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Study on a Magnetic Dynamic Vibration Absorber with Adjustable Natural Frequency

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Abstract

In a previous paper, we proposed a new magnetic dynamic vibration absorber which has a function of tuning the natural frequency to the exciting frequency automatically by adjusting the distance between magnets used as a repelling force system.

In this paper, we analyze a system equipped with the proposed vibration absorber taking into account nonlinear properties of restoring force and to investigate the response of the system through vibration experiments.

Based on these results, we propose an adaptive method for adjusting the distance between magnets such that the amplitude of the primary system is reduced as small as possible.

Vibration experiments assure that the amplitude of the primary system can be suppressed in sufficiently small values by applying the proposed method.

1. Introduction

In the previous paper[1], we proposed a new magnetic dynamic vibration absorber which has a function of tuning the natural frequency of the absorber to the variable exciting frequencies by adjusting the distance between magnets used as repelling force system, and produced a trial vibration absorber using three rare-earth magnets and equipping a control unit for adjusting the distance of the magnets. The trial absorber showed a remarkable absorbing effect compared with conventional passive absorber. However, in the experimental results for automatic adjustment of distance between magnets, a large peak caused by the nonlinear effect of the repelling force was observed around a certain frequency region.

In this paper, the response of a system with a trial magnetic dynamic vibration absorber is analyzed taking into account nonlinear properties of repelling force acting between the permanent magnets. Then, referring these results, a practical method for adjusting the distance between magnets is proposed to suppress the amplitude of primary system as small as possible.

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Finally, vibration experiments are performed for the system with the trial magnetic vibration absorber and effectiveness of the proposed method is confirmed.

2. Repelling force between magnets and restoring force of magnetic vibration absorber

2.1 Characteristics of repelling force between magnets

For a pair of cylindrical magnets confronted the same pole surfaces, the relation between the distance of magnets d and the repelling force $F(d)$ acting between magnets can be calculated analytically[2),3)], and is approximated by eq.(1) for appropriate region of distance between magnets[2)].

$$F(d)=kd^{-n}. \quad (k,n: const.) \tag{1}$$

In the previous paper[1)], the constant values of k and n in eq.(1) were obtained for the magnet used for the absorber (*cf.* Table 1) by applying the method of least squares to the theoretical results in the region of distance (*25* to *55* mm) corresponding to the traveling distance of magnets in the absorber. These values are shown in Table 1. This time, an experiment was performed to obtain the relation between distance of magnets and the repelling force. The results are shown in Fig.1 together with theoretical result and calculated result of eq.(1). Experimental results were obtained only for relatively small repelling force, but these results show good agreement with the theoretical result. This shows that equation (1) represents the analytical result accurately in the region of distance in question.

Table 1 Specification of used magnets

material	Samarium Cobalt
magnetization	8.8 KG
characteristic coefficient of repelling force k	$1.522\text{x}10^{-3}$ Nm^{n}
characteristic exponent of repelling force n	3.022
outside diameter	38 mm
inside diameter	10 mm
thickness	20 mm
mass	0.19 Kg

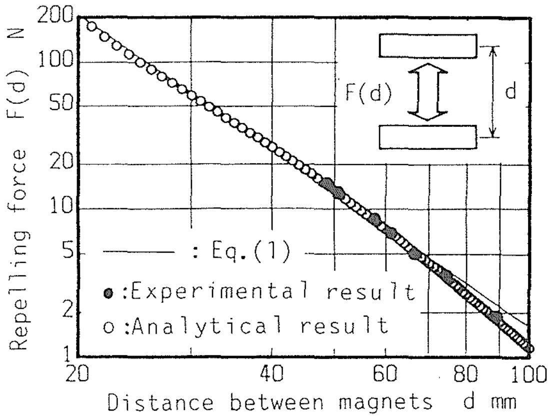


Fig.1 Repelling force vs. distance between magnets

2.2 Nonlinear approximation of restoring force and natural frequency of absorber

In the proposed dynamic absorber[1], three cylindrical magnets are arranged so as to repelling each other (*cf.* Fig.2). In this case, the restoring force acting on the center magnet is represented by the sum of repelling forces subjected from the upper and the lower magnets. Considering the gravitational force, we put d_1 as distance between the center and the upper magnet, and d_2 distance between the center and the lower magnet in the equilibrium state respectively, and y displacement of the center magnet from the equilibrium position. The restoring force for the center magnet can be written as eq.(2) from eq.(1).

