Scorekeeping and Dynamic Logics of Speech Acts

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The gap

Van Benthem & Liu (2007) on commanding
For instance, intuitively, a command
“See to it that φ!”
makes worlds where φ holds preferred over those where it does not - at least, if we accept the preference induced by the issuer of the command.

The need they felt for the proviso here reflects an important logical gap between what an illocutionary act of commanding involves and perlocutionary effects it may have upon our preferences.

Austin’s Distinction (1955, pp.101-3.)

Locutionary Act
He said to me “Shoot her!” meaning by ‘shoot’ shoot and referring by ‘her’ to her.

Illocutionary Act
He urged (advised, ordered, etc.) me to shoot her.

Perlocutionary Act
(a) He persuaded me to shoot her.
(b) He got me to shoot her.
Speech acts as acts

- If the notion of speech act is to be taken seriously, it must be possible to treat speech acts as acts.
- If we succeed in characterizing speech acts in terms of dynamic changes they bring about, it becomes possible to treat them within a general theory of action.
- But how can we do that?

Perlocutinary acts as acts

**Perlocutionary Act**

(a) He persuaded me to shoot her.
(b) He got me to shoot her.

**Austin on perlocutionary acts (1955, p.103)**

According to Austin, perlocutionary acts are acts that really produce “real effects” upon the feelings, thoughts, or actions of addressees, or of speakers, or of other people.

They are recognized only when their effects are recognized.

Illocutinary acts as acts

**Illocutionary Act**

He urged (advised, ordered, etc.) me to shoot her.

**The Problem**

What effects do they have?
What role do they play in our social life?

Austin, Strawson, and Searle

**Austin on illocutionary acts (1955, p.103)**

Austin considered illocutionary acts as acts whose effects are “what we regard as mere conventional consequences”

**After Strawson (1964) and Seale (1969)**

Austin's conception of illocutionary acts as acts whose effects are conventional has been disregarded both by those who follow Strawson and those who follow Searle.
Strawson (1964) on Austin

- Strawson (1964) observed that the kind of conventional effects involved in the examples used by Austin are dependent on special extralinguistic conventions.
- He then argued that there are many other illocutionary acts that do not seem to be dependent on any such special extralinguistic conventions.
- Thus, according to Strawson, Austin made an unwarranted overgeneralization when he attributed conventional effects to illocutionary acts in general.

Searle (1969) on Austin and Strawson

- Searle criticized Grice (and Strawson) for treating meaning as “a matter of intending to perform a perlocutionary acts”, but agreed with Strawson in seeing Austin’s notion of conventional effect as an unwarranted overgeneralization.
- Searle sees conventionality of illocutionary acts as a matter of meaning, and denied the distiction between locutionary acts and illocutionary acts.
- He identified what he called “the illocutionary effect” with “the hearer understanding the utterance of the speaker” (p.46-47).

Conventional effects vs. utterers’ intentions

- Strawson and his followers tried to characterize uses of sentences not in terms of conventional effects, but in terms of utterers’ intentions to produce various effects in addressees along the lines initiated by Grice (1957).
- Utterers’ intentions, however, usually go beyond illocutionary acts by involving reference to perlocutionary effects, while illocutionary acts can be effective even if they failed to produce intended perlocutionary effects.

Beyond the securing of uptake

- Austin considered the securing of uptake of this kind as necessary condition for illocutionary acts, but didn’t considered it to be sufficient.
- Indeed, even typical illocutionary acts such as acts of promising, which both Strawson and Searle see not conventional in what they take to be Austin’s sense, seem to involve more than the mere securing of uptake.
- The social or institutional consequences they have, such as generation of obligations, can be said to be “conventional” in Austin’s sense.
- They are institutional in Searle’s sense.
What Austin's Earlier Answer Enables us to See

**Perlocutionary acts**

Since perlocutionary acts are acts that really produce real effects, they cannot be completed without really producing them.

**Austin 1955, pp.103-4.**

Thus Austin says, “we can say ‘I argue that’ or ‘I warn you that’ but we cannot say ‘I convince you that’ or ‘I alarm you that’”.

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The problem

- Is it possible to develop this conception of illocutionary acts into a general theory of illocutionary acts?
- In order to do so, we have to
  - specify conventional effects of a sufficiently rich variety of illocutionary acts, and
  - develop a theory in which these illocutionary acts are shown to be fully characterized in terms of those conventional effects.

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The plan

- The recent development of Dynamic Epistemic Logics suggests a recipe for developing logics that can capture effects of various speech acts.
- We have developed dynamic logics that can deal with acts of commanding, promising, asserting, conceding, and withdrawing according to this recipe (Yamada 07a, 07b, 08a, 08b, unfinished draft).
- We will review these developments.
- We will then show how the results obtained can be incorporated into a more comprehensive picture of social interaction with the help of the notion of scorekeeping for language games.
The developments of dynamic epistemic logics

The formulas of the form $\phi \rightarrow [\psi]K_i\phi$ are shown to be valid for any $i \in I$ if no operators of the form $K_i$ occur in $\phi$.

- This is too strong for interpreting natural language public announcements.
- A gap similar to the one we have seen is also present here.

The method used in developing DEL can be used to develop logics that deal with a much wider variety of speech acts.

