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Bending of R/C Beam Members with Curved Reinforcement with its Bond-Slip Involved

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Abstract

Herein an extension is made to the writer's earlier proposed system of finite element procedure, developed for analyzing the statical behaviors of ordinary types of r/c members with straight reinforcement, to the cases with curved reinforcement. This method thus modified is otherwise capable of its direct application now being tried by the author to the post-tensioned cases of p. c. beam members since the structures under consideration are analogous to those when the post-tensioning and anchorage of their curved tendons have been completed.

1. Introduction

Post-tensioned types of prestressed concrete beams, typical of concrete structures with curved arrangement of reinforcement have numerous examples so far executed and the corresponding system of design theory has come to be regarded as largely established.

In fact this deals more or less directly with institutional design work involving consideration of introducing stress into tendons and eventual short- and long-term statical behaviors of a structure ; still admitting of generally firmer and more rigorous design bases to be provided by analytical treatment. For instance, the stiffness of a member in its post-tensioned state should be considered with the effects of the lengthwise curved form and the bond-slip of the steel taken into account when their bonding capacity is fully developed with hardening of grout injected into their sheaths.

In this paper to help fill such fundamental requirements, formulation is initially attempted on the governing equations for bending of r/c beams including the above-mentioned points of reinforcement ; this is followed by the development in finite element form of thus obtained sets of equations, eventually to give a numerical solution by the finite element method to an organized series of examples of model structures and, using the result, to examine tendencies observed in the their statical behaviors.

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2. Bending Theory in consideration of Bond-Slip

2. 1 Basic Assumptions

For developing our theory we assumes the following: 1) the same gross concrete sections at any positions along a member axis, as well as the principles of infinitestimal deformation; 2) isotropy and elasticity of the concrete and the retained original plane concrete section after structural deformation; 3) any member section with m layers of reinforcing tendons which form between member supports smooth curves on vertical planes in the span direction and 4) the bond-slip between each layer of tendons and adjacent parts of concrete, defined as their displacement from the assumed plane concrete section on the supposition that there is a linear relation of the displacement with the attendant stress.

2. 2 Displacement Field and Strain-Stress Relation

a) Displacement Field Fig. 1 illustrates the arrangement of an arbitrary layer assigned i of reinforcement in a beam as well as its major longitudinal and sectional measurements referring to adopted x-z coordinates. When based on the above assumptions the displacement field needed to be introduced for formulating the current problem consists of independent variables totaling $(2+m)$, i.e., respective longitudinal and vertical displacements u and w as well as m bond-slip displacements S_1, $S_2 \ldots S_i \ldots S_m$, of any of the m steel layers, i, along its prearranged curve.

b) Concrete Strain and Stress The expression for strain ${}_z\epsilon_c$ of the concrete at a distance of z from the x-axis along the member center line may be possible as:

$$ {}_z\epsilon_c = \epsilon_0 + z\ \phi \qquad 1) $$

where ϵ_0 = longitudinal strain and ϕ = curvature, both being defined as:

$$ \epsilon_0 = du/dx \qquad 2) $$

$$ \phi = -d^2w/dx^2 \qquad 3) $$

Hence, concrete stress ${}_z\sigma_c$ will be:

$$ {}_z\sigma_c = E_c\ {}_z\epsilon_c \qquad 4) $$

with E_c = elastic modulus of concrete.

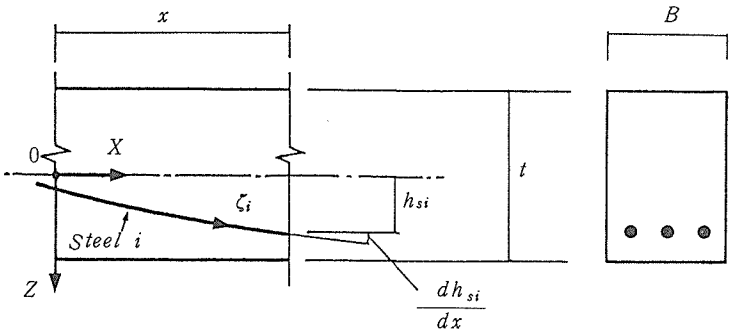


Fig. 1 R/C Member with Curvedly Arranged Reinforcement.

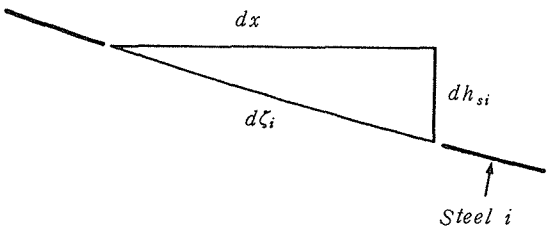


Fig. 2 Relations between $d\zeta_i$, dh_{si} and dx.

c) Strain and Stress in Steel Layer Regarding any arbitrary, i, of the m layers, strain ϵ_{si} in the tangential direction of the curve of reinforcement are representable as :

$$\epsilon_{si} = \epsilon_0 + h_{si}\,\phi + \epsilon_{ssi} \qquad 5)$$

where ϵ_{ssi} = ratio of bond-slip in the above direction and h_{si} = distance of layer i from the x-axis; the latter is specified as follows in terms of x.

$$h_{si} = g_i(x) \qquad 6)$$

And bond-slip ratio ϵ_{ssi} in Eq.(5) comes to be :

$$\varepsilon_{ssi} = \frac{dS_i}{d\zeta_i} = \frac{dS_i}{dx}\frac{dx}{d\zeta_i} = \frac{1}{\sqrt{1+(dh_{si}/dx)^2}}\frac{dS_i}{dx} \qquad 7)$$

where differential $d\zeta_i/dx = \sqrt{1+(dh_{si}/dx)^2}$ is immediately obtained referring to Fig. 2, so that the corresponding steel stress in layer i is defined as

$$\sigma_{si} = E_{si}\,\epsilon_{si} \qquad 8)$$

with E_{si} = elastic modulus of the steel of this layer.

d) Bond Stress On the corresponding previous assumption bond stress τ_i occuring on the surface of contact with reinforcement of layer i is denoted by

$$\tau_i = K_{bi}\,S_i \qquad 9)$$

with K_{bi} = bond coefficient of the steel of layer i.