$$F(y) = k\{(d_1 - y)^{-n} - (d_2 + y)^{-n}\} + mg \tag{2}$$

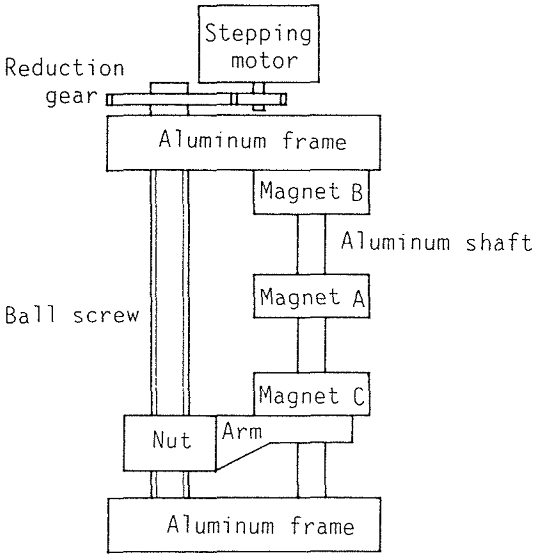


Fig.2 Schematic view of trial magnetic dynamic vibration absorber

where m is a mass of the center magnet, and g is the gravitational acceleration. In eq.(2), d_1 and d_2 are obtained from the following relations.

$$\begin{cases} k(d_1^{-n} - d_2^{-n}) + mg = 0\ , \\ d_1 + d_2 = L\ , \end{cases} \tag{3}$$

where L is the distance between the upper and the lower magnets.

In the previous paper[1], we analyzed the vibration system using linear approximation of the restoring force. In this paper, we approximate the restoring force by expanding the right hand side of eq.(2) into power series of displacement y and retaining terms up to the third power like the analysis performed by Kojima and Yamakawa[4]. Namely, restoring force is expressed by

$$F(d) = \alpha y - \beta y^2 + \gamma y^3, \tag{4}$$

where

$$\begin{cases} \alpha = kn\{d_1^{-(n+1)} + d_2^{-(n+1)}\}\ , \\ \beta = kn(n+1)\{d_1^{-(n+2)} + d_2^{-(n+2)}\}\ , \\ \gamma = kn(n+1)(n+2)\{d_1^{-(n+3)} + d_2^{-(n+3)}\} \end{cases} \tag{5}$$

In Fig.3, the relation between restoring force and displacement of the center magnet calculated from eq.(2) (solid line) and eq.(4) (dotted line) for various distance of magnets L is shown. It can be seen from Fig.3 that when the displacement is small, the restoring force is expressed accurately by eq.(4).

The natural frequency of nonlinear system depends on the amplitude of vibration. However, when the amplitude is sufficiently small, we may estimate the natural frequency by linear approximation. Here we compare the natural frequency calculated by linear approximation ($f_n=\omega_n/2\pi$, $\omega_n=\sqrt{\alpha/m}$) and one obtained from free vibration experiment for small amplitude. The experimental results are shown in Fig.4 together with the calculation result by linear approximation. The solid line is calculated results and mark (o) represents experimental results. Although calculation results are slightly larger than experimental results in the region of large distance of magnets, error of calculation result is less than 7%. From these results, we understand that the natural frequency of the proposed dynamic absorber can be changed about 50 to 15 Hz for 55 to 100 mm of distance between magnets.