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Two points to be noted

- Carefully identify the aspect affected by the kind of speech acts you want to study
- Find the modal logic that characterizes this aspect
- Add dynamic modalities that represent types of those speech acts
- Define model updating operation that interprets the speech acts under study as what update the very aspect
- (If possible) find a complete set of reduction axioms for the resulting dynamic logic.
1 & 2. Identifying the relevant aspect and its logic

Introduction
DEL and A dynamic logic of acts of commanding
Refinements and Variations
Scorekeeping and dynamic logics of speech acts
Conclusion

The language of multi-agent deontic logic

Definition
Take a countably infinite set \( \text{Aprop} \) of proposition letters and a finite set \( I \) of agents, with \( p \) ranging over \( \text{Aprop} \) and \( i \) over \( I \). The multi-agent monadic deontic language \( \mathcal{L}_{\text{MDL}^+} \) is given by:

\[
\varphi ::= T \mid p \mid \neg \varphi \mid \varphi \land \psi \mid \Box \varphi \mid O_i \varphi
\]

- \( O_i \varphi \): It is obligatory upon an agent \( a \) to see to it that \( \varphi \).
- \( P_a \varphi \): \( \neg O_a \neg \varphi \).
- \( F_a \varphi \): \( O_a \neg \neg \varphi \).

\( \mathcal{L}_{\text{MDL}^+} \)-models

Definition
By an \( \mathcal{L}_{\text{MDL}^+} \)-model, we mean a tuple 
\[
M = \langle W^M, \rightarrow^M, \neg^M, \vdash^M, \mid i \in I \mid, V^M \rangle
\]
where:

(i) \( W^M \) is a non-empty set (heuristically, of ‘possible worlds’),
(ii) \( \vdash^M \subseteq W^M \times W^M \),
(iii) \( \neg^M \subseteq \vdash^M \) for each \( i \in I \),
(iv) \( V^M \) is a function that assigns a subset \( V^M(p) \) of \( W^M \) to each proposition letter \( p \in \text{Aprop} \).

Truth definition for \( \mathcal{L}_{\text{MDL}^+} \)

Definition
Let \( M \) be an \( \mathcal{L}_{\text{MDL}^+} \)-model and \( w \) a point in \( M \). If \( p \in \text{Aprop} \), and \( i \in I \), then:

(a) \( M, w \models_{\text{MDL}^+} p \) if \( w \in V^M(p) \)
(b) \( M, w \models_{\text{MDL}^+} T \)
(c) \( M, w \models_{\text{MDL}^+} \neg \varphi \) if it is not the case that \( M, w \models_{\text{MDL}^+} \varphi \)
(d) \( M, w \models_{\text{MDL}^+} (\varphi \land \psi) \) if \( M, w \models_{\text{MDL}^+} \varphi \) and \( M, w \models_{\text{MDL}^+} \psi \)

(to be continued)
**Truth definition for $\mathcal{L}_{MDL^+}$ (continued)**

(e) $M, w \models_{MDL^+} \Box \varphi$ iff for every $v$ such that $(w, v) \in M$, $M, v \models_{MDL^+} \varphi$

(f) $M, w \models_{MDL^+} O_i \varphi$ iff for every $v$ such that $(w, v) \in \sim_i M$, $M, v \models_{MDL^+} \varphi$

A formula $\varphi$ is true in an $\mathcal{L}_{MDL^+}$-model $M$ at a point $w$ of $M$ if $M, w \models_{MDL^+} \varphi$. The semantic consequence relation and the notion of validity can also be defined in the standard way.

**The proof system for MDL$^+$**

**Definition**

The proof system for MDL$^+$ includes (i) all instantiations of propositional tautologies over the present language, (ii) K-axioms for alethic modality and $O_i$-modality for each $i \in I$, (iii) modus ponens, and (iv) necessitation rules for alethic modality and $O_i$-modality for each $i \in I$, in addition to the axiom of the following form for each $i \in I$:

$$(\text{Mix}) \quad P_i \varphi \rightarrow \Diamond \varphi$$

**Example 1: on a hot day in a shared office**

- $p$ The window is open.
- $q$ The air conditioner is running.
- $r$ The temperature is rising.

**Your boss’s act of commanding in MDL$^+$**

In the model $M$, $p \land q \land r$, the window is open, the air conditioner is running, and the temperature is rising. In the model $N$, $O_a p$, $O_a q$, and $O_a r$ hold, indicating that the boss commanded turning on the air conditioner.
3 & 4. Dynamic Extension

\[ [\!_i \varphi] \ O_i \psi \]

Eliminative Command Logic ECL

adding dynamic modalities

translation along reduction axioms

Multi-agent Deontic Logic MDL⁺

\[ O_i \varphi \]

The language of command logic

Definition
Take the same countably infinite set \( \text{Aprop} \) of proposition letters and the same finite set \( I \) of agents as before, with \( p \) ranging over \( \text{Aprop} \), and \( i \) over \( I \). The language \( \mathcal{L}_{ECL} \) of eliminative command logic ECL is given by:

\[
\begin{align*}
\varphi & ::= \top \ | \ p \ | \neg \varphi \ | \ \varphi \land \psi \ | \ \Box \varphi \ | \ O_i \varphi \ | \ [\pi] \varphi \\
\pi & ::= \!_i \varphi
\end{align*}
\]

\[ [\!_a \psi] \ O_a \varphi \]

After every effective act of commanding an agent \( a \) to see to it that \( \psi \), it is obligatory upon \( a \) to see to it that \( \varphi \).

