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Multi-Stable States in Nonlinear Conduction in a Semiconductor at low Temperatures

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Abstract

A model for recombination instability of a semiconductor in the freezeout regime is investigated. It is shown that the nonlinear redistribution of trapped electrons among the ground state, the excited state and the conduction band under impact ionization lead the bistable states in the density of the conduction electron. The distinct relation between the macroscopic instability and the microscopic inoization and recombination processes are also investigated.

1. Introduction

The current instabilities in bulk and inhomogeneous semiconductors have been one of the major subjects in the field of semiconductor physics and technology, and extensive investigations have been carried out by various authors for a long time. In spite of these investigations, the physical origins of several current instabilities still remain unsolved. The current instability accompanied by impact ionization of shallow donors at low temperatures is a typical example of the above mentioned problems.

A new theoretical approach to solve the problems of nonequilibrium instabilities and transition in dissipative structure has been developed by Prigogine, Nicolis and co-workers [1]. Consider a physical system under open or closed condition in such a way that it would be possible to maintain a time-independent macroscopic state. At or near thermal equilibrium, the macroscopic description of such a system gives a unique stationary solution. However, in a highly excited system which is far from equilibrium, the kinetic equations constolling the system become nonlinear. Beyond a certain critical deviation from equilibrium, the continuous extension of the equiligrium solution becomes unstable in response to a small perturbation, and such a perturbation will bring forth newly organized states which are completely different from the equilibrium state.

In the highly excited nonlinear system, there exists the variety of possible solutions depending on the external constrictions imposed on the system. The time-independent homogeneous and inhomogeneous states as well as states which are organized in time or in time and space, namely periodic and wavelike oscillations, might be established.

In this paper, we investigate the occurence of the bistable states in semiconductors at low temperatures applying the above theory of a dynamical system. We pursue the distict relation between the macroscopic current instability and microscopic ionization and recombination processes in semiconductors at low temperatures.

2. Generation-recombination model

Let us consider a partially compensated n-type semiconductor at low temperatures. We perform the present analysis under the following assumptions ; 1) The donor states can be expressed by the combination of the single excited state and the ground state. The highly excited states lying very close to the conduction band are neglected because of their large probabilities of the reemission of electrons into the conduction band. 2) The effect of the impurity conduction is neglected for simplicity. 3) The broadening of the impurity levels due to random fluctuation of the impurity potentials and the tailing of the conduction band are also neglected. 4) We limit ourselves to treat the homogeneous steady states.

The phenomenological rat equations[2] for the recombination-generation (g-r) processes are given by

$$dn/dt = \mathrm{X}_1 n\ n_1 + (\mathrm{Xls} + \mathrm{X}_1^* n) n_2 - \mathrm{Tls}(\mathrm{Na} + n) n = \mathrm{F}_0 \quad (1)$$

$$dn_1/dt = -(\mathrm{X}^* + \mathrm{X}_1 n) n_1 + \mathrm{T}^* n_2 = \mathrm{F}_1 \quad (2)$$

$$n_2 = (\mathrm{N_d} - \mathrm{N_a}) - n - n_1 \quad (3),$$

where n is the conduction electron density, n_1 is the concentration of the ground state with trapped electron, n_2 is the concentration of the excited state with trapped electron, respectively. Tls and Xls are the thermal recombination and generation coefficients of the excited state, T^* and X^* are the transition probabilities of the electron between the excited state and ground state, X_1^* and X_1 are the impact ionization coefficients of the excited state and the ground state, respectively. Nd and Na are the donor and acceptor concentrations. The above g-r processes are represented schematically in Fig.1. The direct recombination of conduction electron with the ground state of donors are expected to be negligible compared with the other recombination processes. The appearance of the n^2 term in Eq. (1) and the n_1^2 term in Eq. (2) identify these as nonlinear differential equations.

In order to describe the real physical processes in a semiconductor by the above rate equations, it is necessary to determine the various g-r rate coefficients involved in these equations taking account of the microscopic interations in these processes.

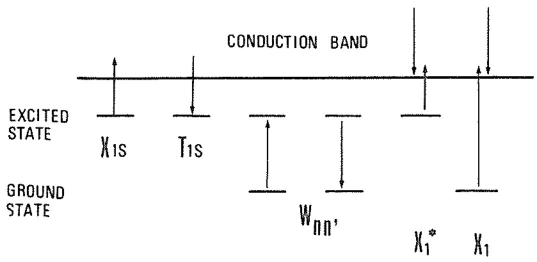

Fig. 1 Generation-recombination processes involving the conduction band, donor ground state and excited state.

The dominant mechanisum in electron-ionized donor recombination is that the recombination energy is released in the form of phonons. We discuss the impact ionization and phonon-mediate recombination processes in the following section.