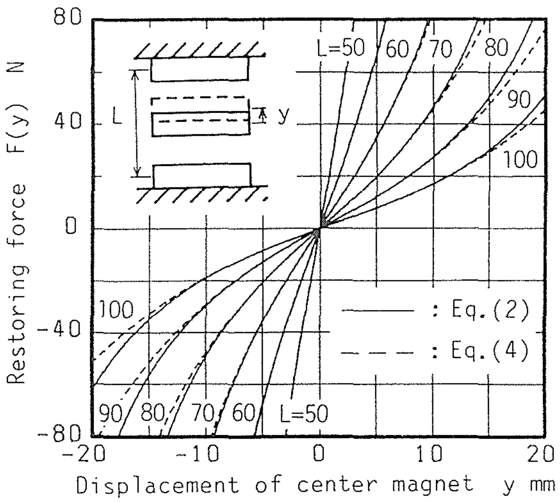


Fig.3 Restoring force vs. displacement of center magnet

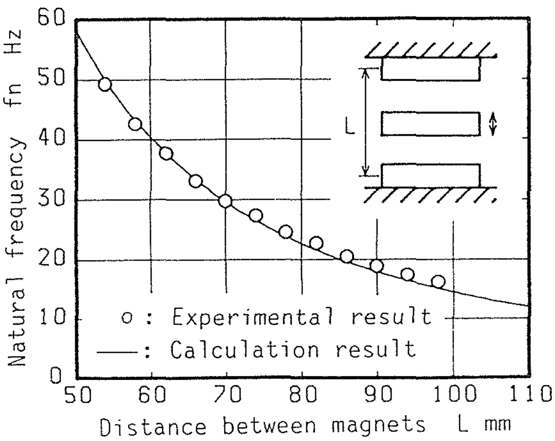


Fig.4 Natural frequency of absorber vs. distance between magnets

3. Vibration analysis

3.1 Mechanical model of system and equation of motion

The mechanical model of a vibration system with a proposed absorber is shown in Fig.5. In Fig.5, constants α, β and γ given in eq.(5) can be changed by adjusting the distance of magnets. Equations of motion of the system are written as:

$$\begin{cases} M\ddot{x}_1+m\ddot{y}_1+c_1(\dot{x}_1-\dot{u}_1)+K(x_1-u_1)=0\,, \\ m\ddot{y}_1+c_2\dot{z}_1+\alpha z_1-\beta z_1^2+\gamma z_1^3=0\,, \\ z_1=y_1-x_1, \qquad u_1=u_0\sin\omega t\,, \end{cases} \tag{6}$$

where, M is mass of primary system, K is spring constant of primary system, c_1 is damping coefficient of primary system, m is mass of vibration absorber, c_2 is damping coefficient of absorber and z_1 is relative displacement between primary system (x_1) and absorber (y_1).

We introduce here the following nondimensional quantities to abbreviate the subsequent treatment.

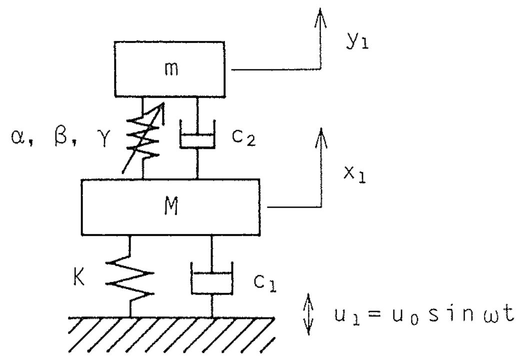


Fig.5 Mechanical model of system with trial dynamic vibration absorber

$$\tau=\omega t,\ \sigma=\frac{\omega_n}{\Omega_n},\ \eta=\frac{\omega}{\Omega_n},\ \mu=\frac{m}{M},\ \xi_1=\frac{c_1}{2M\Omega_n},\ \xi_2=\frac{c_2}{2m\Omega_n},$$

