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*HOKKAIDO UNIVERSITY*
Scorekeeping and Dynamic Logics of Speech Acts

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### The gap

**Van Benthem & Liu (2007) on commanding**

For instance, intuitively, a command

“See to it that φ!”

makes worlds where φ holds preferred over those where it does not - at least, if we accept the preference induced by the issuer of the command.

The need they felt for the proviso here reflects an important logical **gap** between what an illocutionary act of commanding involves and perlocutionary effects it may have upon our preferences.

### Austin's Distinction (1955, pp.101-3.)

**Locutionary Act**

He said to me “Shoot her!” meaning by ‘shoot' shoot and referring by ‘her’ to her.

**Illocutionary Act**

He urged (advised, ordered, etc.) me to shoot her.

**Perlocutionary Act**

(a) He persuaded me to shoot her.
(b) He got me to shoot her.
Speech acts as acts

- If the notion of speech act is to be taken seriously, it must be possible to treat speech acts as acts.
- If we succeed in characterizing speech acts in terms of dynamic changes they bring about, it becomes possible to treat them within a general theory of action.
- But how can we do that?

Perlocutionary acts as acts

**Perlocutionary Act**
(a) He persuaded me to shoot her.
(b) He got me to shoot her.

**Austin on perlocutionary acts (1955, p.103)**
According to Austin, perlocutionary acts are acts that really produce “real effects” upon the feelings, thoughts, or actions of addressees, or of speakers, or of other people.

They are recognized only when their effects are recognized.

Illocutionary acts as acts

**Illocutionary Act**
He urged (advised, ordered, etc.) me to shoot her.

**The Problem**
What effects do they have?
What role do they play in our social life?

Austin, Strawson, and Searle

**Austin on illocutionary acts (1955, p.103)**
Austin considered illocutionary acts as acts whose effects are “what we regard as mere conventional consequences”

**After Strawson (1964) and Seale (1969)**
Austin’s conception of illocutionary acts as acts whose effects are conventional has been disregarded both by those who follow Strawson and those who follow Searle.
Strawson (1964) on Austin

- Strawson (1964) observed that the kind of conventional effects involved in the examples used by Austin are dependent on special extralinguistic conventions.
- He then argued that there are many other illocutionary acts that do not seem to be dependent on any such special extralinguistic conventions.
- Thus, according to Strawson, Austin made an unwarranted overgeneralization when he attributed conventional effects to illocutionary acts in general.

Searle (1969) on Austin and Strawson

- Searle criticized Grice (and Strawson) for treating meaning as “a matter of intending to perform a perlocutionary acts”, but agreed with Strawson in seeing Austin’s notion of conventional effect as an unwarranted overgeneralization.
- Searle sees conventionality of illocutionary acts as a matter of meaning, and denied the distiction between locutionary acts and illocutionary acts.
- He identified what he called “the illocutionary effect” with “the hearer understanding the utterance of the speaker” (p.46-47).

Conventional effects vs. utterers’ intentions

- Strawson and his followers tried to characterize uses of sentences not in terms of conventional effects, but in terms of utterers’ intentions to produce various effects in addressees along the lines initiated by Grice (1957).
- Utterers’ intentions, however, usually go beyond illocutionary acts by involving reference to perlocutionary effects, while illocutionary acts can be effective even if they failed to produce intended perlocutionary effects.

Beyond the securing of uptake

- Austin considered the securing of uptake of this kind as necessary condition for illocutionary acts, but didn’t considered it to be sufficient.
- Indeed, even typical illocutionary acts such as acts of promising, which both Strawson and Searle see not conventional in what they take to be Austin’s sense, seem to involve more than the mere securing of uptake.
- The social or institutional consequences they have, such as generation of obligations, can be said to be “conventional” in Austin’s sense.
- They are institutional in Searle’s sense.
What Austin's Earlier Answer Enables us to See

Perlocutionary acts
Since perlocutionary acts are acts that really produce real effects, they cannot be completed without really producing them.

Illocutionary acts
Illocutionary acts are completed when the “mere conventional” effects are produced.

Austin 1955, pp.103-4.
Thus Austin says, “we can say ‘I argue that’ or ‘I warn you that’ but we cannot say ‘I convince you that’ or ‘I alarm you that’.

The problem
- Is it possible to develop this conception of illocutionary acts into a general theory of illocutionary acts?
- In order to do so, we have to
  - specify conventional effects of a sufficiently rich variety of illocutionary acts, and
  - develop a theory in which these illocutionary acts are shown to be fully characterized in terms of those conventional effects.

The plan
- The recent development of Dynamic Epistemic Logics suggests a recipe for developing logics that can capture effects of various speech acts.
- We have developed dynamic logics that can deal with acts of commanding, promising, asserting, conceding, and withdrawing according to this recipe (Yamada 07a, 07b, 08a, 08b, unfinished draft).
- We will review these development.
- We will then show how the results obtained can be incorporated into a more comprehensive picture of social interaction with the help of the notion of scorekeeping for language games.

