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# Windows for Pre-Emphasis and De-Emphasis for Block Coding of Images

Hideo KITAJIMA*, Tetsuo SHIMONO**, Tomoaki SHIRAKAWA*, and Yoshihiko OGAWA*

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**Abstract**

This paper proposes the use of a signal domain window to suppress blockiness that occasionally degrades the reconstruction of a block-coded signal. Effects of a window and its inverse window on signal-to-noise ratios, and required bit rates are analyzed. A likely window is also given. New quantization error measures are also introduced to augment the conventional Euclidean norm for vectors.

## 1. Introduction

Given a large array of signal elements to be encoded, we often divide it into small subarrays. The processing requirements are typically so involved that the original data array should be divided to be conquered. The statistical model of the signal may not be fully known to us. To make our life even harder, statistical error measures are not acceptable under some circumstances. If a particular realization of the quantized signal array is accompanied with large errors in some of its elements, it will not be accepted even if the statistically determined norm of errors is small enough. This problem can be attacked by breaking up the given signal array into small subarrays and fine-tuning the quantizer so that it can employ the best tactics to handle each of them. We will call this adaptive block quantization.

While adaptive quantization may be a viable approach to image coding, where each subarray is a small rectangle array or block of pixels, there emerges a new problem inherent to block processing. Errors in one block occasionally highlight it so distinctly that the human observer sees a rectangular object that actually does not exist. This phenomenon is often referred to as "blockiness."

To circumvent blockiness Wu et al.(1) scrambled pixels in neighboring subblocks, through regular decimation of pixels, to get a new set of blocks. In(2) the SCT(4) has been applied to overlapping subblocks to yield acceptable performance. Scrambling of DCT coefficients based on M-sequences has been reported to be effective for suppressing blockiness(3).

Some form of smoothing at the block boundaries could wipe out blockiness with possible

* Electronic Circuits, Department of Electronic Engineering
** Hokkaido Tokai University

smearing of legitimate edges……an unacceptable side effect. Since the blockiness is caused by none other than abrupt changes in the quantization noise in the block boundary, it will certainly be suppressed if the noise changes more smoothly in the block-to-block transition regions. We can achieve this by gradually chopping off the error magnitudes as we approach the block boundary and letting them grow gradually as we leave the boundary, going deep into the adjacent block. This must be attained without any sacrifice to the signal quality within the blocks.

Assume that a quantizer is fed with an array of analog data and outputs an quantized array of data with an array of quantization errors whose variances are uniform over the entire array. With this quantizer we can realize the above-mentioned error graduation by the use of window-based pre-emphasis on near-boundary elements of the signal array going into the quantizer and subsequent matched de-emphasis on the array coming out of the quantizer.

In this paper we propose the use of a signal domain window to suppress blockiness. We analyze in Section 2 effects of a window and its inverse window on signal-to-noise ratios, and required bit rates for the windowed signal. New quantization error measures are introduced to augment the conventional Euclidean norm for vectors. A likely window is proposed and evaluated in Section 3. Secoion 4 has some concdluding remarks.

## 2. Signal-to-Noise Ratios and Bit Rates for Windowed Signals

In the following we focus our attention on a one-dimensional problem. The arguments will be readily extended to cover two-dimensional cases or three-dimensional cases, as circumstances dictate.

In dealing with a random signal vector, we often find it necessary to express in terms of a scalar quantity per element signal power. If a like value is known for a noise vector accompanying the signal, we can compute an average signal-to-noise ratio or SNR based on them. The conventional per element signal power is defined as the ensemble avaerage of the square of the Euclidean norm of the signal vector divided by its dimension or the number of elements. The power measure is widely used since the norm is invariant under an orthogonal, linear transform.

The nathematical tractability of the Euclidean power measure breaks down, however, when it is used with non-orthogonal linear transforms. We will therefore introduce auxiliary signal-to-noise ratios for windowed signals. Bit rates for windowed signals will also be analyzed.

### 2.1. Conventional SNR

Let the original random signal vector be given by

$$x=(x_1,\ x_2,\ \cdots\cdots x_N) \tag{1}$$

where $x_i$ are zero-mean random variables with variances $\sigma_i^2$. Their sum gives the total signal power :

$$\text{Signal} = \sum_{i=1}^{N} \sigma_i^2 \tag{2}$$

We operate a window function W on x as follows:

$$W\{x\} = Wx \tag{3}$$

where

$$W = \text{diag}(w_1, \ w_2, \ \cdots\cdots, \ w_N)$$

with real constants $w_i$ yet to be defined for pre-emphasis operations.