2. 3 Axial Force and Bending Moment

Both axial force and bending moment are given by the following equation from the relations in Eqs. (1) through (8) :

$$N = B\int_{-t/2}^{t/2} z\sigma_c\,dz + \sum^{m}\{A_{si}\sqrt{1+(dh_{si}/dx)^2}\,(\sigma_{si} - {}_{hsi}\sigma_c)\} \qquad 10)$$

$$M = B\int_{-t/2}^{t/2} z\sigma_c\,z\,dz + \sum^{m}\{A_{si}h_{si}\sqrt{1+(dh_{si}/dx)^2}\,(\sigma_s - {}_{hsi}\sigma_c)\} \qquad 11)$$

in which B =beam width and A_{si} = sectional area of steel layer i.

In the above equations ${}_{h_{si}}\sigma_c$ is such an amount of concrete stress concurrent with steel stress of layer i as is necessary for the considered reduction of concrete section by the amount of steel area. Substitution of relations in Eqs.(4), (5) and (8) in the two equations just above with due arrangement provides :

$$N = \left[AE_c + \sum^{m}\{A^{\star}_{si}E_{si}\sqrt{1+(dh_{si}/dx)^2}\}\right]\frac{du}{dx} - \sum^{m}\{A^{\star}_{si}E_{si}h_{si}\sqrt{1+(dh_{si}/dx)^2}\}\frac{d^2w}{dx^2} + \sum^{m}\left(A_{si}E_{si}\frac{dS_i}{dx}\right) \qquad 12)$$

$$M=\sum^{m}\{A^{\star}_{si}E_{si}h_{si}\sqrt{1+(dh_{si}/dx)^2}\}\frac{du}{dx}-\left[I_oE_c+\sum^{m}\{A^{\star}_{si}E_{si}h^2_{si}\sqrt{1+(dh_{si}/dx)^2}\}\right]\frac{d^2w}{dx^2}+\sum^{m}\left\{A_{si}E_{si}h_{si}\frac{dS_i}{dx}\right\} \quad 13)$$

where A = area of beam section, I_0 = moment of inertia of gross concrete section and $A^{\star}_{si} = A_{si}(1 - E_c/E_{si})$ = transformed steel area of layer i.

2. 4 Total Potential Energy Functional

The functional of total potential energy may generally be expressed as[1),2),3)] :

$$\Pi = (U_{cn} + U_{st} + U_{bs}) - V \quad 14)$$

provided V = external potential energy and U_{cn}, U_{st} and U_{bs} = internal energies with the ensuing expressions.

These three, respectively related to the concrete, the steel and the bond-slip action of a p. c. beam, become :

$$U_{cn}=\frac{1}{2}\int_0^L\int_{t/2}^{t/2}BE_c\ {}_z\varepsilon^2_c dz\, dx-\frac{1}{2}\int_0^L\left\{\sum^{m}A_{si}E_c\sqrt{1+(dh_{si}/dx)^2}\,\varepsilon^2_{si}\right\}dx \quad 15)$$

$$U_{st}=\frac{1}{2}\int_0^{L_{si}}\left\{\sum^{m}A_{si}E_{si}\varepsilon^2_{si}\right\}d\zeta_i \quad 16)$$

$$U_{bs}=\frac{1}{2}\int_0^{L_{si}}\left\{\sum^{m}A_{bsi}K_{bi}S^2_i\right\}d\zeta i \quad 17)$$

where L = member length, L_{si} = whole length of the curve of the tendons of layer i, K_{bi} = bond coefficient of the steel of layer i and A_{bsi} = area of its bond surface per unit length of it.

In the above, U_{st} and U_{bs} for integrals respecting curved segment ζ_i of the steel layer axis can be transformed into the other forms referring to x ; i.e. as follows :

$$U_{st}=\frac{1}{2}\int_0^L\left\{\sum^{m}A_{si}E_{si}\varepsilon^2_{si}\sqrt{1+(dh_{si}/dx)^2}\right\}dx \quad 18)$$

$$U_{bs}=\frac{1}{2}\int_0^L\left\{\sum^{m}A_{bsi}K_{bi}S^2_i\sqrt{1+(dh_{si}/dx)^2}\right\}dx \quad 19)$$