Introduction
DEL and A dynamic logic of acts of commanding
Refinements and Variations
Scorekeeping and dynamic logics of speech acts
Conclusion

Tomoyuki Yamada
Scorekeeping and Dynamic Logics of Speech Acts
The developments of dynamic epistemic logics

\[ \text{[ϕ]} K_i \varphi \]

Dynamic Epistemic Logics DEL

adding dynamic modalities

translation along reduction axioms

Multi-agent Epistemic Logics EL

\[ K_i \varphi \]


Two points to be noted

The formulas of the form \( \varphi \rightarrow [\varphi] K_i \varphi \) are shown to be valid for any \( i \in I \) if no operators of the form \( K_i \) occur in \( \varphi \).

- This is too strong for interpreting natural language public announcements.
- A gap similar to the one we have seen is also present here.

The method used in developing DEL can be used to develop logics that deal with a much wider variety of speech acts.

The recipe (Yamada, to appear)

1. Carefully identify the aspect affected by the kind of speech acts you want to study
2. Find the modal logic that characterizes this aspect
3. Add dynamic modalities that represent types of those speech acts
4. Define model updating operation that interprets the speech acts under study as what update the very aspect
5. (if possible) Find a complete set of reduction axioms for the resulting dynamic logic.

This recipe works for acts of commanding

(Yamada, 2007a)
1 & 2. Identifying the relevant aspect and its logic

\[ [\alpha] \sigma \]

Eliminative Command Logic ECL

adding dynamic modalities

Multi-agent Deontic Logic MDL+

\( O_i \sigma \)

The language of multi-agent deontic logic

Definition

Take a countably infinite set \( A_{prop} \) of proposition letters and a finite set \( I \) of agents, with \( p \) ranging over \( A_{prop} \) and \( i \) over \( I \). The multi-agent monadic deontic language \( \mathcal{L}_{MDL}^+ \) is given by:

\[ \varphi ::= \top \mid p \mid \neg \varphi \mid \varphi \land \psi \mid \Box \varphi \mid O_i \varphi \]

\( O_a \varphi \) It is obligatory upon an agent \( a \) to see to it that \( \varphi \).

\( P_a \varphi \) \( \neg O_a \neg \varphi \).

\( F_a \varphi \) \( O_a \neg \neg \varphi \).

\( \mathcal{L}_{MDL}^+ \)-models

Definition

By an \( \mathcal{L}_{MDL}^+ \)-model, we mean a tuple 
\[ M = ( W^M, \models^M, \{ \models^M_i \mid i \in I \}, V^M ) \] where:

(i) \( W^M \) is a non-empty set (heuristically, of ‘possible worlds’),

(ii) \( \models^M \subseteq W^M \times W^M \),

(iii) \( \models^M_i \subseteq \models^M \) for each \( i \in I \),

(iv) \( V^M \) is a function that assigns a subset \( V^M(p) \) of \( W^M \) to each proposition letter \( p \in A_{prop} \).

Truth definition for \( \mathcal{L}_{MDL}^+ \)

Definition

Let \( M \) be an \( \mathcal{L}_{MDL}^+ \)-model and \( w \) a point in \( M \). If \( p \in A_{prop} \), and \( i \in I \), then:

(a) \( M, w \models_{MDL^+} p \) iff \( w \in V^M(p) \)

(b) \( M, w \models_{MDL^+} \top \)

(c) \( M, w \models_{MDL^+} \neg \varphi \) iff it is not the case that 
\[ M, w \models_{MDL^+} \varphi \]

(d) \( M, w \models_{MDL^+} (\varphi \land \psi) \) iff \( M, w \models_{MDL^+} \varphi \) and \( M, w \models_{MDL^+} \psi \)

(to be continued)
Truth definition for $\mathcal{L}_{\text{MDL}^+}$ (continued)

(e) $M, w \models_{\text{MDL}^+} \Box \varphi$ iff for every $v$ such that $(w, v) \in M$, $M, v \models_{\text{MDL}^+} \varphi$

(f) $M, w \models_{\text{MDL}^+} O_i \varphi$ iff for every $v$ such that $(w, v) \in \sim_i M$, $M, v \models_{\text{MDL}^+} \varphi$

A formula $\varphi$ is true in an $\mathcal{L}_{\text{MDL}^+}$-model $M$ at a point $w$ of $M$ if $M, w \models_{\text{MDL}^+} \varphi$. The semantic consequence relation and the notion of validity can also be defined in the standard way.

Example 1: on a hot day in a shared office

- $p$ The window is open.
- $q$ The air conditioner is running.
- $r$ The temperature is rising.