3. Impact ionization and phonon-mediate recombination

3. 1 Impact ionization coefficient and electron temperature

The impact ionization coefficient for a trap with depth E_l can be computed from a treatment by Cohen and Landsbberg[3]. The calculation of the probability of an impact ionization in time t after the perturbation has been switched on is based on the well-known result $T=(2t^2/\hbar^2)|U|^2(1\text{-}\cos x)/x^2$, where x is $(t/\hbar)$ times the change of energy in the transition. Here, U is the screened Coulomb potential interaction between electrons in the conduction band.

The impact ionization coefficient X_l for a given trapped state with (N, 1) where N is the principal quantum number, is given by

$$X_l = D\,I(+) - I(-) \qquad (4\,a),$$

where

$$I(+) = [k_d(k_d-k_0)(k_d+k_0)^2/2q + (1/q^3) + (9k_d^2+7k_dk_0)/(4q^2)]$$
$$\times exp[-q(k_0-k_d)^2]$$
$$\times[15k_d/4q^{5/2}) + (5k_d^3-3k_0^2k_d)/(q^{3/2}) + (k_d(k_d^2-k_0^2)^2/(q^{1/2})]$$
$$\times\int_a^\infty exp(-x^2)\,dx \qquad (4\,b),$$

$$a = q^{1/2}(k_0-k_d),$$
$$D = [(21+1)(N-1-1)!/(n+1)!][(1/2)(1+N-1)!/(1/2)(N-1-1)!]^2$$
$$\times 2^{21+3}\pi^{1/2}q^{1/2}(Nm_ee^4/\varepsilon^2\hbar^3k_0^7k_d) \qquad (4\,c),$$
$$q = h^2/2m_ek_bT_e,$$
$$k_d = (m_e\hbar)\mu E,$$
$$k_0 = 2m_eE_l/\hbar,$$

and I(−) is obtained from I(+) by replacing each k_d by $-k_d$. Here, m_e is the effective mass of the electron, k_b is the Boltzmann constant, μ is the mobility of the conduction electron, ε is the dielectric constant and $\hbar=h/2$, where h is the Planck constant.

In the above expressions, a displaced Maxwellian-type distribution with an effective temperature T_e is assumed for the distribution of the conduction electrons. This is a rather crude approximation, especially for the electrons in the low electric field region at low temperatures where almost all electrons are trapped in the donor states (freeze-out regime). However, this is a necessary compromise to obtain the convergent results for the present probem.

At low temperatures, the dominant mechanism of momentum relaxation of the conduction electron is the impurity scattering, and the definite dependence of X_l on the applied electric field can be estimated from the field dependence of the mobility.

3. 2 Capture cross section for generation and recombination

The quantum theory for the phonon-mediate generation and recombination processes between the hydrogenic donor states and the conduction band has been developed by Ascarelli and Rodorigues[4]. The cross section for the capture electron in a state having principal quantum number n of donor impurity is given by

$$\sigma(\mathrm{n})=(\pi^2\hbar^2\mathrm{b_n})/2\mathrm{m_e}(\mathrm{k_b T})^2\cdot exp(\mathrm{I_n}/\mathrm{k_b T}) \qquad (5),$$

where I_n is the ionization energy of the n-th excited donor state and the b_n is the probability per unit time for the thermal ionization of an electron in the n-th state.

b_n is approximately expressed as

$$b_n=(256\ \mathrm{E_1^2 m_e}\hbar^2 c_s)/(\pi\rho a^{*6}(k_b\mathrm{T})^4 n^3)\mathrm{I}(n,\ p,\ g) \qquad (6)$$

where

$$p=\mathrm{E_l}/k_b\mathrm{T}$$

$$g=2\hbar c_s/a^* k_b\mathrm{T}$$

$$\mathrm{I}(n,\ p,\ g)=\int_0^{n^2/h} dt\ t^3 exp(-t^{-1}=g^2t^2/n).$$

Here, a is the effective radius of the first Bohr orbit and E_l is the deformation potential constant and the c_s is the averaged sound velocity.

The transition probability between n and n' donor states is given by

$$\mathrm{W}_{n\ n'}=(64\ \mathrm{E_1^2}h^4 c_s^3/(\pi\rho \mathrm{a}^{*8}\mathrm{E_1^5})(n'\ n)^5/(n-n')(n+n')$$
$$\times[1-exp-h\omega_{\mathrm{n\ n'}}/k_b\mathrm{T})]^{-1} \qquad (7).$$

4. Numerical results and discussion

We take a partially compensated n-GaAs at 4.2K as a typical example for the present probiem and perform the numerical calculations of the g-r rate coefficients for the two-level donor system which consists of 2S excited state ($E_i=1.5$ meV) and the ground state ($E_i=5.9$ meV).

The impact ionization coefficient as a function of the electron tmperature is shown in Fig. 2. The field dependence of the impact ionization coefficients of the excited state and the ground state are shown in Fig. 3.

The homogeneous steady states of the conduction electrons are obtained from Eqs. (1), (2) and (3). The stablility of these steady states can be derived from the following stability equation ;

$$\begin{vmatrix} \partial F_0^0/\partial n-\omega & \partial F_0^0/\partial n_1 \\ \partial F_1^0/\partial n & \partial F^0/\partial n_1-\omega \end{vmatrix}=0 \qquad (8),$$

where the superscript 0 indicates the derivative at the specific steady state. Unless the real part of the eigen values of ω is positive, the obtained steady states remain stable against small perturbations.