Accordingly, substitution of relations in Eqs. (1) through (8) in the Eqs. (14), (15), (18) and (19) with rearrangement results in the equation in functional form of total potential energy Π :

$$\begin{aligned}\Pi=&\frac{1}{2}\int_0^L\Bigg[\left[AE_c+\sum^{m}\{A^{\star}_{si}E_{si}\sqrt{1+(dh_{si}/dx)^2}\}\right]\left(\frac{du}{dx}\right)^2\\&-2\left[\sum^{m}\{A^{\star}_{si}E_{si}h_{si}\sqrt{1+(dh_{si}/dx)^2}\}\right]\frac{du}{dx}\frac{d^2w}{dx^2}+2\sum^{m}\left\{A_{si}E_{si}\left(\frac{du}{dx}\frac{dS_i}{dx}\right.\right.\\&\left.\left.-h_{si}\frac{d^2w}{dx^2}\frac{dS_i}{dx}\right)\right\}+\left[I_oE_c+\sum^{m}\left\{A^{\star}_{si}E_{si}h_{si}\sqrt{1+(dh_{si}/dx)^2}\right\}\right]\left(\frac{d^2w}{dx^2}\right)^2\\&+\sum^{m}\left\{\frac{A_{si}E_{si}}{\sqrt{1+(dh_{si}/dx)^2}}\left(\frac{dS_i}{dx}\right)^2\right\}+\sum^{m}\left\{A_{bsi}K_{bi}S^2_i\sqrt{1+(dh_{si}/dx)^2}\right\}\Bigg]dx\\&-\int_0^L\{P_u(du/dx)+q_zw\}\,dx\end{aligned} \quad 20)$$

2. 5 Governing Differential Equations

The set of differential equations governing the bending of an r/c beam when the bond-slip of its reinforcement is taken into account can be obtained by taking the first variation of the above functional or Eq. (20) ; so as to follow :

$$\Big\{AE_c+\sum^m(A^\star_{si}E_{si}\sqrt{1+(dh_{si}/dx)^2})\frac{d^2u}{dx^2}+\sum^m\Big(\frac{A^\star_{si}E_{si}}{\sqrt{1+(dh_{si}/dx)^2}}\frac{dh_{si}}{dx}\frac{d^2h_{si}}{dx^2}\Big)\frac{du}{dx}$$
$$-\sum^m(A^\star_{si}E_{si}h_{si}\sqrt{1+(dh_{si}/dx)^2})\frac{d^3w}{dx^3}$$
$$-\sum^m\Big(A^\star_{si}E_{si}\frac{dh_{si}}{dx}\sqrt{1+(dh_{si}/dx)^2}+\frac{A^\star_{si}E_{si}h_{si}}{\sqrt{1+(dh_{si}/dx)^2}}\frac{dh_{si}}{dx}\frac{d^2h_{si}}{dx^2}\Big)\frac{d^2w}{dx^2}$$
$$+\sum^m\Big(A_{si}E_{si}\frac{d^2S_i}{dx^2}\Big)\Big\}=0 \qquad 21)$$

$$-\sum^m(A^\star_{si}E_{si}h_{si}\sqrt{1+(dh_{si}/dx)^2})\frac{d^3u}{dx^3}-2\sum^m\Big(A^\star_{si}E_{si}\frac{dh_{si}}{dx}\sqrt{1+(dh_{si}/dx)^2}$$
$$+\frac{A^\star_{si}E_{si}h_{si}}{\sqrt{1+(dh_{si}/dx)^2}}\frac{dh_{si}}{dx}\frac{d^2h_{si}}{dx^2}\Big)\frac{d^2u}{dx^2}-\sum^m\Big(A^\star_{si}E_{si}\frac{d^2h_{si}}{dx^2}\sqrt{1+(dh_{si}/dx)^2}$$
$$+\frac{2A^\star_{si}E_{si}}{\sqrt{1+(dh_{si}/dx)^2}}\Big(\frac{dh_{si}}{dx}\Big)^2\frac{d^2h_{si}}{dx^2}+\frac{A^\star_{si}E_{si}h_{si}}{\sqrt{1+(dh_{si}/dx)^2}}\Big\{\Big(\frac{d^2h_{si}}{dx^2}\Big)^2+\frac{dh_{si}}{dx}\frac{d^3h_{si}}{dx^3}\Big\}$$
$$-\frac{A^\star_{si}E_{si}h_{si}}{\{1+(dh_{si}/dx)^2\}^{\frac{3}{2}}}\Big(\frac{dh_{si}}{dx}\Big)^2\Big(\frac{d^2h_{si}}{dx^2}\Big)^2\Big)\frac{du}{dx}-\sum^m\Big(A_{si}E_{si}h_{si}\frac{d^3S_i}{dx^3}\Big)$$
$$-2\sum^m\Big(A_{si}E_{si}\frac{dh_{si}}{dx}\frac{d^2S_i}{dx^2}\Big)-\sum^m\Big(A_{si}E_{si}\frac{d^2h_{si}}{dx^2}\frac{dS_i}{dx}\Big)$$
$$+\Big\{I_oE_c+\sum^m(A^\star_{si}E_{si}h_{si}\sqrt{1+(dh_{si}/dx)^2})\frac{d^4w}{dx^4}$$
$$+2\sum^m\Big(2A^\star_{si}E_{si}h_{si}\frac{dh_{si}}{dx}\sqrt{1+(dh_{si}/dx)^2}+\frac{A^\star_{si}E_{si}h^2_{si}}{\sqrt{1+(dh_{si}/dx)^2}}\frac{dh_{si}}{dx}\frac{d^2h_{si}}{dx^2}\Big)\frac{d^3w}{dx^3}$$
$$+\sum^m(2A^\star_{si}E_{si}\Big(\frac{dh_{si}}{dx}\Big)^2\sqrt{1+(dh_{si}/dx)^2}+2A^\star_{si}E_{si}h_{si}\frac{d^2h_{si}}{dx^2}\sqrt{1+(dh_{si}/dx)^2}$$
$$+\frac{4A^\star_{si}E_{si}h_{si}}{\sqrt{1+(dh_{si}/dx)^2}}\Big(\frac{dh_{si}}{dx}\Big)^2\frac{d^2h_{si}}{dx^2}+\frac{A^\star_{si}E_{si}h^2_{si}}{\sqrt{1+(dh_{si}/dx)^2}}\Big\{\Big(\frac{d^2h_{si}}{dx}\Big)^2+\frac{dh_{si}}{dx}\frac{d^3h_{si}}{dx^3}\Big\}$$
$$-\frac{A^\star_{si}E_{si}h^2_{si}}{\{1+(dh_{si}/dx)^2\}^{\frac{3}{2}}}\Big(\frac{dh_{si}}{dx}\Big)^2\Big(\frac{d^2h_{si}}{dx^2}\Big)^2\Big)\frac{d^2w}{dx^2}=q_z \qquad 22)$$