Your boss’s act of commanding in $\text{MDL}^+$

The proof system for $\text{MDL}^+$

Definition

The proof system for $\text{MDL}^+$ includes (i) all instantiations of propositional tautologies over the present language, (ii) K-axioms for alethic modality and $O_i$-modality for each $i \in I$, (iii) modus ponens, and (iv) necessitation rules for alethic modality and $O_i$-modality for each $i \in I$, in addition to the axiom of the following form for each $i \in I$:

$$(\text{Mix}) \quad P_i \varphi \to \Diamond \varphi$$
3 & 4. Dynamic Extension

\[ O_i \phi \]
Eliminative Command Logic ECL

Adding dynamic modalities

Translation along reduction axioms

Multi-agent Deontic Logic MDL+

The language of command logic

**Definition**
Take the same countably infinite set \(Aprop\) of proposition letters and the same finite set \(I\) of agents as before, with \(p\) ranging over \(Aprop\), and \(i\) over \(I\). The language \(\mathcal{L}_{ECL}\) of eliminative command logic ECL is given by:

\[
\varphi ::= \top | p | \neg \varphi | \varphi \land \psi | \Box \varphi | O_i \varphi | [\pi] \varphi
\]

\[ \pi ::= !a\varphi \]

\[ [!a\psi] O_a \varphi \]
After every effective act of commanding an agent \(a\) to see to it that \(\psi\), it is obligatory upon \(a\) to see to it that \(\varphi\).

The truth definition for \(\mathcal{L}_{ECL}\)

**Definition**
Let \(M\) be an \(\mathcal{L}_{MDL+}\)-model and \(w\) a point in \(M\). If \(p\in Aprop\), and \(i\in I\), then the truth definition for \(\mathcal{L}_{ECL}\) is given by expanding that of \(\mathcal{L}_{MDL+}\) mutatis mutandis with the following new clause:

\[
(g) \quad M, w \models_{ECL} [!i\chi] \varphi \text{ iff } M^{i,\chi}, w \models_{MDL+} \varphi,
\]
where \(M^{i,\chi}\) is the \(\mathcal{L}_{MDL+}\)-model obtained from \(M\) by replacing \(\sim^M\) with \(\{ (x, y) \in \sim^M \mid M, y \models_{ECL} \chi \}\).

Your boss's act of commanding in ECL

\[ M \]
\[ \diamond p \land \diamond q \land \diamond r \]
\[ [!a p] O_a p \]
\[ M_{a,p} \]
\[ \diamond p \land \diamond q \land \diamond r \]
\[ O_a p \]
\[ \dashrightarrow [!a p] \]

Some interesting principles

**CUGO Principle**

If $\phi$ is a formula of $\mathcal{L}_{MDL^+}$ and is free of occurrences of modal formulas of the form $O_i$, then $[!\phi]O_i\phi$ is valid.

**Dead End Principles**

$[!((\phi \land \neg \phi))]O_i\phi$ is valid.

**Restricted Sequential Conjunction**

If $\phi$ and $\psi$ are formulas of $\mathcal{L}_{MDL^+}$ and are free of occurrences of modal formulas of the form $O_i$, then $[!\phi][!\psi]\chi \leftrightarrow [!((\phi \land \psi))\chi$ is valid.

---

**5. Finding reduction axioms**

Eliminative Command Logic Logic $ECL$

Adding dynamic modalities translation along

Translation along reduction axioms

Multi-agent Deontic Logic MDL$^+$

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The proof system for ECL

**Definition**

The proof system for ECL includes all the axioms and all the rules of the proof system for MDL$^+$, and in addition, the following rule and axioms:

$(!\text{-nec})$

$$\frac{\psi}{[!\phi]\psi}$$

(for each $i \in I$)

(To be continued)
Translation from $\mathcal{L}_{ECL}$ to $\mathcal{L}_{MDL^+}$

**Definition**

- $t(\rho) = \rho$
- $t(\top) = \top$
- $t(\lnot \varphi) = \lnot t(\varphi)$
- $t(\varphi \land \psi) = t(\varphi) \land t(\psi)$
- $t(\Box \varphi) = \Box t(\varphi)$
- $t(O_{i, j} \varphi) = O_i t(\varphi)$

$\mathcal{L}_{ECL}$ is strongly complete with respect to $\mathcal{L}_{MDL^+}$-models.

Proof of the weak completeness of ECL

**Completeness of ECL**

- Lemma: $\models_{ECL} \eta$ \implies $\models_{ECL} t(\eta)$

- Corollary: $\models_{ECL} t(\eta)$ \iff $\models_{MDL^+} t(\eta)$ is $\mathcal{L}_{MDL^+}$-models.