The elements of W{x}have variances

$$\{w_1^2 \sigma_1^2, \ w_2^2 \sigma_2^2, \ \cdots\cdots w_N^2 \sigma_N^2\} \tag{4}$$

the sum of which gives the total signal power in the window domain:

$$\text{Signal}_W = \sum_{i=1}^{N} w_i^2 \sigma_i^2 \tag{5}$$

Let a quantization noise vector be added to W{x}whose variances are

$$\{\sigma_{\text{noise}1}^2, \ \sigma_{\text{noise}2}^2, \ \cdots\cdots \sigma_{\text{noise}N}^2\} \tag{6}$$

SNRs for individual signal elements in the window domain are

$$\{\frac{\sigma_1^2}{\sigma_{\text{noise}1}^2} w_1^2, \ \frac{\sigma_2^2}{\sigma_{\text{noise}2}^2} w_2^2, \ \cdots\cdots, \ \frac{\sigma_N^2}{\sigma_{\text{noise}N}^2} w_N^2\} \tag{7}$$

The average signal-to-noise ratio based on the Euclidean norm is given by

$$\text{SNR1} = \frac{\text{Signal}_W}{\text{Noise}_W} \tag{8}$$

where $\text{Signal}_W$ is as defined in (5) and $\text{Noise}_W$ is the total noise power given by

$$\text{Noise}_W = \sum_{i=1}^{N} \sigma_{\text{noise } i}^2 \tag{9}$$

To take W{x}back to the original signal domain, we multiply it by $W^{-1}$, where the inverse window is readily available:

$$W^{-1} = \text{diag}(w_1^{-1}, \ w_2^{-1}, \ \cdots\cdots w_N^{-1}) \tag{10}$$

The signal vector is accompanied with quantization noise whose variances are given by

$$\{w_1^{-2} \sigma_{\text{noise}1}^2, \ w_2^{-2} \sigma_{\text{noise}2}^2, \ \cdots\cdots, \ w_N^{-2} \sigma_{\text{noise}N}^2\} \tag{11}$$

They are summed to give the total noise power in the signal domain:

$$\text{Noise} = \sum_{i=1}^{N} w_i^{-2} \sigma_{\text{noise } i}^2 \tag{12}$$

While the individual SNRs given by (7) are invariant under the inverse window operation, the Euclid-sense average SNR after the inverse window, denoted as SNR2, generally has a value different from SNR1. Namely

$$\mathrm{SNR2} = \frac{\mathrm{Signal}}{\mathrm{Noise}} \tag{13}$$

where Signal is the average signal power defined in (2).

### 2.2. SNR by Geometric Average

The conventional per element power of the random signal vector (1) is computed through dividing (3) by the number of elements N :

$$P_x = \frac{1}{N} \sum_{i=1}^{N} \sigma_i^2 \tag{14}$$

It is well known that the above quantity is invariant if the signal vector undergoes a unitary transform such as DFT, DCT, etc. This feature is utilized in transform signal processing.

A drawback with (14) is that it can not reflect the correlation structure of the signal vector ; it uses only the trace of the covariance matrix of the signal. We feel that highly correlated signal elements have a smaller per element signal power than uncorrelated elements having the same variances.

The power measure has another serious problem when it is used with windows ; it is, in general, not invariant under such a non-orthogonal transform as (3). In the following we shall define a new average signal power measure.

Consider a zero-mean source represented by a random vector

$$u = (u_1,\ u_2,\ \cdots\cdots u_N) \tag{15}$$

whose elements are linearly independent and whose covariance matrix is given by

$$C_u = E\{uu^t\} \tag{16}$$

Proposition :