The truth definition for \( \mathcal{L}_{ECL} \)

Definition
Let \( M \) be an \( \mathcal{L}_{MDL⁺} \)-model and \( w \) a point in \( M \). If \( p \in \text{Aprop} \), and \( i \in I \), then the truth definition for \( \mathcal{L}_{ECL} \) is given by expanding that of \( \mathcal{L}_{MDL⁺} \) mutatis mutandis with the following new clause:

\[
\begin{align*}
(\text{g}) \quad M, w \models_{ECL} [\!_i \chi] \varphi \iff M_{i,\chi}, w \models_{ECL} \varphi \\
\end{align*}
\]

where \( M_{i,\chi} \) is the \( \mathcal{L}_{MDL⁺} \)-model obtained from \( M \) by replacing \( \cup^M \) with \( \{ (x, y) \in \cup^M \mid M, y \models_{ECL} \chi \} \).

Your boss’s act of commanding in ECL

Let \( M \) be a \( \mathcal{L}_{MDL⁺} \)-model and \( w \) a point in \( M \). If \( p \in \text{Aprop} \), and \( i \in I \), then the truth definition for \( \mathcal{L}_{ECL} \) is given by expanding that of \( \mathcal{L}_{MDL⁺} \) mutatis mutandis with the following new clause:

\[
\begin{align*}
(\text{g}) \quad M, w \models_{ECL} [\!_i \chi] \varphi \iff M_{i,\chi}, w \models_{ECL} \varphi \\
\end{align*}
\]

where \( M_{i,\chi} \) is the \( \mathcal{L}_{MDL⁺} \)-model obtained from \( M \) by replacing \( \cup^M \) with \( \{ (x, y) \in \cup^M \mid M, y \models_{ECL} \chi \} \).
Some interesting principles

CUGO Pinciple

If $\varphi$ is a formula of $L_{MDL^+}$ and is free of occurrences of modal formulas of the form $O_i$, then $[\![\varphi]\!]_O \varphi$ is valid.

Dead End Principles

$[\![\varphi \land \neg \varphi]_O \varphi$ is valid.

Restricted Sequential Conjunction

If $\varphi$ and $\psi$ are formulas of $L_{MDL^+}$ and are free of occurrences of modal formulas of the form $O_i$, then $[\![\varphi \land \psi]_O \varphi$ is valid.

5. Finding reduction axioms

Eliminative Command Logic ECL

Adding dynamic modalities translation along reduction axioms

Multi-agent Deontic Logic $MDL^+$

The proof system for ECL

Definition

The proof system for ECL includes all the axioms and all the rules of the proof system for $MDL^+$, and in addition, the following rule and axioms:

\[(!\text{-nec}) \quad \frac{\psi}{[\![\varphi]\!]_O \psi} \quad \text{(for each } i \in I)\]

(To be continued)
### Translation from $\mathcal{L}_{ECL}$ to $\mathcal{L}_{MDL^+}$

#### Definition

| $t(p)$ | $=p$ | $t([\langle i \rangle \varphi]p) = p$ |
| $t(\top)$ | $= \top$ | $t([\langle i \rangle \varphi] \top) = \top$ |
| $t(\neg \varphi)$ | $= \neg t(\varphi)$ | $t([\langle i \rangle \varphi] \neg \psi) = \neg t([\langle i \rangle \varphi] \psi)$ |
| $t(\varphi \land \psi)$ | $= t(\varphi) \land t(\psi)$ | $t([\langle i \rangle \varphi] (\psi \land \chi)) = t([\langle i \rangle \varphi] \psi) \land t([\langle i \rangle \varphi] \chi)$ |
| $t(\Box \varphi)$ | $= \Box t(\varphi)$ | $t([\langle i \rangle \varphi] \Box \psi) = \Box t([\langle i \rangle \varphi] \psi)$ |
| $t(O_{ij} \varphi)$ | $= O_i t(\varphi)$ | $t([\langle i \rangle \varphi] O_j \psi) = O_t t([\langle i \rangle \varphi] \psi)$ (for any $i \neq j$) |
| $t([\langle i \rangle \varphi] O_j \psi)$ | $= t([\langle i \rangle \varphi] t([\langle i \rangle \varphi] \chi))$ | $t([\langle i \rangle \varphi] t([\langle i \rangle \varphi] \chi))$ (for any $j \in I$) |

### Completeness of ECL

#### Theorem

**ECL is strongly complete with respect to MDL$^+$-models.**

#### Corollary

If $\xi \in \mathcal{L}_{MDL^+}$, then $M, w \models_{MDL^+} \xi$ iff $M, w \models_{ECL} \xi$.

#### Lemmas

For any $\eta \in \mathcal{L}_{ECL}$, $t(\eta) \in \mathcal{L}_{MDL^+}$.
For any $\eta \in \mathcal{L}_{ECL}$, $M, w \models_{ECL} \eta$ iff $M, w \models_{ECL} t(\eta)$.
For any $\eta \in \mathcal{L}_{ECL}$, $t(\eta) \in \mathcal{L}_{MDL^+}$.