$$A_{si}E_{si}\Big(\frac{d^2u}{dx^2}-h_{si}\frac{d^3w}{dx^3}-\frac{dh_{si}}{dx}\frac{d^2w}{dx^2}+\frac{1}{\sqrt{1+(dh_{si}/dx)^2}}\frac{d^2S_i}{dx^2}$$
$$-\frac{1}{\{1+(dh_{si}/dx)^2\}^{\frac{3}{2}}}\frac{dh_{si}}{dx}\frac{d^2h_{si}}{dx^2}\frac{dS_i}{dx}\Big)=A_{bsi}K_{bi}S_i\sqrt{1+(dh_{si}/dx)^2} \qquad 23)$$

with $i = 1,...m$.

3. Finite Element Analysis

3. 1 Assumed Curve of Reinforcement

On our regarding the curved form of arranged reinforcement as a quadratic in x measured along the member axis, vertical distance h_{si} of the steel of layer i from this axis at point x is given by the next equation :

$$h_{si} = a_i + b_i x + c_i x^2 \tag{24}$$

where a_i, b_i and c_i = coefficients defining the quadratic.

Here the quadratic assumption required to hold only within indivisual elements in a span, not necessarily over its length, provided pertinent segments of the steel form any smoothly continuous curve between supports.

3. 2 Displacement Function

Currently we assume displacments u, w and S as follows.

$$u = [1\ x]\{\alpha_{u1}\ \alpha_{u2}\}^{\mathrm{T}} = [f_u]\{\alpha_u\} \tag{25}$$

$$w = [1\ x\ x^2\ x^3]\{\alpha_{w1}\ \alpha_{w2}\ \alpha_{w3}\ \alpha_{w4}\}^{\mathrm{T}} = [f_w]\{\alpha_w\} \tag{26}$$

$$S_i = [1\ x]\{\alpha_{si1}\ \alpha_{si2}\}^{\mathrm{T}} = [f_s]\{\alpha_{si}\} \tag{27}$$

where $[f_u]$,$[f_w]$ and $[f_s]$ = shape functions of the respective displacements while $\{\alpha_u\}$,$\{\alpha_w\}$ and $\{\alpha_s\}$ = generalized displacements corresponding thereto. And the attendant strains as their first derivatives with one second are :

$$u' = du/dx = [0\ 1]\{\alpha_u\} = [f'_u]\{\alpha_u\} \tag{28}$$

$$w' = dw/dx = [0\ 1\ 2x\ 3x^2]\{\alpha_w\} = [f'_w]\{\alpha_w\} \tag{29}$$

$$w'' = d^2w/dx^2 = [0\ 0\ 2\ 6x]\{\alpha_w\} = [f''_w]\{\alpha_w\} \tag{30}$$

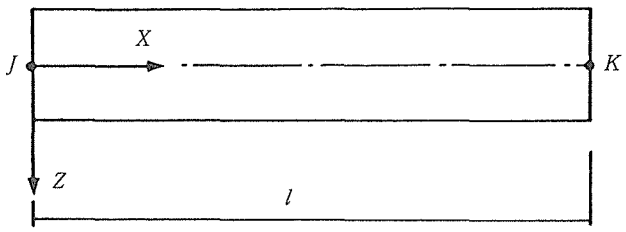
$$S'_i = dS_i/dx = [0\ 1]\{\alpha_{si}\} = [f'_s]\{\alpha_{si}\} \tag{31}$$

Node displacement vectors of this beam element (Fig. 3) result in :

$$\{\Delta_u\} = \{u_j\ u_k\}^{\mathrm{T}} = [C_u]\{\alpha_u\} \tag{32}$$

$$\{\Delta_w\} = \{w_j\ w'_j\ w_k\ w'_k\}^{\mathrm{T}} = [C_w]\{\alpha_w\} \tag{33}$$

$$\{\Delta_{si}\} = \{S_{ij}\ S_{ik}\}^{\mathrm{T}} = [C_s]\{\alpha_{si}\} \tag{34}$$



$\{\Delta u\} = \{u_i\ u_k\}^T$, $\{\Delta w\} = \{w_j\ w'_j\ w_k\ w'_k\}^T$
$\{\Delta s\} = \{S_{j1}\ S_{j2} \cdots S_{jm}\ S_{k1}\ S_{k2} \cdots S_{km}\}^T$

Fig. 3 R/C Beam Element including Steel Bond-Slip Effect.