$$a=1-\frac{\beta^2}{3\alpha\gamma},\ b=\frac{\beta}{\gamma u_0},\ e=\frac{\gamma u_0^2}{\alpha},\ H=\frac{(1+2a)\,b}{9},$$

$$u=\frac{u_1}{u_0},\ x=\frac{x_1}{u_0},\ y=\frac{y_1}{u_0},\ q=y-x-\frac{b}{3},$$

where we put $\Omega_n=\sqrt{K/M}$ *and* $\omega_n=\sqrt{\alpha/m}.$ Equations (6) can be rewritten as follows.

$$\begin{cases} (1+\mu)\,\eta^2\ddot{x}+\mu\eta^2\ddot{q}+2\xi_1\eta\dot{x}+x=\sqrt{1+(2\xi_1\eta)^2}\ \sin\tau\ , \\ \eta^2\ddot{x}+\eta^2\ddot{q}+2\xi_2\eta\dot{q}+\alpha\sigma^2 q+e\sigma^2 q^3+\sigma^2 H=0\ , \end{cases} \tag{7}$$

where dot means here the derivative with respect to variable τ.

3.2 Solution of equations of motion and stability analysis of solution

It is difficult to obtain the exact solution of eqs.(7), so we solve these equations by using the method of harmonic balance assuming steady state harmonic vibration. First, we write the solution of eqs.(7) as follows:

$$\begin{cases} x=X\sin(\tau+\phi)\ , \\ q=P+Q\sin(\tau+\psi)\ , \end{cases} \tag{8}$$

where X, P, and Q are considered as slowly varying functions with respect to time (τ). We put their time derivatives as:

$$\begin{cases} \dot{X}=A(X,\ P,\ Q), \\ \dot{P}=B(X,\ P,\ Q), \\ \dot{Q}=C(X,\ P,\ Q), \end{cases} \tag{9}$$

where A, B and C are all small quantities in the first order.

Substituting x and q of eqs.(8) into eqs.(7) and retaining the first order small quantities by considering eqs.(9), then comparing terms of $\sin\tau$ and $\cos\tau$ in each equation, we obtain the following equations.

$$\begin{cases} 2\zeta_2\eta B+\sigma^2(E_1P+H)=0\,, \\ 2QI_1C+\{E_2^2+(2\zeta_2\eta)^2\}\ Q^2-\eta^4X^2=0\,, \\ 2QI_2C+2XI_3A+E_3^2+E_4^2\eta^2-\{1+(2\zeta_1\eta)^2\}X^2=0\,, \end{cases} \tag{10}$$

where we put:

$$\begin{cases} E_1=a+e(P^2+3Q^2/2)\ , \\ E_2=\eta_e^2-\eta^2\ , \\ E_3=\xi X^2-\mu E_2Q^2\ , \\ E_4=2(\zeta_1X^2+\mu\zeta_2Q^2\ , \\ I_1=2\zeta_2\eta(\eta_e^2+\eta^2)\ , \\ I_2=2\mu\eta\{\zeta_2E_3+\eta_e^2E_4\}\ , \\ I_3=2\eta(E_4-\zeta_1E_3)\ , \\ \xi=1-(1+\mu)\eta^2\ , \\ \eta_e^2=\sigma^2\{a+3e(P^2+Q^2/4\}\ . \end{cases} \tag{11}$$

Equations (10) are linear equations with respect to A, B and C, so we can easily obtain these quantities as functions of X, P and Q.

$$\begin{cases} A=\dfrac{\{1+(2\zeta_1\eta)^2\}X^2-E_3^2-\eta^2E_4^2}{2XI_3}-\dfrac{QI_2}{XI_3}C\ , \\ B=-\dfrac{\sigma^2(E_1P+H)}{2\zeta_2\eta}\ , \\ C=\dfrac{\eta^4X^2-\{E_2^2+(2\zeta_2\eta)^2\}\ Q^2}{2QI_1}\ . \end{cases} \tag{12}$$