### Proof of the weak completeness of ECL

#### Completeness of ECL

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#### Completeness of MDL$^+$

1. Introduction
2. DEL and A dynamic logic of acts of commanding
3. Refinements and Variations
   - Conflicting commands
   - Acts of commanding and promising
   - Obligations and preferences
   - Assertions, concessions and their withdrawals
4. Scorekeeping and dynamic logics of speech acts
5. Conclusion
A refinement (Yamada 2007b)

\[ O_{(i, j)} \varphi \] It is obligatory upon an agent \( i \) with respect to an authority \( j \) to see to it that \( \varphi \).

\[ [\langle (i, j) \psi \rangle]_{X} \] After an authority \( j \) commands an agent \( i \) to see to it that \( \psi \), \( \chi \) holds.

Contradictory commands from two distinct authorities

A dilemma

\[ [\langle (a, b) \rho \rangle]_{\langle (a, c) \rangle} [O_{(a, b)} \rho \wedge O_{(a, c)} \lnot \rho) . \]

Note that this does not lead to deontic explosion.

Example 2: Conflicting commands from your boss and your guru

A contingent dilemma

\[ [\langle (a, b) \rho \rangle]_{\langle (a, c) \rangle} [O_{(a, b)} \rho \wedge O_{(a, c)} \lnot \rho) \wedge \lnot (\rho \wedge q) . \]

\( p \) You will attend the conference in São Paulo on 11 June 2010.
\( q \) You will join the demonstration in Sapporo on 11 June 2010.

Some results (Yamada, 2007b)

CUGO Principle

If \( \varphi \) is a formula of MDL\(^+\) II and is free of modal operators of the form \( O_{(i, j)} \), \([\langle (i, j) \psi \rangle]_{X} O_{(i, j)} \varphi \) is valid.

Theorem

There is a complete axiomatization of ECLII.
Introduction
DEL and A dynamic logic of acts of commanding
Refinements and Variations
Scorekeeping and dynamic logics of speech acts
Conclusion

Conflicting commands
Acts of commanding and promising
Obligations and preferences
Assertions, concessions and their withdrawals

Announcement
A further refinement and extension (Yamada 2008a)

ECL

DMDL+ III

EPL

Dynamic Epistemic Upgrade Logic DEUL

Some results (Yamada, 2008a)

CUGO Principle
If \( \varphi \) is a formula of MDL+III and is free of modal operators of the form \( O_{(i,j,k)} \), \([Com_{(i,j)} \varphi]O_{(i,j,i)} \varphi \) is valid.

PUGO Principle
If \( \varphi \) is a formula of MDL+III and is free of modal operators of the form \( O_{(i,j,i)} \), \([Prom_{(i,j)} \varphi]O_{(i,j,i)} \varphi \) is valid.

Theorem
There is a complete axiomatization of DMDL+III.

Example 3: a command and a promise can lead to a dilemma

A contingent dilemma
\[
[Prom_{(a,b)}(p)] [Com_{(c,a)}(q)] (O_{(a,b)}(p) \land O_{(a,c)}(q) \land \neg (p \land q))
\]

\( p \) You will attend the conference in São Paulo on 11 June 2010.
\( q \) You will join the demonstration in Sapporo on 11 June 2010.

The same strategy works for changing preferences (van Benthem and Liu, 2007) (Liu, 2008)
The language of EPL

**Definition**

Take a set $A_{prop}$ of proposition letters, and a set $I$ of agents, with $p$ ranging over $A_{prop}$ and $i$ over $I$. The epistemic preference language is given by:

$$\varphi ::= \bot \mid p \mid \neg \varphi \mid \varphi \land \psi \mid U \varphi \mid K_i \varphi \mid [\text{pref}]_i \varphi$$

Intuitively, $[\text{pref}]_i \varphi$ means that all worlds $i$ considers at least as good as the current one satisfy $\varphi$.

$U$ is the so-called “universal modality”, and $U \varphi$ means that $\varphi$ holds at every world.

Combining preference upgrades and deontic updates (Yamada 2008b)

The language of DEUL

**Definition**

Take the same set $A_{prop}$ of proposition letters, and the same set $I$ of agents as before, with $p$ ranging over $A_{prop}$ and $i$ over $I$. The dynamic epistemic preference language is given by:

$$\varphi ::= \bot \mid p \mid \neg \varphi \mid \varphi \land \psi \mid U \varphi \mid K_i \varphi \mid [\text{pref}]_i \varphi \mid [\pi] \varphi$$

$\pi$ is the type of acts of publicly announcing that $\varphi$, and $\not{\pi} \varphi$ is the type of acts of publicly suggesting $\varphi$.