- Weak Completeness:
  - If $\xi \in \mathcal{L}_{MDL^+}$, then $M, w \models_{MDL^+} \xi$ iff $M, w \models_{ECL} \xi$.
  - Corollary:
    - For any $\eta \in \mathcal{L}_{ECL}$, $t(\eta) \in \mathcal{L}_{MDL^+}$.
    - For any $\eta \in \mathcal{L}_{ECL}$, $M, w \models_{ECL} \eta$ iff $M, w \models_{ECL} t(\eta)$.
    - For any $\eta \in \mathcal{L}_{ECL}$, $\models_{ECL} \eta \iff t(\eta)$.

**Refinements and Variations**

- Conflicting commands
- Acts of commanding and promising
- Obligations and preferences
- Assertions, concessions and their withdrawals

**Conclusion**
A refinement (Yamada 2007b)

\[ O_{(i,j)} \varphi \] It is obligatory upon an agent \( i \) with respect to an authority \( j \) to see to it that \( \varphi \).

\[ [\diamondsuit_{(i,j)} \psi] \chi \] After an authority \( j \) commands an agent \( i \) to see to it that \( \psi \), \( \chi \) holds.

Example 2: Conflicting commands from your boss and your guru

A contingent dilemma

\[ [\diamondsuit_{(a,b)} \psi] [\diamondsuit_{(a,c)} \varphi] \left( O_{(a,b)} \psi \land O_{(a,c)} \varphi \right) \land \neg (\psi \land \varphi) . \]

- \( p \) You will attend the conference in São Paulo on 11 June 2010.
- \( q \) You will join the demonstration in Sapporo on 11 June 2010.

Contradictory commands from two distinct authorities

A dilemma

\[ [\diamondsuit_{(a,b)} \psi] [\diamondsuit_{(a,c)} \varphi] \left( O_{(a,b)} \psi \land O_{(a,c)} \varphi \right) . \]

Note that this does not lead to deontic explosion.

Some results (Yamada, 2007b)

CUGO Principle

If \( \varphi \) is a formula of MDL\(^+\) II and is free of modal operators of the form \( O_{(i,j)} \diamondsuit_{(i,j)} \varphi \), \( [\diamondsuit_{(i,j)} \varphi] O_{(i,j)} \varphi \) is valid.

Theorem

There is a complete axiomatization of ECLII.
A further refinement and extension (Yamada 2008a)

- ECL (dynamicification)
- DMDL⁺ III (dynamification)
- MDL⁺ (refinement)

It is obligatory upon an agent \( i \) with respect to an obligee \( j \) in the name of \( k \) to see to it that \( \varphi \).

\[ \text{Com}_{(i,j)} \varphi \] Act of commanding.

\[ \text{Prom}_{(i,j)} \varphi \] Act of promising.

Some results (Yamada, 2008a)

**CUGO Principle**

If \( \varphi \) is a formula of MDL⁺ III and is free of modal operators of the form \( O_{(i,j,k)} \), \( [\text{Com}_{(i,j)} \varphi] O_{(i,j,k)} \varphi \) is valid.

**PUGO Principle**

If \( \varphi \) is a formula of MDL⁺ III and is free of modal operators of the form \( O_{(i,j)} \), \( [\text{Prom}_{(i,j)} \varphi] O_{(i,j)} \varphi \) is valid.

**Theorem**

There is a complete axiomatization of DMDL⁺ III.

Example 3: a command and a promise can lead to a dilemma

A contingent dilemma

\[ [\text{Prom}_{(a,b)}] [\text{Com}_{(c,a)}] (O_{(a,b)} p \land O_{(a,c)} q) \land \neg (p \land q) . \]

\( p \) You will attend the conference in São Paulo on 11 June 2010.

\( q \) You will join the demonstration in Sapporo on 11 June 2010.

The same strategy works for changing preferences (van Benthem and Liu, 2007) (Liu, 2008)

**Dynamic Epistemic Upgrade Logic DEUL**

adding dynamic modalities

translation along reduction axioms

**Epistemic Preference Logic EPL**
The language of EPL

Definition

Take a set $\text{Aprop}$ of proposition letters, and a set $I$ of agents, with $p$ ranging over $\text{Aprop}$ and $i$ over $I$. The epistemic preference language is given by:

$$\varphi ::= \bot \mid p \mid \neg \varphi \mid \varphi \land \psi \mid U \varphi \mid K_i \varphi \mid [\text{pref}]_i \varphi$$

Intuitively, $[\text{pref}]_i \varphi$ means that all worlds $i$ considers at least as good as the current one satisfy $\varphi$. $U$ is the so-called “universal modality”, and $U \varphi$ means that $\varphi$ holds at every world.