Steady state solutions X_0, P_0 and Q_0 are obtained from the following equations by substituting $A=B=C=0$ into eqs.(10) or eqs.(12).

$$\begin{cases} E_{10}P_0+H=0\,, \\ \{E_{20}^2+(2\zeta_2\eta)^2\}Q_0^2-(\eta^2X_0)^2=0\,, \\ E_{30}^2+\eta^2E_{40}^2-\{1+(2\zeta_1\eta)^2\}X_0^2=0\,, \end{cases} \tag{13}$$

where E_{i0} $(i=1\sim4)$ represent values of E_i substituted by steady state solution (X_0, P_0, Q_0). Backbone curve can be obtained from the following equations by substituting $\zeta_1=\zeta_2=0$ and $u_0=0$ into eqs.(13).

$$\begin{cases} E_{10}P_0+H=0\,, \\ E_{20}Q_0-\eta^2X_0=0 \\ \xi X_0-\mu\eta^2Q_0=0 \end{cases} \tag{14}$$

To investigate the stability of steady state solution, putting $X = X_0 + \delta X$, $P = P_0 + \delta P$ and $Q = Q_0 + \delta Q$, and expanding eqs.(9) around steady state solution, we obtain the following equations of variations.

$$\begin{cases} \delta\dot{X} = A_X \delta X + A_P \delta P + A_Q \delta Q \ , \\ \delta\dot{P} = B_X \delta X + B_P \delta P + B_Q \delta Q \ , \\ \delta\dot{Q} = C_X \delta X + C_P \delta P + C_Q \delta Q \ , \end{cases} \tag{15}$$

$$\begin{cases} A_X = \dfrac{1 + (2\zeta_1\eta)^2 - 2\xi E_{30} - 4\zeta_1\eta^2 E_{40}}{I_{30}} - \dfrac{Q_0 I_{20}}{X_0 I_{30}} C_X \ , \\ A_P = \dfrac{6\mu\sigma^2 e P_0 Q_0^2 E_{30}}{X_0 I_{30}} - \dfrac{Q_0 I_{20}}{X_0 I_{30}} C_P \ , \\ A_Q = \dfrac{\mu Q_0 \{ (4E_{20} + 3\sigma^2 e Q_0^2) E_{30} - 8\zeta_2\eta^2 E_{40} \}}{2 X_0 I_{30}} - \dfrac{Q_0 I_{20}}{X_0 I_{30}} C_Q \ , \\ B_X = 0 \ , \quad B_P = -\dfrac{\sigma^2 (E_{10} + 2eP_0^2)}{2\zeta_2\eta} \ , \quad B_Q = -\dfrac{3\sigma^2 e P_0 Q_0}{2\zeta_2\eta} \ , \\ C_X = \dfrac{\eta^4 X_0}{Q_0 I_{10}} \ , \quad C_P = -\dfrac{6\sigma^2 e P_0 Q_0 E_{20}}{I_{10}} \ , \\ C_Q = -\dfrac{2\{ E_{20}^2 + (2\zeta_2\eta)^2 \} + 3\sigma^2 e Q_0^2 E_{20}}{2 I_{10}} \ . \end{cases} \tag{16}$$

where suffix X, P and Q mean derivatives with respect to each variable. Substituting $\delta X = x_0 e^{\lambda\tau}$, $\delta P = p_0 e^{\lambda\tau}$, *and* $\delta Q = q_0 e^{\lambda\tau}$ into eqs.(15), we obtain the characteristic equation:

$$\begin{vmatrix} A_X - \lambda & A_P & A_Q \\ B_X & B_P - \lambda & B_Q \\ C_X & C_P & C_Q - \lambda \end{vmatrix} = 0 \ . \tag{17}$$