The total, real power of u be represented by a power product defined by

$$PP_u = \det C_u \tag{17}$$

We shall define geometric average, per-element signal power based on the power product by

$$P_u = \sqrt[N]{PP_u} = \sqrt[N]{\det C_u} \tag{18}$$

Note

$$\det C_u = \beta_1\, \beta_2 \cdots\cdots \beta_N$$

where $\beta_i$ are characteristic values of $C_u$ ; $\beta_i$ represent the variance power spectrum. In other words they are the variances or power of the signal elements observed in the KLT domain. Hence the name "power product." With the above (18) can be written as

$$P_u = \sqrt[N]{\beta_1\, \beta_2 \cdots\cdots \beta_N} \tag{19}$$

Taking the logarithm of $P_u$ we get

$$\log P_u = \frac{1}{N} \log[\det C_u] = \frac{1}{N} \log[\beta_1 \beta_2 \cdots\cdots \beta_N]$$

Namely

$$\log P_u = \frac{1}{N} (\log \beta_1 + \log \beta_2 \cdots\cdots \log \beta_N) \tag{20}$$

We observe that $P_u$ in[dB]is equal to the variance power spectrum components in[dB] arithmetically averaged in the KLT domain.

We are now ready to define an average SNR based on the geometric average power. Assume that the random signal vector x in contaminated by a random noise vector n added to it. We shall define an average SNR by

$$\mathrm{SNR3} = \frac{P_x}{P_n} \tag{21}$$

where $P_x$ and $P_n$ are geometric average power of x and n, respectively. Taking the logarithm of the SNR and using the definitions of $P_x$ and $P_n$ we get

$$\mathrm{SNR3}[\mathrm{dB}] = 10 \log_{10} \mathrm{SNR3} = \frac{10}{N} \log_{10} \frac{\det C_x}{\det C_n} \tag{22}$$

where $C_x$ and $C_n$ are the covariance matrices of x and n, respectively. Effects of the window on SNR3 will be discussed in the next section.

### 2.3. Bit Rate Analysis

We begin our discussion with rate analysis for the conventional, windowless case. If quantization of the signal vector x is done in the KLT domain, the average required number of bits will be

$$R(d_1, d_2, \cdots\cdots d_N) = \frac{1}{2} \sum_{i=1}^{N} \log_2 \frac{\lambda_i}{d_i} \tag{23}$$

where $\lambda_i$ and $d_i$ are the variances of the transform-domain signal elements and quantization noise elements. Since $\det C_x$ and $\det C_n$ can be written respectively as

$$\det C_x = \lambda_1 \lambda_2 \cdots\cdots \lambda_N$$
$$\det C_n = d_1 d_2 \cdots\cdots d_N$$

where we have made a reasonable assumption that the quantizer generates uncorrelated noise. The above relations follow readily from the orthogonality of the KLT matrix. With the above relations put into the right-hand side of R, we get

$$R(d_1, d_2, \cdots\cdots d_N) = R(\mathrm{SNR3}) = \frac{1}{2} \log_2 \frac{\mathrm{dot}\ C_x}{\det C_n} \tag{24}$$

with SNR3 replacing the sequence of arguments in the leftmost R ; the rate achieves the SNR given by (21).

With the window

$$W = \mathrm{diag}(w_1, w_2, \cdots\cdots w_N)$$

the windowed signal (3) has the covariance matrix

$$C_w = WC_x W \tag{25}$$

with its determinant

$$\det C_w = w_1^2 w_2^2 \cdots\cdots w_N^2 \cdot \det C_x \tag{26}$$

Let quantization be done through the KLT performed on (3). We denote the covariance matrix of the quantization noise vector in the window domain by $C_{nw}$. We set

$$\det C_{nw} = w_1^2 w^2{}_2 \cdots\cdots w_N^2 \cdot \det C_n \tag{27}$$

so that we can obtain the same overall SNR3 (21).

The average number of bits required to attain the same SNR3 is given by

$$R_W(\mathrm{SNR3}) = \frac{1}{2}\log_2 \frac{w_1^2 w_2^2 \cdots\cdots w_N^2 \det C_x}{w_1^2 w_2^2 \cdots\cdots w_N^2 \det C_n} = \frac{1}{2}\log_2 \frac{\det C_x}{\det C_n} = R(\mathrm{SNR3}) \tag{28}$$

as seen from (24). Thus the introduction of the window W presents no burden on the bit rate to achieve the same SNR3.