Combining preference upgrades and deontic updates

(Yamada 2008b)

The language of DEUL

Definition

Take the same set $\text{Aprop}$ of proposition letters, and the same set $I$ of agents as before, with $p$ ranging over $\text{Aprop}$ and $i$ over $I$. The dynamic epistemic preference language is given by:

$$\varphi ::= \bot \mid p \mid \neg \varphi \mid \varphi \land \psi \mid U \varphi \mid K_i \varphi \mid [\text{pref}]_i \varphi \mid [\pi] \varphi$$

$$\pi ::= \varphi \mid \not\varphi$$

$[\varphi]$: the type of acts of publicly announcing that $\varphi$, $[\not\varphi]$: the type of acts of publicly suggesting $\varphi$.

The language of DPL

Definition

Take a set $\text{Aprop}$ of proposition letters, and a set $I$ of agents, with $p$ ranging over $\text{Aprop}$ and $i, j$ over $I$. The deontic preference language is given by:

$$\varphi ::= \bot \mid p \mid \neg \varphi \mid \varphi \land \psi \mid U_i \varphi \mid [\text{pref}]_i \varphi \mid O_{(i,j)} \varphi$$
The language of DDPL

**Definition**

Take a set $A_{prop}$ of proposition letters, and a set $I$ of agents, with $p$ ranging over $A_{prop}$ and $i, j$ over $I$. The dynamic deontic preference language is given by:

$$\varphi ::= \bot \mid p \mid \neg \varphi \mid \varphi \land \psi \mid U \varphi \mid [\text{pref}]_{i} \varphi \mid O_{(i,j)} \varphi \mid [\pi] \varphi$$

$$\pi ::= \# \psi \mid !_{(i,j)} \varphi$$

**Theorem**

There is a complete axiomatization of DDPL.

The following formulas are satisfiable.

$$O_{(i,j)} p \land U(p \rightarrow \langle \text{pref} \rangle_{i} \neg p).$$

$$[!_{(i,j)} p] U(p \rightarrow \langle \text{pref} \rangle_{i} \neg p).$$

$$\langle \text{pref} \rangle_{i} \varphi$$ is an abbreviation of $\neg [\text{pref}]_{i} \neg \varphi$.

Walton & Krabbe (1995)

**Three Kinds of propositional commitments**

- commitments incurred by making concessions
- commitments called assertions
- participant’s dark-side commitments

Since dark-side commitments are hidden commitments and supposed to be fixed, we will ignore them.

We call the remaining two kinds of commitments c-commitments and a-commitments respectively.
A-commitments and c-commitments

According to Walton and Krabbe (1995, p.186)
Propositional commitments constitute a special case of commitments to a course of action.

- an agent who has an a-commitment to the proposition \( p \) is obliged to defend it if the other party in the dialogue require her to justify it.
- an agent who has a c-commitments to \( p \) is only obliged to allow the other party to use it in the arguments.

As anyone who asserts that \( p \) will be obliged to allow the other party to use it in the arguments, a-commitments imply c-commitments.

The language of MPCL

Definition
Take a countably infinite set \( A_{prop} \) of proposition letters, and a finite set \( I \) of agents, with \( p \) ranging over \( A_{prop} \), and \( i \) over \( I \). The language \( L_{MPCL} \) of the multi-agent propositional commitment logic MPCL is given by:

\[ \varphi ::= T \mid p \mid \neg \varphi \mid \varphi \land \psi \mid [a-cmt]_i \varphi \mid [c-cmt]_i \varphi \]

\([a-cmt]_i \varphi\): an agent \( i \) has an a-commitment to the proposition \( \varphi \).
\([c-cmt]_i \varphi\): an agent \( i \) has a c-commitment to the proposition \( \varphi \).

P-commitments are different from knowledge

The following formulas are not valid.

\([a-cmt]_i \varphi \rightarrow \varphi\)
\([c-cmt]_i \varphi \rightarrow \varphi\)

Cf. \( K_i \varphi \rightarrow \varphi \)

P-commitments are different from belief

The following formulas are not valid.

\(\neg[a-cmt]_i \bot\)
\(\neg[c-cmt]_i \bot\)

Cf. \( \neg B_i \bot \)
By an \( \mathcal{L}_{MPCL} \)-model, we mean a tuple \( M = (W^M, \{D^M_i \mid i \in I\}, \{W^M_i \mid i \in I\}, V^M) \) where:

- (i) \( W^M \) is a non-empty set (heuristically, of ‘possible worlds’),
- (ii) \( D^M_i \subseteq W^M \times W^M \) for each \( i \in I \),
- (iii) \( W^M_i \subseteq D^M_i \) for each \( i \in I \),
- (iv) \( V^M \) is a function that assigns a subset \( V^M(p) \) of \( W^M \) to each proposition letter \( p \in \text{Aprop} \).