Expanding the left hand side of eq.(17) with respect to λ, we obtain:

$$\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0 \ . \tag{18}$$

where

$$\begin{cases} a_0 = A_X (B_Q C_P - B_P C_Q) + A_P (B_X C_Q - B_Q C_X) + A_Q (B_P C_X - B_X C_P) \ , \\ a_1 = (A_X B_P - A_P B_X) + (B_P C_Q - B_Q C_P) + (C_Q A_X - C_X A_Q) \\ a_2 = -(A_X + B_P + C_Q) \ . \end{cases} \tag{19}$$

Stability condition of steady state solution is expressed as:

$$a_0 > 0 \ , \ a_1 > 0 \ , \ a_2 > 0 \ , \ a_1 a_2 - a_0 > 0 \ . \tag{20}$$

3.3 Response of vibration system and a practical method for adjusting distance between magnets

In the previous paper[1], we showed the response of the primary system for the case in which the distance between magnets is constant and exciting frequency is changed. In the real system, on the contrary, exciting frequency is constant and the distance between magnets is adjusted so as to minimize the amplitude of the primary system. So, in this paper, we examine change of amplitude of the primary system caused by changing the distance between magnets while keeping the exciting frequency and amplitude constant.

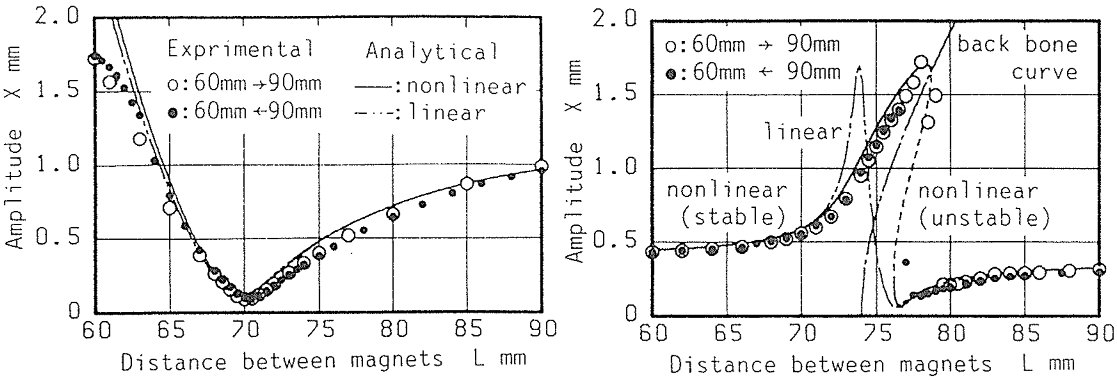
Table 2 Parameters of primary system and absorber

parameter	primary system		vibration absorber	
	symbol	value	symbol	value
mass	M	2.49 Kg	m	0.19 Kg
natural frequency	$\frac{\Omega_n}{2\pi}$	33.4 Hz	$\frac{\omega_n}{2\pi}$	variable
damping ratio	$\zeta_1 = \frac{c_1}{2M\Omega_n}$	0.023	$\zeta_2 = \frac{c_2}{2m\Omega_n}$	0.015

In Table 2, parameters and their values of the primary system and the absorber used for subsequent experiments and analyses are listed.

Figure 6(a) shows the results for exciting frequency 30 Hz and amplitude 0.1 mm. Calculation results of nonlinear and linear approximation represent almost the same results, and show good agreement with the experimental results except for small distance between magnets. In this case, jumping phenomenon does not appear even if the exciting amplitude becomes large. When the exciting frequency is larger than 30 Hz, almost the same results were obtained.