The language of DPL

**Definition**

Take a set $A_{prop}$ of proposition letters, and a set $I$ of agents, with $p$ ranging over $A_{prop}$ and $i, j$ over $I$. The deontic preference language is given by:

$$\varphi ::= \bot \mid p \mid \neg \varphi \mid \varphi \land \psi \mid U \varphi \mid K_i \varphi \mid [\text{pref}]_i \varphi \mid O_{(i,j)} \varphi$$
The language of DDPL

Definition

Take a set $A_{prop}$ of proposition letters, and a set $I$ of agents, with $p$ ranging over $A_{prop}$ and $i, j$ over $I$. The dynamic deontic preference language is given by:

$$
\varphi ::= \bot \mid p \mid \neg \varphi \mid \varphi \land \psi \mid U\varphi \mid [pref]i\varphi \mid O_{i,j}\varphi \mid [\pi]\varphi
$$

$$
\pi ::= \#i\varphi \mid !_{i,j}\varphi
$$

Some results (Yamada, 2008b)

Theorem

There is a complete axiomatization of DDPL.

The following formulas are satisfiable.

$$
O_{i,j}p \land U(p \rightarrow \langle pref \rangle \neg p) \ .
$$

$$
|_{i,j}\varphi \mid U(p \rightarrow \langle pref \rangle \neg p) \ .
$$

$\langle pref \rangle \varphi$ is an abbreviation of $\neg [pref]i\neg \varphi$.

The same recipe works for acts of asserting and conceding (Yamada, Unfinished Draft)

Dynamified Multiagent Propositional Commitment Logic

DMPCL

adding dynamic modalities

translation along reduction axioms

Multi-agent Propositional Commitment Logic

MPCL

Walton & Krabbe (1995)

Three Kinds of propositional commitments

- commitments incurred by making concessions
- commitments called assertions
- participant’s dark-side commitments

Since dark-side commitments are hidden commitments and supposed to be fixed, we will ignore them.

We call the remaining two kinds of commitments $c$-commitments and $a$-commitments respectively.
A-commitments and c-commitments

According to Walton and Krabbe (1995, p.186)

Propositional commitments constitute a special case of commitments to a course of action.

- an agent who has an a-commitment to the proposition \( p \) is obliged to defend it if the other party in the dialogue require her to justify it
- an agent who has a c-commitments to \( p \) is only obliged to allow the other party to use it in the arguments.

As anyone who asserts that \( p \) will be obliged to allow the other party to use it in the arguments, a-commitments imply c-commitments.

P-commitments are different from knowledge

The following formulas are not valid.

\[ [a\text{-cmt}]_i \varphi \rightarrow \varphi \]
\[ [c\text{-cmt}]_i \varphi \rightarrow \varphi \]
Cf. \( K_i \varphi \rightarrow \varphi \)

The language of MPCL

Definition

Take a countably infinite set \( \text{Aprop} \) of proposition letters, and a finite set \( l \) of agents, with \( p \) ranging over \( \text{Aprop} \), and \( i \) over \( I \). The language \( \mathcal{L}_{\text{MPCL}} \) of the multi-agent propositional commitment logic MPCL is given by:

\[ \varphi : = \top \mid p \mid \neg \varphi \mid \varphi \land \psi \mid [a\text{-cmt}]_i \varphi \mid [c\text{-cmt}]_i \varphi \]

\[ [a\text{-cmt}]_i \varphi : \text{an agent } i \text{ has an a-commitment to the proposition } \varphi ,\]
\[ [c\text{-cmt}]_i \varphi : \text{an agent } i \text{ has a c-commitment to the proposition } \varphi . \]

P-commitments are different from belief

The following formulas are not valid.

\[ \neg [a\text{-cmt}]_i \bot \]
\[ \neg [c\text{-cmt}]_i \bot \]
Cf. \( \neg B_i \bot \)
**Definition**

By an \( \mathcal{L}_{\text{MPCL}} \)-model, we mean a tuple 
\[ M = ( W^M, \{ M^i \mid i \in I \}, \{ V^M_{i} \mid i \in I \}, V^M ) \]
where:

(i) \( W^M \) is a non-empty set (heuristically, of ‘possible worlds’),
(ii) \( M^i \subseteq W^M \times W^M \) for each \( i \in I \),
(iii) \( V^M_{i} \subseteq \text{AP} \) for each \( i \in I \),
(iv) \( V^M \) is a function that assigns a subset \( V^M(p) \) of \( W^M \)
to each proposition letter \( p \in \text{AP} \).

**Truth definition for \( \mathcal{L}_{\text{MPCL}} \) (crucial part)**

In addition to the standard clauses for proposition letters and Boolean operations,

(e) \( M, w \models_{\text{MPCL}} [a\text{-cmt}]_i \varphi \) iff for every \( v \) such that 
\[ \langle w, v \rangle \in M^i, M, v \models \varphi \]

(f) \( M, w \models_{\text{MPCL}} [c\text{-cmt}]_i \varphi \) iff for every \( v \) such that 
\[ \langle w, v \rangle \in M^i, M, v \models \varphi \]

**The Proof system for MPCL**

The proof system for MPCL includes (i) all instantiations of propositional tautologies over the present language, (ii) \( K \)-axioms for \([a\text{-cmt}]_i\)-modality and \([c\text{-cmt}]_i\)-modality for each \( i \in I \), (iii) modus ponens, and (iv) necessitation rules for \([a\text{-cmt}]_i\)-modality and \([c\text{-cmt}]_i\)-modality for each \( i \in I \), in addition to the axiom of the following form for each \( i \in I \):

\[ (\text{Mix}) \quad [a\text{-cmt}]_i \varphi \rightarrow [c\text{-cmt}]_i \varphi \]