**Definition**

The proof system for MPCL includes (i) all instantiations of propositional tautologies over the present language, (ii) K-axioms for \([a\text{-}cmt]_i\)-modality and \([c\text{-}cmt]_i\)-modality for each \( i \in I \), (iii) modus ponens, and (iv) necessitation rules for \([a\text{-}cmt]_i\)-modality and \([c\text{-}cmt]_i\)-modality for each \( i \in I \), in addition to the axiom of the following form for each \( i \in I \):

\[
[a\text{-}cmt]_i \varphi \rightarrow [c\text{-}cmt]_i \varphi
\]

**Theorem (Completeness of MPCL)**

MPCL is strongly complete with respect to \( \mathcal{L}_{MPCL} \)-models.
The language of DMPCL

### Definition

Take the same countably infinite set \textit{Aprop} of proposition letters and the same finite set \( I \) of agents as before, with \( p \) ranging over \textit{Aprop}, and \( i \) over \( I \). The language \( \mathcal{L}_{\text{DMPCL}} \) of dynamified multi-agent propositional commitment logic DMPCL is given by:

\[
\varphi ::= \top \mid p \mid \neg \varphi \mid \varphi \land \psi \mid [a\text{-cmt}]_i \varphi \mid [c\text{-cmt}]_i \varphi \mid [\pi] \varphi
\]

\[
\pi ::= \text{assert}_i \varphi \mid \text{concede}_i \varphi
\]

### The proof system for \( \mathcal{L}_{\text{DMPCL}} \)

The proof system for DMPCL includes all the axioms and all the rules of the proof system for MPCL, and in addition, necessitation rules for assertion modality and concession modality for each \( i \in I \), and the following axioms:

\begin{align*}
(A1) \quad & \text{assert}_i \varphi \rightarrow \varphi \\
(A2) \quad & \text{assert}_i \top \rightarrow \top \\
(A3) \quad & \neg \text{assert}_i \varphi \rightarrow \neg \varphi \\
(A4) \quad & \text{assert}_i \varphi \land \text{assert}_i \psi \rightarrow \text{assert}_i (\varphi \land \psi) \\
(A5) \quad & \text{assert}_i \varphi \rightarrow \text{assert}_i \varphi \land \text{assert}_i \psi \\
(A6) \quad & \text{assert}_i \varphi \rightarrow \text{assert}_i \varphi \land \text{assert}_i \psi \\
(A7) \quad & \text{concede}_i \varphi \land \text{concede}_i \psi \rightarrow \text{concede}_i \varphi \\
(A8) \quad & \text{concede}_i \varphi \rightarrow \text{concede}_i \varphi \land \text{concede}_i \psi \\
(C1) \quad & \text{concede}_i \varphi \rightarrow \varphi \\
(C2) \quad & \text{concede}_i \top \rightarrow \top \\
(C3) \quad & \neg \text{concede}_i \varphi \rightarrow \neg \varphi \\
(C4) \quad & \text{concede}_i \varphi \land \text{concede}_i \psi \rightarrow \text{concede}_i (\varphi \land \psi) \\
(C5) \quad & \text{concede}_i \varphi \rightarrow \text{concede}_i \varphi \land \text{concede}_i \psi \\
(C6) \quad & \text{concede}_i \varphi \land \text{concede}_i \psi \rightarrow \text{concede}_i \varphi \\
(C7) \quad & \text{concede}_i \varphi \land \text{concede}_i \psi \rightarrow \text{concede}_i \varphi \land \text{concede}_i \psi
\end{align*}

### Translation from \( \mathcal{L}_{\text{DMPCL}} \) to \( \mathcal{L}_{\text{MPCL}} \)

The translation function that takes a formula from \( \mathcal{L}_{\text{DMPCL}} \) and yields a formula in \( \mathcal{L}_{\text{MPCL}} \) is defined as follows:

\[
\tau(\top) = \top \quad \tau(p) = p \quad \tau(\neg \varphi) = \neg \tau(\varphi) \\
\tau(\varphi \land \psi) = \tau(\varphi) \land \tau(\psi) \quad \tau([\pi] \varphi) = [\tau(\pi)] \tau(\varphi) \\
\tau([a\text{-cmt}]_i \varphi) = [a\text{-cmt}]_i \tau(\varphi) \quad \tau([c\text{-cmt}]_i \varphi) = [c\text{-cmt}]_i \tau(\varphi)
\]
Some results

**Proposition**

If $\varphi \in \mathcal{L}_{MPCL}$ is free of modalities indexed by $i$, the following formulas are valid:

\[
\begin{align*}
[\text{assert}_i \varphi][\text{a-cmt}]_i \varphi \\
[\text{assert}_i \varphi][\text{c-cmt}]_i \varphi \\
[\text{concede}_i \varphi][\text{c-cmt}]_i \varphi
\end{align*}
\]

**Theorem**

There is a complete axiomatization of DMPCL.