Figure 6(b) shows the results for exciting frequency 25 Hz and amplitude 0.15 mm. In this case, jumping phenomenon is observed. Nonlinear calculation results predict this phenomenon unlike linear calculation, but the calculation results are shifted to smaller distance of magnets as a whole and the value of estimated distance giving the minimum amplitude is smaller than the experimental value by several mm. When the exciting frequency is smaller than 30 Hz, almost the same results were obtained. From these results, it is understand that we have to adjust the distance of magnets to the value larger than the estimated value by several mm for minimizing the amplitude of the primary system.



(a) u^0=0.15mm, f=30Hz (b) u_0=0.15mm, f=25Hz

Fig.6 Response of primary system for various distance between magnets

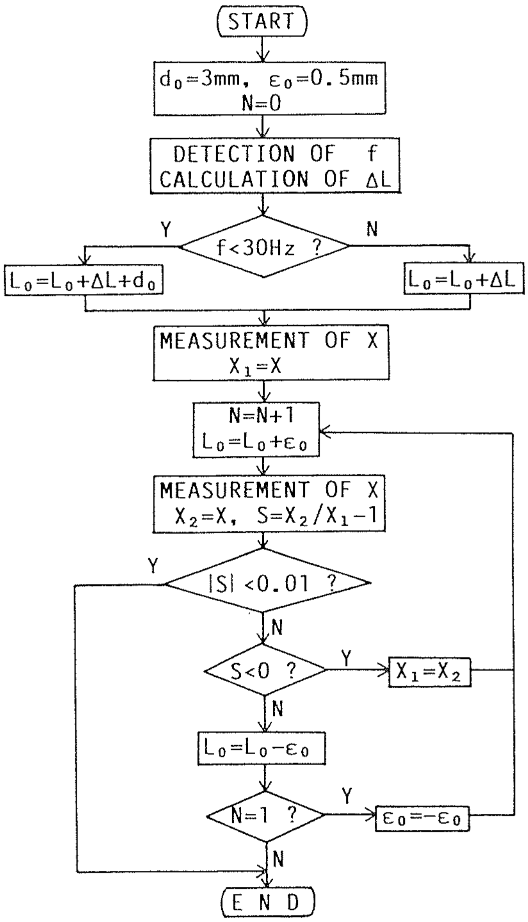


Fig.7 Flow of procedure for adjusting distance between magnets

However, the amplitude of the primary system is sufficiently small in the vicinity of distance of magnets giving the minimum amplitude in any case, so we only need to set the distance between magnets at almost the minimum position.

Referring these results, we propose here a practical adjusting method for the proposed absorber to give almost the minimum amplitude. In this method, we first set the distance at the value near the minimum position. Namely, when the exciting frequency is larger than 30 Hz, we set the distance at the value calculated by linear approximation, and when the exciting frequency is smaller than 30 Hz, we set the distance at slightly larger value than linear estimation. Then we search the minimum position with moving the magnets by small distance while observing the amplitude of the primary system. Flow of this procedure is shown in Fig.7.

4. Vibration experiment for a system with a trial magnetic dynamic vibration absorber

4.1 Experimental method

To investigate the effectiveness of the proposed adjusting method, a trial magnetic dynamic vibration absorber is mounted on a model of the primary system which is composed of a brass beam with built in ends and an aluminum frame, and vibration experiments are performed (*cf.* Fig.8). In vibration experiments, the primary system is excited with several values of the amplitude and frequencies over the range of 15 to 45 Hz, then the distance between magnets of the absorber is adjusted by using the proposed method and the amplitude of the primary system is measured for each frequency. For comparison purpose, vibration experiments in which the distance between magnets is adjusted by manual operation are performed to find optimal distance and true minimum amplitude of the primary system. Experimental results obtained from the proposed method are compared with those from manual adjustment and with the amplitude calculated by linear approximation.