3. 3 Finite Element Equations

Finally we can derive the required finite element equations for the bending analysis of an r/c beam in the title, from Eqs. (25) through (34) and the relation in functional Eq. (20) ; now to be given by the expressions in the following :

$$\begin{pmatrix} K_{uu} & K_{uw} & K_{us} \\ K_{us}^{T} & K_{ww} & K_{ws} \\ K_{us}^{T} & K_{ws}^{T} & K_{ss} \end{pmatrix} \begin{Bmatrix} \Delta_u \\ \Delta_w \\ \Delta_s \end{Bmatrix} = \begin{Bmatrix} P_u \\ P_w \\ 0 \end{Bmatrix} \qquad 35)$$

where, of the partial stiffness matrices, $[K_{uu}]$, $[K_{uw}]$ and $[K_{ww}]$ are to have specific descriptions of :

$$[K_{uu}]=[C_u^{-1}]^T\left[AE_c\int_0^l[f'_u]^T[f'_u]dx+\sum^m\int_0^l[f'_u]^T\{A_{si}^{\star}E_{si}\sqrt{1+b_i^2+4b_ic_ix+4c_i^2x^2}\}[f'_u]dx\right][C_u^{-1}]$$

$$[K_{uw}]=-[C_u^{-1}]^T\left[\sum^m\int_0^l\{A_{si}^{\star}E_{si}(a_i+b_ix+c_ix^2)\sqrt{1+b_i^2+4b_ic_ix+4c_i^2x^2}\}[f'_u]^T[f''_w]dx\right][C_w^{-1}]$$

$$[K_{ww}]=[C_w^{-1}]^T\left[I_oE_c\int_0^l[f''_w]^T[f''_w]dx\right.$$

$$\left.+\sum^m\int_0^l\{A_{si}^{\star}E_{si}(a_i+b_ix+c_ix^2)^2\sqrt{1+b_i^2+4b_ic_ix+4c_i^2x^2}\}[f''_w]^T[f''_w]dx\right][C_w^{-1}] \qquad 36)$$

and the rest, $[K_{us}]$,$[K_{ws}]$ and $[K_{ss}]$, are easily obtained by assembling partial matrices $[k_{usi}]$ $[k_{wsi}]$ and $[k_{ssi}]$, which respectively constitute the above three referring to arbitrary steel layer i, successively in sequence of array of the displacement vectors for all the steel layers.

$$[k_{usi}]=[C_u^{-1}]^T\left[\int_0^lA_{si}E_{si}[f'_u]^T[f'_s]dx\right][C_s^{-1}]$$

$$[k_{ssi}]=[C_s^{-1}]^T\left[\int_0^lA_{si}E_{si}(1+b_i^2+4b_ic_ix+4c_i^2x^2)^{-1/2}[f'_s]^T[f'_s]dx\right.$$

$$\left.+\int_0^lA_{bsi}K_{bi}\sqrt{1+b_i^2+4b_ic_ix+4c_i^2x^2}\,[f_s]^T[f_s]dx\right][C_s^{-1}]$$

$$[k_{wsi}]=-[C_w^{-1}]^T\left[\int_0^lA_{si}E_{si}(a_i+b_ix+c_ix^2)[f''_w]^T[f'_s]dx\right][C_s^{-1}] \qquad 37)$$

4. Numerical Examples and Examined Results

On the same r/c model, in Fig. 4, under a uniform load, as the example discussed in our previous report[3] we intend to examine the effect of the difference in the adopted form of the steel arrangement curve and in the bond coefficient on the statical behavior of the structure.

Here we assume seven cases of steel arrangement curves each given by a single equation covering a whole span with $d_e = 4, 7, 15, 19, 23$ and $27cm$, while $d_e = 4cm$ refers to the case of straightly arranged reinforcement[2]. Further, for each of the above curves seven values stepwise varied of bond coefficients are used as $K_b = 1.0, 10, 10^2, 10^3, 10^4, 10^5$ and $10^6\ kgf/cm^3$, with the steel perfectly anchored at both member ends.

In Fig. 5 through 11 are the main parts of the results of the analysis using each of the

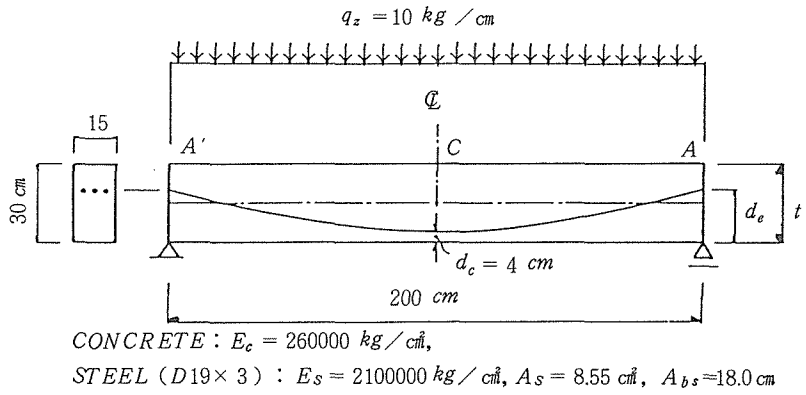


Fig. 4 General Description of Model R/C Beams.

above *de*'s. In each figure the two upper diagrams (a) and (b) respectively compare curves of deflection and bending moment in case of the assumed largest and smallest of the above-mentioned bond coefficients ; and in the two lower diagrams of (c) and (d), relating respectively to both bending and bond stresses, comparison is also made between their curves each resulting from one of all these seven coefficient values different by progressive degrees.