The language of DMPCL$^+$

**Definition**

Take the same countably infinite set $A_{prop}$ of proposition letters and the same finite set $I$ of agents as before, with $p$ ranging over $A_{prop}$, and $i$ over $I$. The language $\mathcal{L}_{DPCMT^+}$ of dynamified multi-agent propositional commitment logic with withdrawals DMPCL$^+$ is given by:

\[
\varphi ::= \top \mid p \mid \neg \varphi \mid \varphi \land \psi \mid [\text{a-cmt}]_i \varphi \mid [\text{c-cmt}]_i \varphi \mid [\pi] \varphi
\]

\[
\pi ::= \text{assert}_i \varphi \mid \text{concede}_i \varphi \mid \cup \text{assert}_i \varphi \mid \cup \text{concede}_i \varphi
\]
A positive commitment act sequence

If $\sigma$ is a sequence of moves in an argumentation, it may involve not only acts of asserting and conceding but also acts of withdrawing.

For the sake of simplicity, we will only consider a special kind of sequences, namely, a sequence $\sigma = \langle \pi_1, \pi_2, \cdots, \pi_n \rangle$ of speech acts $\pi_j$ ($1 \leq j \leq n$) such that each $\pi_j$ is either of the form $\text{assert} \varphi$ for some $i \in I$ or of the form $\text{concede} \varphi$ for some $i \in I$. We call such a sequence a positive commitment act sequence, or a pca-sequence for short.

Reduced positive commitment act sequence

**Definition**

Let $\sigma$ be a (possibly empty) positive commitment act sequence $\langle \pi_1, \cdots, \pi_n \rangle$ such that each $\pi_j$ ($1 \leq j \leq n$) is of the form $\text{assert} \varphi$ or of the form $\text{concede} \varphi$ for some $i \in I$. We define the reduced sequence $\sigma = \langle \Box \text{assert} \varphi (\sigma \vdash \Box \text{assert} \varphi) \rangle$ obtained by withdrawing every occurrence of an act of type $\text{assert} \varphi (\text{concede} \varphi)$ from $\sigma$ as follows:

(To be continued)

Reduced pca-sequence (continued)

(Ai) if $\sigma$ is empty, $\sigma = \langle \Box \text{assert} \varphi \rangle = \sigma$

(Aii) if $\sigma = \langle \pi_1, \cdots, \pi_n \rangle$, and $\pi_n = \text{assert} \varphi$,

$\sigma = \langle \Box \text{assert} \varphi = \langle \pi_1, \cdots, \pi_n-1 \rangle \vdash \Box \text{assert} \varphi \rangle$

(Aiii) if $\sigma = \langle \pi_1, \cdots, \pi_n \rangle$, and $\pi_n \neq \text{assert} \varphi$,

$\sigma = \langle \Box \text{assert} \varphi = \langle \pi_1, \cdots, \pi_n-1 \rangle \vdash \Box \text{assert} \varphi, \pi_n \rangle$

(Ci) if $\sigma$ is empty, $\sigma = \langle \Box \text{concede} \varphi \rangle = \sigma$

(Cii) if $\sigma = \langle \pi_1, \cdots, \pi_n \rangle$, and $\pi_n = \text{concede} \varphi$,

$\sigma = \langle \Box \text{concede} \varphi = \langle \pi_1, \cdots, \pi_n-1 \rangle \vdash \Box \text{concede} \varphi \rangle$

(Ciii) if $\sigma = \langle \pi_1, \cdots, \pi_n \rangle$, and $\pi_n \neq \text{concede} \varphi$,

$\sigma = \langle \Box \text{concede} \varphi = \langle \pi_1, \cdots, \pi_n-1 \rangle \vdash \Box \text{concede} \varphi, \pi_n \rangle$.

The Problem of Notation

Given a pca-sequence $\sigma = \langle \pi_1, \cdots, \pi_n \rangle$, the model obtained by updating $M$ with $\sigma$ is denoted by $(\cdots (M_{\pi_1}) \cdots)_{\pi_n}$ in the notation of the truth definition for $L_{\text{DMPC}_L}$.

This notation leads to a paradox when we deal with withdrawals. Let abbreviate $(\cdots (M_{\pi_1}) \cdots)_{\pi_n}$ as $M_\sigma$. Now there may be another model $N$ and a pcs-sequence $\tau$ such that $N_\tau = M_\sigma$. Then we may have

$(N_\tau)_{\pi_n} = M_\tau$, but $(\cdots (N_{\pi_1}) \cdots)_{\pi_n} \neq (M_{\pi_1})_{\pi_n}$.
**Truth Definition 1/5**

**Definition**

Let $M$ be an $L_{MPCL}$-model, $\sigma$ a positive commitment act sequence, and $w$ a point in $M$. If $p \in Aprop$, and $i \in I$, then:

(a) $M, \sigma, w \vDash_{MPCL} p$ \iff $w \in V^M(p)$

(b) $M, \sigma, w \vDash_{MPCL} T$

(c) $M, \sigma, w \vDash_{MPCL} \neg \varphi$ \iff it is not the case that $M, \sigma, w \vDash_{MPCL} \varphi$