**Theorem (Completeness of MPCL)**

MPCL is strongly complete with respect to \( \mathcal{L}_{\text{MPCL}} \)-models.
Definition

The language of DMPCL is given by:

\[ \varphi ::= \top \mid \rho \mid \neg \varphi \mid \varphi \land \psi \mid [a\text{-cmt}]_i \varphi \mid [c\text{-cmt}]_i \varphi \mid [\pi] \varphi \]

\[ \pi ::= \text{assert}_i \varphi \mid \text{concede}_i \varphi \]

The truth definition for \( \mathcal{L}_{\text{DMPCL}} \)

Let \( M \) be an \( \mathcal{L}_{\text{MPCL}} \)-model and \( w \) a point in \( M \). If \( \rho \in \text{Aprop} \), and \( i \in I \), then the truth definition for \( \mathcal{L}_{\text{DMPCL}} \) is given by expanding that of \( \mathcal{L}_{\text{MPCL}} \) mutatis mutandis with the following new clause:

\[
\text{(g)} \quad M, w \models_{\text{DMPCL}} [\text{assert}_i \varphi] \varphi \quad \text{iff} \quad M_{\text{assert}_i \varphi}, w \models_{\text{MPCL}} \varphi
\]

\[
\text{(h)} \quad M, w \models_{\text{DMPCL}} [\text{concede}_i \varphi] \varphi \quad \text{iff} \quad M_{\text{concede}_i \varphi}, w \models_{\text{MPCL}} \varphi,
\]

where \( M_{\text{assert}_i \varphi} \) is the \( \mathcal{L}_{\text{MPCL}} \)-model obtained from \( M \) by replacing \( \mathcal{D}_M^i \) with \( \{(x,y) \in \mathcal{D}_M^i \mid M, y \models_{\text{MPCL}} \varphi \} \) and \( \mathcal{M}_{\text{assert}_i \varphi} \) is the \( \mathcal{L}_{\text{MPCL}} \)-model obtained from \( M \) by replacing \( \mathcal{D}_M^i \) with \( \{(x,y) \in \mathcal{D}_M^i \mid M, y \models_{\text{MPCL}} \varphi \} \).
Some results

**Proposition**

If $\varphi \in \mathcal{L}_{\text{MPCL}}^{i}$ is free of modalities indexed by $i$, the following formulas are valid:

\[\text{assert}_i\varphi] [a\text{-cmt}]_i \varphi\]
\[\text{assert}_i\varphi] [c\text{-cmt}]_i \varphi\]
\[\text{concede}_i\varphi] [c\text{-cmt}]_i \varphi\].

**Theorem**

There is a complete axiomatization of DMPCL.

The language of DMPCL$^+$

**Definition**

Take the same countably infinite set $A_{\text{prop}}$ of proposition letters and the same finite set $I$ of agents as before, with $p$ ranging over $A_{\text{prop}}$, and $i$ over $I$. The language $\mathcal{L}_{\text{DPCMT}^+}$ of dynamified multi-agent propositional commitment logic with withdrawals DMPCL$^+$ is given by:

$\varphi ::= \top \mid p \mid \neg \varphi \mid \varphi \land \psi \mid [a\text{-cmt}]_i \varphi \mid [c\text{-cmt}]_i \varphi \mid [\pi]_i \varphi$

$\pi ::= \text{assert}_i \varphi \mid \text{concede}_i \varphi \mid \Diamond \text{assert}_i \varphi \mid \Diamond \text{concede}_i \varphi$

Does the same strategy work for acts of asserting and conceding combined with acts of withdrawing?

Dynamified Multiagent Propositional Commitment Logic with withdrawals DMPCL$^+$

adding dynamic modalities available?

Multi-agent Propositional Commitment Logic MPCL

An update by withdrawing?

A sequence of acts: $\ldots$, assert$\chi$, assert$\xi$, assert$\eta$, $\ldots$

$\downarrow \Diamond$ assert$\xi$

A reduced sequence: $\ldots$, assert$\chi$, assert$\eta$, $\ldots$

The set of propositional commitments agents bear after $j$'s act of withdrawing of the form $\Diamond$ assert$\xi$ will be, other things being equal, the same as the set of propositional commitments they would bear if $j$ had not asserted that $\xi$. 

Tomoyuki Yamada | Scorekeeping and Dynamic Logics of Speech Acts
A positive commitment act sequence

If \( \sigma \) is a sequence of moves in an argumentation, it may involve not only acts of asserting and conceding but also acts of withdrawing.

For the sake of simplicity, we will only consider a special kind of sequences, namely, a sequence \( \sigma = (\pi_1, \pi_2, \ldots, \pi_n) \) of speech acts \( \pi_j \) (1 \( \leq \) j \( \leq \) n) such that each \( \pi_j \) is either of the form \( \text{assert}_j \varphi \) for some \( i \in I \) or of the form \( \text{concede}_j \varphi \) for some \( i \in I \). We call such a sequence a positive commitment act sequence, or a pca-sequence for short.