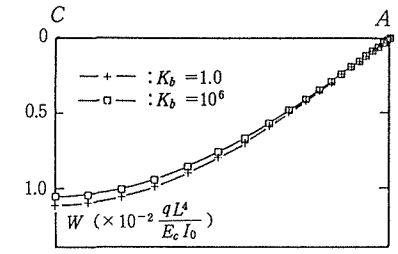
Noticeable aspects of the foregoing solution results will be shown respectively for each of the items headed with (a) through (d) in the figures. Namely regarding :

(a) deflection curves —— comparison of those curves in all the figures shows increasing deflections with increased distance d_e from *4* to *27cm* attended with larger differences between the two concurrent curves each for K_b of *1.0* and 10^6, which tendencies are more explicitly shown in Fig. 12 where the transition of deflection with changed values of bond coefficient are shown for each case of reinforcement curve ;

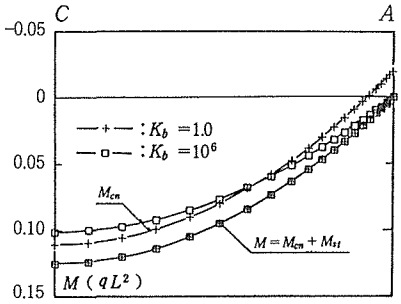
(b) bending moment curves —— in each associated diagram labelled (b), obtained in cases of the above upper and lower of bond coeffients, shown as well are moment M_c carried by the concrete other than total moment M $(= M_c + M_{st})$, calculated from internal forces, in order to identify the portion transferred by the steel, M_{st}. These analytical results confirm that the total moment is constant throughout all cases and M_c and accordingly M_{st} vary significantly depending on the form of a steel arrangement curve ;

(c) curves of the steel stress —— covering K_b-values of *1.0* through 10^6, the pertinent diagrams designated c) above show varying curves of the steel stress with changed bond coefficient K_b in the above-mentioned way ; and that the steel stress may be regarded as that in either a bondless state through the member length or a near perfectly anchored state respectively when K_b is within *10* or more than $10^4\ kgf/cm^3$ and :

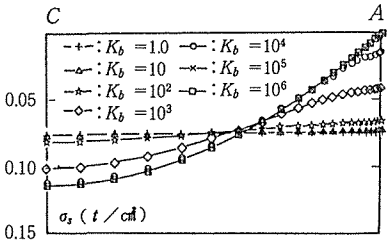
(d) bond stress —— marked with (d) and shown also as to seven degrees of bond coefficient, each corresponding diagram distinctly shows the transition of the distribution curve of the bond stress as the corresponding coefficient has changing values from those in virtually bondless states to those in states of assumably near perfect anchorage.



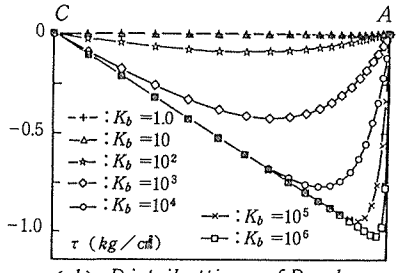
(a) Distributions of Deflection over a Span



(b) Bending Moment Distributions over a Span

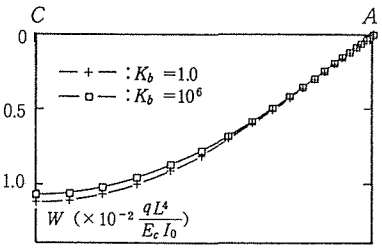


(c) Distributions of Axial Stress in Reinforcement

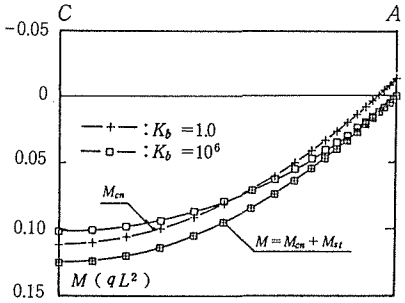


(d) Distributtions of Bond Stress on Reinforcement

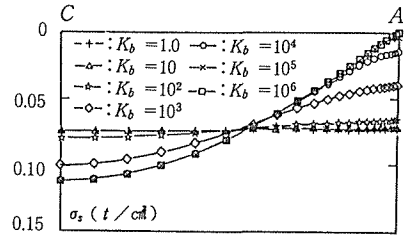
Fig. 5 Results for Model Beam with de = 4cm



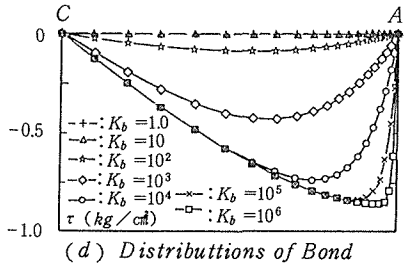
(a) Distributions of Deflection over a Span



(b) Bending Moment Distributions over a Span

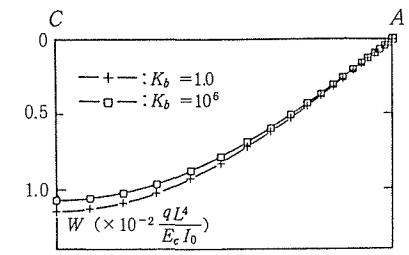


(c) Distributions of Axial Stress in Reinforcement

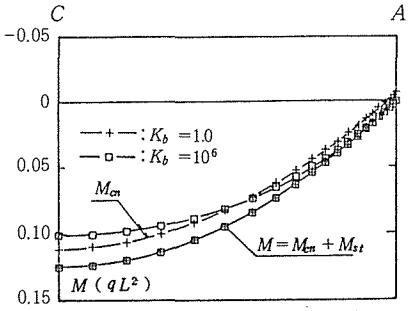


(d) Distributtions of Bond Stress on Reinforcement

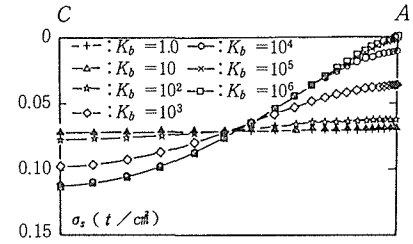
Fig. 6 Results for Model Beam with de = 7cm



