of the mold volume, which results in an increase of the feeder head level, initially. In any event, feeding due to the contraction in liquid state or that in the solid state does not effect the soundness of the ingot.

As in the case of calculated results for unidirectional experiments, if 0.67 fraction solid is considered as the limiting fraction solid for direct feeding, the calculated results for the cylindrically solidified ingots also show good agreement with the measured results.

6. Conclusions

The contraction during solidification of an ingot is prolonged from pouring till cooling down to room temperature, which makes it impossible to experimentally separate the feed for the liquid-solid coexisting region. The partitioning became possible by theoretical analysis of the feeding behavior utilizing the numerical method established by the authors. Feeding of the ingot, represented as the fall in the feeder head level of the solidifying ingots, was partitioned and the calculated results of the fall in the feeder head level were compared with the actual measured ones. Analyses were carried out for unidirectionally and cylindrically solidified ingots. In both cases, the comparison of measured and calculated results shows that, the total calculated feed and the actual measured feed are in good agreement if 0.67 fraction solid is used as the limit for direct feeding during solidification. It shows that 0.67 fraction solid represents the limit to direct feeding from the bulk liquid. Furthermore it also shows that feeding from 0 to 0.67 fraction solid, in the process of liquid -solid transformation, plays the most important role in the feeding behavior of the ingot.

Literature

1. B. P. Winter, R. D. Rehlke and P. K. Trojan ; Trans. AFS, vol. 91, 1983, p. 81.
2. R. Ichikawa and M. Hashiura ; Imono, vol. 39, 1967, p. 883.
3. F. Siddiqui, T. Takahashi, M. Kudoh and K. Ohsasa ; Bull. Fac. Eng., Hokkaido University, No 136, 1987, p. 21.
4. M. F. U. Siddiqui ; A Study of feeding Behavior of Binary Alloy Ingots, Doctoral Thesis, Department of Metallurgical Engineering, Faculty of Engineering, Hokkaido University, Mar 1987.