Reduced pca-sequence (continued)

(Ai) if \( \sigma \) is empty, \( \sigma \models \text{assert}_i \varphi = \sigma \)

(Aii) if \( \sigma = (\pi_1, \ldots, \pi_{n-1}) \), and \( \pi_n = \text{assert}_i \varphi \),

\[ \sigma \models \text{assert}_i \varphi = (\pi_1, \ldots, \pi_{n-1}) \models \text{assert}_i \varphi \]

(Aiii) if \( \sigma = (\pi_1, \ldots, \pi_{n-1}) \), and \( \pi_n \neq \text{assert}_i \varphi \),

\[ \sigma \models \text{assert}_i \varphi = (\pi_1, \ldots, \pi_{n-1}) \models \text{assert}_i \varphi, \pi_n \]

(Ci) if \( \sigma \) is empty, \( \sigma \models \text{concede}_i \varphi = \sigma \)

(Cii) if \( \sigma = (\pi_1, \ldots, \pi_{n-1}) \), and \( \pi_n = \text{concede}_i \varphi \),

\[ \sigma \models \text{concede}_i \varphi = (\pi_1, \ldots, \pi_{n-1}) \models \text{concede}_i \varphi \]

(Ciii) if \( \sigma = (\pi_1, \ldots, \pi_{n-1}) \), and \( \pi_n \neq \text{concede}_i \varphi \),

\[ \sigma \models \text{concede}_i \varphi = (\pi_1, \ldots, \pi_{n-1}) \models \text{concede}_i \varphi, \pi_n \].

The Problem of Notation

Given a pca-sequence \( \sigma = (\pi_1, \ldots, \pi_n) \), the model obtained by updating \( M \) with \( \sigma \) is denoted by \( (\ldots (M_{\pi_1}) \ldots)_{\pi_n} \) in the notation of the truth definition for \( \mathcal{L}_{\text{DMPCL}} \).

This notation leads to a paradox when we deal with withdrawals. Let abbreviate \( (\ldots (M_{\pi_1}) \ldots)_{\pi_n} \) as \( M_{\pi_n} \). Now there may be another model \( N \) and a pcs-sequence \( \tau \) such that \( N_{\tau} = M_{\pi_n} \). Then we may have

\[ (N_{\tau})_{\sigma} \models \text{concede}_i \varphi \neq (M_{\pi_n})_{\pi_n} \models \text{concede}_i \varphi. \]
**Truth Definition 1/5**

**Definition**

Let $M$ be an $L_{MPCL}$-model, $\sigma$ a positive commitment act sequence, and $w$ a point in $M$. If $p \in Aprop$, and $i \in I$, then:

(a) $M, \sigma, w \vdash_{MPCL} p$ iff $w \in V^M(p)$
(b) $M, \sigma, w \vdash_{MPCL} T$
(c) $M, \sigma, w \vdash_{MPCL} \neg \varphi$ iff it is not the case that $M, \sigma, w \vdash_{MPCL} \varphi$
(d) $M, \sigma, w \vdash_{MPCL} (\varphi \land \psi)$ iff $M, \sigma, w \vdash_{MPCL} \varphi$ and $M, \sigma, w \vdash_{MPCL} \psi$

(To continue)

**Truth Definition 2/5**

(e) $M, \sigma, w \vdash_{MPCL}[a-cmt]_i \varphi$ iff for all $v$ s. t. $(w, v) \in \triangleright_M^i \sigma$,

$$M, \sigma, v \vdash_{MPCL} \varphi$$

(f) $M, \sigma, w \vdash_{MPCL}[c-cmt]_i \varphi$ iff for all $v$ s. t. $(w, v) \in \triangleright_M^i \sigma$,

$$M, \sigma, v \vdash_{MPCL} \varphi$$

(g) $M, \sigma, w \vdash_{MPCL}[assert]_i \varphi$ iff $M, \langle \sigma, assert_i \rangle$, $w \vdash_{MPCL} \varphi$

(h) $M, \sigma, w \vdash_{MPCL}[concede]_i \varphi$ iff $M, \langle \sigma, concede_i \rangle$, $w \vdash_{MPCL} \varphi$

(To continue)

**Truth Definition 3/5**

(i) $M, \sigma, w \vdash_{MPCL}[\cup assert]_i \varphi$ iff $M, \sigma \cup assert_i \varphi$,

$$w \vdash_{MPCL} \varphi$$

(j) $M, \sigma, w \vdash_{MPCL}[\cup concede]_i \varphi$ iff $M, \sigma \cup concede_i \varphi$,

$$w \vdash_{MPCL} \varphi$$

where $\triangleright_M^i \sigma$ and $\triangleright_M^i \sigma$ are

where $\triangleright_M^i \sigma$ and $\triangleright_M^i \sigma$ are

(To continue)