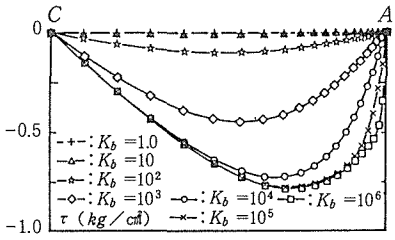
(a) *Distributions of Deflection over a Span*



(b) *Bending Moment Distributions over a Span*

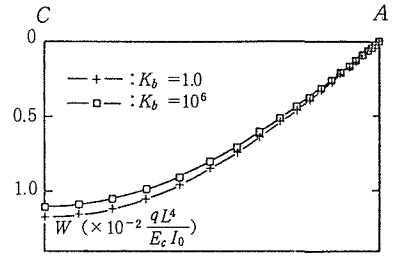


(c) *Distribution of Axial Stress in Reinforcement*

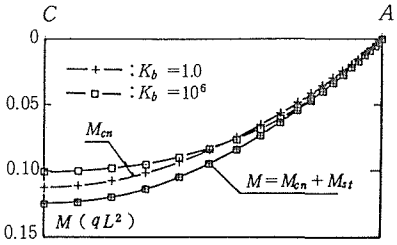


(d) *Distributtions of Bond Stress on Reinforcement*

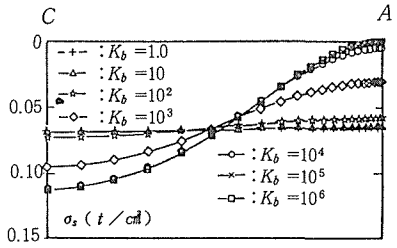
Fig. 7 Results for Model Beam with de = 11cm



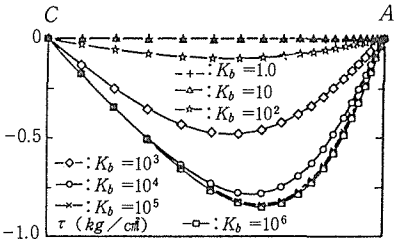
(a) *Distributions of Deflection over a Span*



(b) *Bending Moment Distributions over a Span*

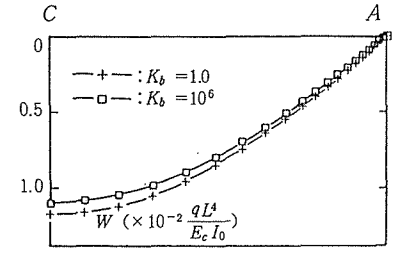


(c) *Distributions of Axial Stress in Reinforcement*

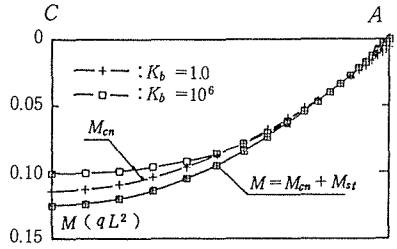


(d) *Distributtions of Bond Stress on Reinforcement*

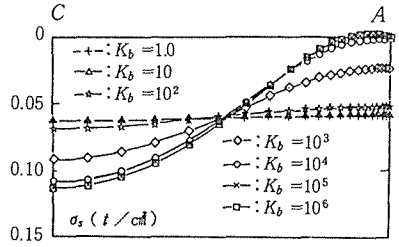
Fig. 8 Results for Model Beam with de = 15cm



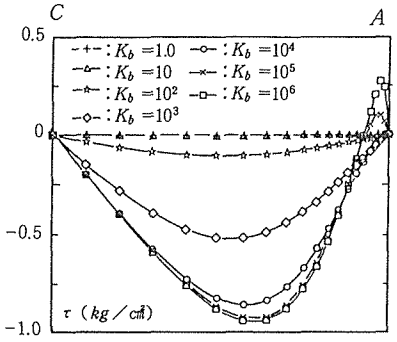
(a) Distributions of Deflection over a Span



(b) Bending Moment Distributions over a Span

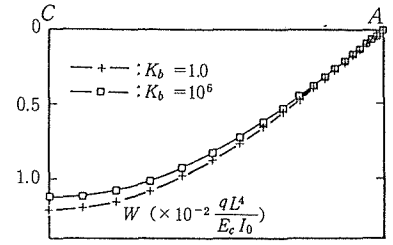


(c) Distributions of Axial Stress in Reinforcement

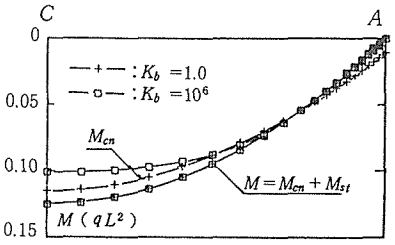


(d) Distributtions of Bond Stress on Reinforcement

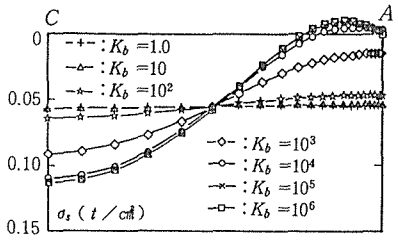
Fig. 9 Results for Model Beam with de = 19cm