The normalized current density-electric field characteristics are shown in Fig. 4. It is seen that the heavily compensated n-GaAs at low temperatures shows an s-type negative differential conductivity SNDC) induced by impact ionization. The appearence of the two-stable steady states…bistable states…in the current densty is due to the nonlinear

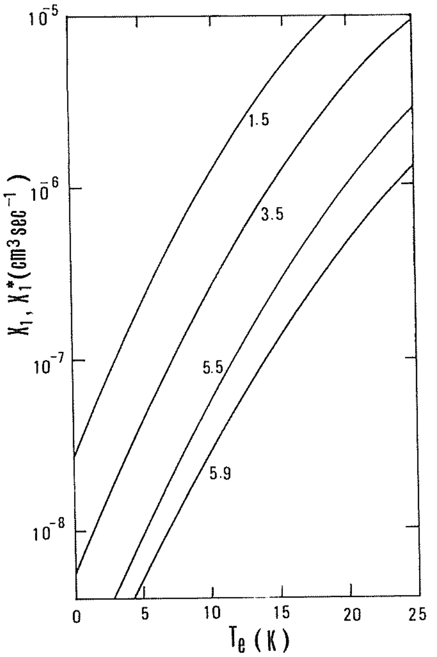

Fig. 2 Impact ionization coefficient as a function of electron temperature for different trap levels (unit of meV).

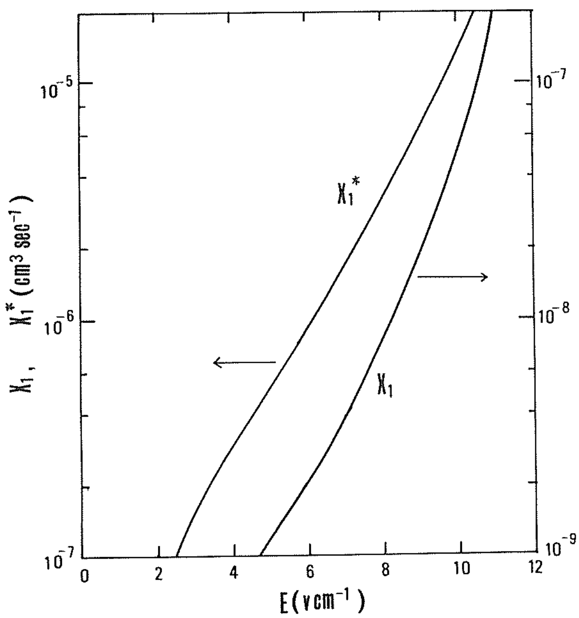

Fig. 3 Impact ionization coefficient of the excited and the ground states as a function of the electric field.

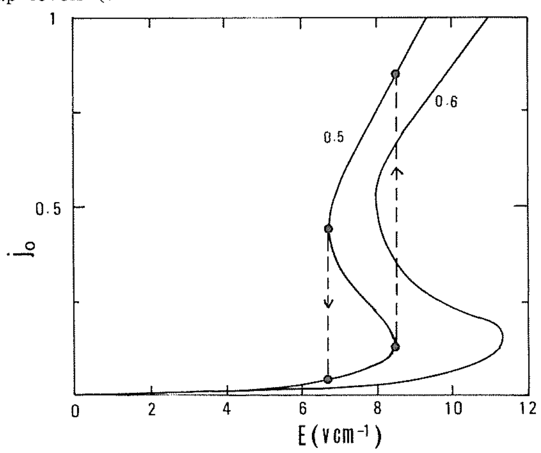

Fig. 4 Normalized current density-electric field characteristics. Here, j_0 is plotted in the unit of e (N_d-N_a). The parameter in the figure is the compensation ratio $((N_d-N_a)/N_d)$. ○ marks indicate the bistable states.

redistribution of the electrons among the donor states and the conduction band under impact ionization.

The observation of SNDC in GaAs has been reported by several authors[5]. The occurrence of SNDC in GaAs at low temperatures strongly depended on the concentration of the impurities involved in the specimen used in the experiments. In the impure specimens, the effect of the impurity conduction, the broadening of the impurity levels due to the fluctuation of random potentials of impurities, and the tailing of the conduction band edge modify the g-r processes significantly, and it was shown that a more sophisticated model is necessary to explain the experimental results. In spite of the strongly simplified approximations of the present treatment, the numerical results are in reasonable agreement with the experimental results with pure-GaAs.

In this paper, we limit ourselves to treat the homogeneous steady states in a nonlinear system. However, any homogeneous steady state having SNDC can no longer maintain the specially homogeneous state and will induce the nonequilibrium phase transition to a certain inhomogeneous state, such as filamentary structure. The detailed investigation for these phase transitions will be published elsewhere.

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