**Truth Definition 4/5**

where $\triangleright_M^i \sigma =$

- $\triangleright_M^i$ if $\sigma$ is empty,
- $\{ (x, y) \in \triangleright_M^i \mid \langle \pi_1, \ldots, \pi_{n-1} \rangle \}$

if $\sigma = \langle \pi_1, \ldots, \pi_n \rangle$ and $\pi_n = assert_i \psi$

- $\triangleright_M^i \langle \pi_1, \ldots, \pi_{n-1} \rangle$

if $\sigma = \langle \pi_1, \ldots, \pi_n \rangle$ and $\pi_n \neq assert_i \psi$

and

(To continue)
Truth Definition 5/5

\[ \mathcal{M} \models \sigma = \]

\[ \mathcal{M} \mathcal{M} \text{ if } \sigma \text{ is empty,} \]

\[ \{ (x, y) \in \mathcal{M} \mid \langle \pi_1, \ldots, \pi_{n-1} \rangle \} \]

\[ \mathcal{M}, \langle \pi_1, \ldots, \pi_n \rangle \models_{\text{DMPCL}} \psi \] if \( \sigma = \langle \pi_1, \ldots, \pi_n \rangle \) and

\[ \text{either } \pi_n = \text{assert}_i \psi \text{ or } \pi_n = \text{concede}_i \psi, \]

\[ \mathcal{M} \models \langle \pi_1, \ldots, \pi_{n-1} \rangle \]

if \( \sigma = \langle \pi_1, \ldots, \pi_n \rangle \), \( \pi_n \neq \text{assert}_i \psi \), and \( \pi_n \neq \text{concede}_i \psi. \]

A result and an open problem

A result

Acts of withdrawing behave slightly differently from contraction studied in belief revision. Let \( B \) be a set of beliefs of an agent, say \( a \). Then in the AGM approach, contraction \( \ominus \) is supposed to satisfy the postulate that \( \varphi \notin B \ominus \varphi \) if \( \lnot \varphi \), but we have \( M, \sigma \models_{\text{DMPCL}} \text{assert}_a q \) and \( \text{assert}_a (q \rightarrow p) \).

An open problem

The completeness problem of \( \text{DMPCL}^+ \) is still open.

Scorekeeping

- The notion of scorekeeping is introduced into the discussion of language by Lewis (1979) and utilized by Brandom (1994) in developing his theory of meaning based on Wittgenstein’s notion of meaning as use.
- In Brandom’s version, each agent is considered as a deontic scorekeeper, and “the significance of an assertion of \( p' \) is considered as “a mapping that associates with one social deontic score—characterising the stage before that speech act is performed, according to some scorekeeper—the set of scores for conversational stage that results from the assertion, according to the same scorekeeper” (ibid., 190).
**Scorekeeping for argumentation games**

- We will only consider “the official score” kept by an idealised scorekeeper, and examine how DMPCL+ can be applied to such official scorekeeping in an argumentation game.
- In order to do so, we need a special model that can represent the initial stage of the game.

**Definition**

Given a countably infinite set \( \text{Aprop} \) of proposition letters, and the set \( I \) with \( p \) ranging over \( \text{Aprop} \) and \( i \) over \( I \). Then, the initial stage model is the tuple \( M^0 = (W^0, \{ D_i^0 \mid i \in I \}, \{ C_i^0 \mid i \in I \}, V^0) \) where:

(i) \( W^0 \) is the power set \( \mathcal{P}(\text{Aprop}) \) of \( \text{Aprop} \),
(ii) \( D_i^0 = W^0 \times W^0 \) for each \( i \in I \),
(iii) \( C_i^0 = W^0 \times W^0 \) for each \( i \in I \),
(iv) \( V^0 \) is the function that assigns a subset \( V^0(p) = \{ w \in W^0 \mid p \in w \} \) of \( W^0 \) to each proposition letter \( p \in \text{Aprop} \).

**What we have at the initial stage**

For any agent \( i \), for any proposition letter \( p \), and for any point \( w \in W^0 \), if \( \sigma \) is empty, we have

- \( M^0, \sigma, w \not\models_{\text{DMPCL}^+} [a\text{-cmt}]p \)
- \( M^0, \sigma, w \not\models_{\text{DMPCL}^+} [a\text{-cmt}]\neg p \)
- \( M^0, \sigma, w \not\models_{\text{DMPCL}^+} [c\text{-cmt}]p \)
- \( M^0, \sigma, w \not\models_{\text{DMPCL}^+} [c\text{-cmt}]\neg p \)

Thus each agent has no substantial propositional commitments at the initial stage.

**What DMPCL+ enables us to do**

- Then we can reason about what propositional commitments agents will bear at each stage after each of their acts of asserting, conceding or withdrawing.
- This doesn’t gives us the whole score of each play of an argumentation game. The official scorekeeper may have to record other factors such as penalties for withdrawing, the relation between the moves made by the participants, etc.
- But DMPCL+ can be said to capture the evolution of the score at least partially.
The notion of, or the metaphor of, scorekeeping can be extended to more complex language games where not only acts of asserting, conceding, and withdrawing but also acts of commanding, promising, etc. are involved along with non-verbal actions.

Then the dynamic logics of speech acts can be used to reason about the changes and non-changes brought about by speech acts.

The dynamic logics of speech acts can thus partially characterize the scorekeeping function for a language game, where speech acts they deal with are involved as moves made by participants at various stages.

The conventional or institutional effects of an illocutionary act can then be seen in the score that characterizes the stage after it is performed.

In contrast, the real effects of a perlocutionary act may be seen in the responses agents make or be concealed in their feelings, thoughts, etc.