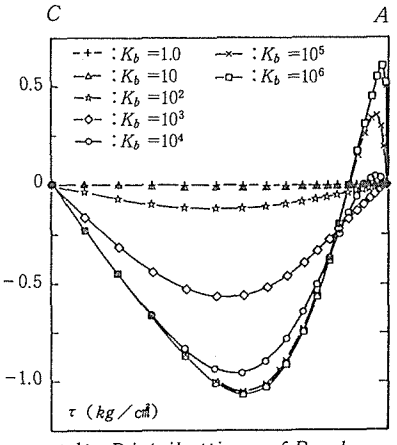
(a) Distributions of Deflection over a Span



(b) Bending Moment Distributions over a Span

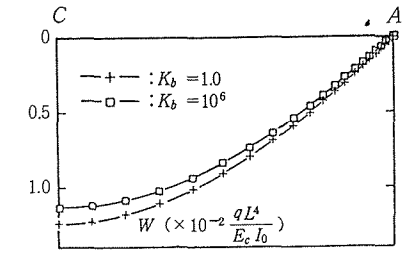


(c) Distributions of Axial Stress in Reinforcement

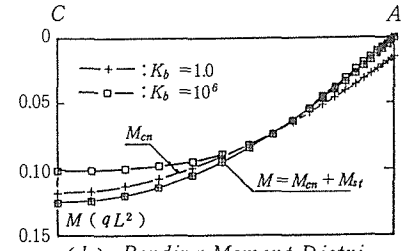


(d) Distributtions of Bond Stress on Reinforcement

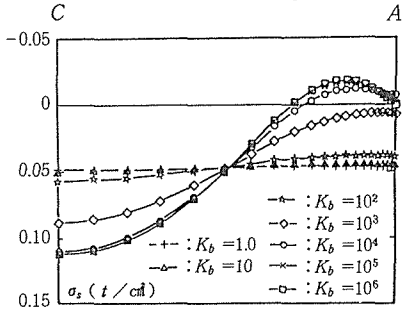
Fig. 10 Results for Model Beam with de = 23cm



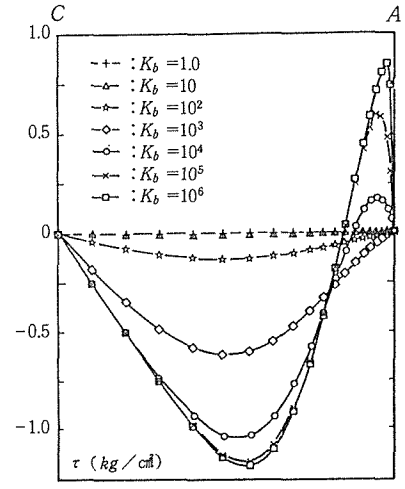
(a) *Distributions of Deflection over a Span*



(b) *Bending Moment Distributions over a Span*



(c) *Distributions of Axial Stress in Reinforcement*



(d) *Distributtions of Bond Stress on Reinforcement*

Fig. 11 Results for Model Beam with de = 27cm

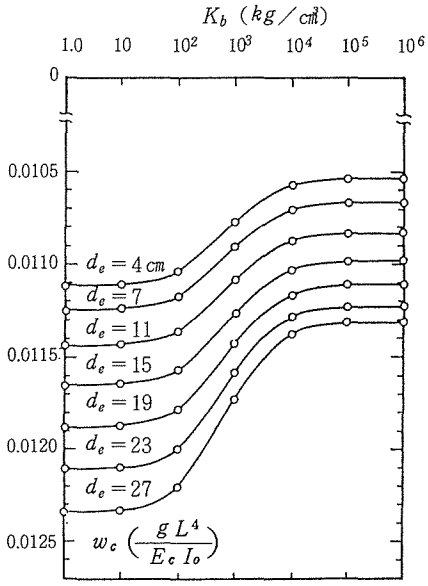


Fig. 12 Aspect of Transition of Mid-Span Deflection w_c of Model Beam

5. Summary and Conclusions

Throughout we have discussed the bending action of r/c beam members with curvilinearly placed reinforcement while its bond-slip is considered on the assumption of maintained sectional planarity in structural deformation. For a theoretical basis of the subject matter we initially derives governing differential equations for the bending of members of the introduced types.

Thereupon, finite element formulation of the problem is attempted and by using the resulting equation system seven simply supported examples are solved in order to examine the effect of the difference in the form of steel arrangement and the bond coefficient on statical behaviors of the

models ; with the result that :

(1) the form of a steel arrangement curve has a large effect on member stiffness and distributions of either axial steel stress or bond stress ; and that ;

(2) the effect on statical behaviors of the difference in bond coefficient is virtually independent of steel arrangement form ; with there arising bondless states and states close to perfect anchorage respectively for values of K_b within 10 and over 10^4 kgf/cm^3.

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2) Inomata, T. : Design and Construction of Prestressed Concrete Structures, Gihodo Publ. Co., 1985.

3) Ueda, M. : Finite Element Analysis of Flexural Deformation of Reinforced Concrete Beam Members Taking account of the Bond-Slip of Reinforcement, Summaries of Techn. Paps. for Ann. Meeting of AIJ (in Kinki Prefecture), Oct. 1987.

4) Ueda, M. and Dobashi, Y. : Analysis of Deformations in Bending of Reinforced Concrete Beams Including Bond-Slip of Reinforcement. Proceedings of Japan Society of Civil Engineers, No.372, Vol. 5, Aug. 1986.