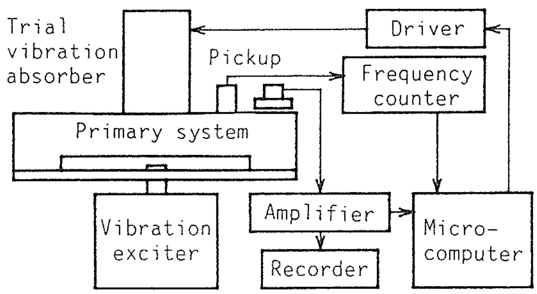


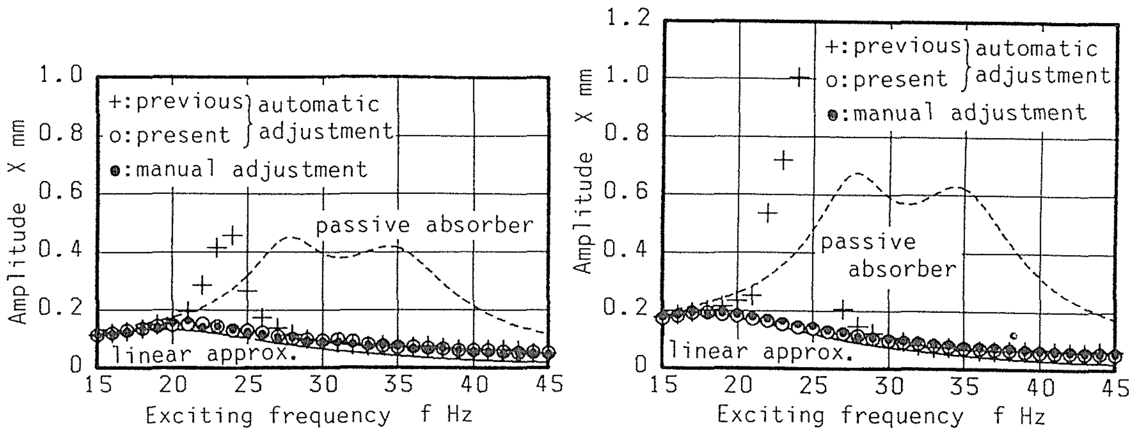
Fig.8 Diagram of experimental setup

4.2 Results of experiments

Figure.9(a) shows the results for exciting amplitude of 0.1 mm. The result for automatic adjustment by the present method (hollow circle) shows good agreement with manual operation (solid circle) and represents much higher absorbing effect compared with the passive absorber (dotted line). The result of linear estimation of amplitude (solid line) predicts the present result and the result of manual operation in good approximation.

In Fig.9(b), results for exciting amplitude of 0.15 mm are plotted. The result for automatic adjustment by the present method (hollow circle) does not have large peak value around 24 Hz which appeared in the previous result (+) and shows almost the same absorbing effect as the manual operation. Other properties are almost the same as Fig.9(a).

From these results, it is confirmed that by applying the proposed adjusting method, for any case, the amplitude of the primary system is suppressed in sufficiently small value, which can be predicted by linear approximation of restoring force.



(a)u_0=0.10mm (b)u_0=0.15mm

Fig.9 Response of primary system with trial dynamic vibration absorber

5. Conclusions

The results of the present study are summarized as follows:

(1) By analyzing response of a system with a proposed magnetic dynamic vibration absorber considering nonlinear properties of repelling force between magnets, it could be predicted that when exciting frequency is lower than 30 Hz, jumping phenomenon of amplitude appears in the vicinity of the minimum amplitude.

(2) Comparing analytical and experimental results, it was found that distance between magnets which gives the minimum amplitude estimated by linear calculation is smaller than the real value by several mm, then it could be happened that extremely large amplitude appears when adjusting the distance between magnets to the value estimated by linear calculation.

(3) Based on the above mentioned results, we proposed a practical adjusting method of the distance between magnets so as to suppress the amplitude of the primary system to almost the minimum value by detecting the amplitude and searching its minimum value while changing the distance.

(4) It was confirmed that by applying the proposed adjusting method, the amplitude of the primary system can be suppressed in almost the minimum value obtained by the manual operation.

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