With the inverse window $W^{-1}$ the signal power product is brought back to det $C_x$, and the noise power product (27) is converted to $C_n$. Hence the overall SNR is invariant under the inverse window if the average SNR is measured in terms of the geometric average. Also remark that the SNR in (22) is directly linked to the required bit rate given in (28) :

$$\mathrm{SNR[dB]} = 20 \log_{10} 2 \cdot R(\mathrm{SNR3}) \cong 6R(\mathrm{SNR3}) \tag{29}$$

## 3. A Candidate Window and its Performance

The window should be designed so that artifacts generated in the block boundary regions are reduced to accepteable levels for the human observer. An ideal approach would come from the full understanding of the human visual system. We shall, however, abopt an ad hoc window here for simplicity.

### 3.1. A Candidate Window

It can be shown that the inverse, de-emphasis window having a constant slope in the transition region can suppress the maximum leap in errors in the transition region below an acceptable level. We submit without derivation the matching pre-emphasis window :

$$w_i = \begin{cases} \dfrac{p-1/2}{i-1/2}, & 1 \leq i \leq p \\ 1, & p < i < N-p+1 \\ \dfrac{p-1/2}{N-i+1/2}, & N-p+1 \leq i \leq N \end{cases} \tag{30}$$

In the above p is a positive integer chosen to satisfy

$$p \geq \frac{e_m}{\Delta_m} + \frac{1}{2}$$

where $e_m$ is the peak error and $\Delta_m$ the permissible maximum leap in the signal domain quantization errors. p defines the depth of the transition region in which errors are made to change gradually.

### 3.2. Statistical Performance of the Window

In the preceding subsection a candidate window was proposed. While its performance in the worst case is as stated there, we are also interested its average performance.

Let the covariance matrix of the quantization noise vector be

$$C_q = \mathrm{diag}(\sigma_{q1}^2,\ \sigma_{q2}^2,\ \cdots\cdots \sigma_{qN}^2) \tag{31}$$

where quantization is so be done in the KLT domain. Then back in the window domain, the noise will have the covariance matrix

$$C_{nw} = T^t\, C_q\, T \tag{32}$$

where T represents the forward KLT. Nothing that $T = U^t$ with $U$ = the characteristic matrix of $C_W$ (as defined in (25)), we can show that window-domain noise variances are

$$\sigma_{noise\ i}^2 = (C_{nw})_{ii} = \sigma_{q1}^2\, u_{i1}^2 + \sigma_{q2}^2\, u_{i2}^2 + \cdots\cdots \sigma_{qN}^2\, u_{iN}^2 \tag{33}$$

where

$$u^{ij} = (U)^{ij}$$

The elements in (33) represent the variances of noise elements in the window domain. If multiplied by the inverse window function, they yield signal domain noise variances given by (11), i. e.,

$$\sigma_{ni}^2 = w_i^{-1} (C_{nw})_{ii}$$

Signal domain noise variances are compared in Fig. 1 for windowed and window-less cases. The results have been obtained through assuming

$$N = 16$$
$$(C_x)_{ij} = \rho^{|i-j|},\quad \rho = 0.95,\quad p = 4$$

for the window. SNRs by various definitions are compared in Table 1.

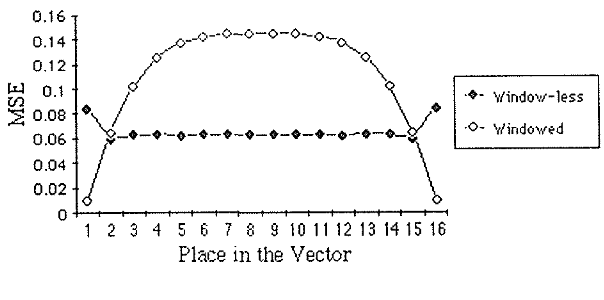


**Fig. 1** MSE distribution for the window-less and windowed signal vectors.

| | Window-less | Windowed |
|---|---|---|
| SNR1 | 11.8 dB | 15.4 dB |
| SNR2 | 11.8 dB | 9.6 dB |
| SNR3 | 3.0 dB | 3.0 dB |

**Table 1.** Average SNRs.

We observe that the window suppresses quantization errors near both ends at the cost of higher mean-squared errors for the inside elements.

## 4. Conclusion

We have discussed effects of windows for block-based image coding. For ease of notation and discussion, only one-dimensional cases have been considered. The extension to two-dimensional and three-dimensional cases will be obvious. Since the window deforms the covariance matrix of an input signal, the popular DCT, which may simulate the KLT for the input signal, will be useless in the window domain. As the reader may have found, further work must be done for selecting appropriate windows for actual application of the theory.

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