(d) $M, \sigma, w \vDash_{MPCL} (\varphi \land \psi)$ \iff $M, \sigma, w \vDash_{MPCL} \varphi$ and $M, \sigma, w \vDash_{MPCL} \psi$

---

**Truth Definition 2/5**

(f) $M, \sigma, w \vDash_{MPCL} [c-cmt]_\varphi$ \iff for all $v$ s. t. $(w, v) \in \bigtriangledown^M_i \sigma$,

$$M, \sigma, v \vDash_{MPCL} \varphi$$

(g) $M, \sigma, w \vDash_{MPCL} [assert]_\varphi$ \iff if for all $v$ s. t. $(w, v) \in \bigtriangledown^M_i \sigma$,

$$M, \langle \sigma, \text{assert}_\chi \rangle, w \vDash_{MPCL} \varphi$$

(h) $M, \sigma, w \vDash_{MPCL} [concede]_\varphi$ \iff if $M, \langle \sigma, \text{concede}_\chi \rangle$,

$$w \vDash_{MPCL} \varphi$$

---

**Truth Definition 3/5**

(i) $M, \sigma, w \vDash_{MPCL} [\cup \text{assert}_\chi]_\varphi$ \iff $M, \sigma \vDash \cup \text{assert}_\chi$,

$$w \vDash_{MPCL} \varphi$$

(j) $M, \sigma, w \vDash_{MPCL} [\cup \text{concede}_\chi]_\varphi$ \iff $M, \sigma \vDash \cup \text{concede}_\chi$,

$$w \vDash_{MPCL} \varphi$$

where $\bigtriangledown^M_i \sigma$ and $\bigtriangledown^M_i \sigma$ are

(To continue)

---

**Truth Definition 4/5**

where $\bigtriangledown^M_i \sigma =$

- $\bigtriangledown^M_i \sigma$ if $\sigma$ is empty,
- $\{ (x, y) \in \bigtriangledown^M_i \langle \pi_1, \ldots, \pi_{n-1} \rangle \mid M, \langle \pi_1, \ldots, \pi_{n-1} \rangle, y \vDash_{MPCL} \psi \}$

if $\sigma = \langle \pi_1, \ldots, \pi_n \rangle$ and $\pi_n = \text{assert}_\psi$,

- $\bigtriangledown^M_i \langle \pi_1, \ldots, \pi_{n-1} \rangle$

if $\sigma = \langle \pi_1, \ldots, \pi_n \rangle$ and $\pi_n \neq \text{assert}_\psi$,

and

(To continue)
A result and an open problem

A result
Acts of withdrawing behave slightly differently from contraction studied in belief revision. Let $B$ be a set of beliefs of an agent, say $a$. Then in the AGM approach, contraction $\Diamond$ is supposed to satisfy the postulate that $\varphi \not\in B \land \Diamond \varphi$, but we have $M, \sigma \models \Diamond \text{assert}_a \varphi$, $w \models \text{DMPLC}^+ [\text{a-cmt}]_a \varphi$ if $\sigma$ include $\text{assert}_a q$ and $\text{assert}_a (q \rightarrow p)$.

An open problem
The completeness problem of $\text{DMPLC}^+$ is still open.
Scorekeeping for argumentation games

- We will only consider “the official score” kept by an idealised scorekeeper, and examine how DMPCL\(^+\) can be applied to such official scorekeeping in an argumentation game.
- In order to do so, we need a special model that can represent the initial stage of the game.

**Definition**

Given a countably infinite set A\text{prop} of proposition letters, and the set I = \{a, b\}, with p ranging over A\text{prop} and i over I. Then, the initial stage model is the tuple ...

What we have at the initial stage

For any agent i, for any proposition letter p, and for any point w ∈ W\(^0\), if σ is empty, we have

- \(M^0, σ, w \not\models_{\text{DMPCL}^+} \ [a\text{-cmt}]p\)
- \(M^0, σ, w \not\models_{\text{DMPCL}^+} \ [a\text{-cmt}]\neg p\)
- \(M^0, σ, w \not\models_{\text{DMPCL}^+} \ [c\text{-cmt}]p\)
- \(M^0, σ, w \not\models_{\text{DMPCL}^+} \ [c\text{-cmt}]\neg p\)

Thus each agent has no substantial propositional commitments at the initial stage.

What DMPCL\(^+\) enables us to do

- Then we can reason about what propositional commitments agents will bear at each stage after each of their acts of asserting, conceding or withdrawing.
- This doesn’t give us the whole score of each play of an argumentation game. The official scorekeeper may have to record other factors such as penalties for withdrawing, the relation between the moves made by the participants, etc.
- But DMPCL\(^+\) can be said to capture the evolution of the score at least partially.
The notion of, or the metaphor of, scorekeeping can be extended to more complex language games where not only acts of asserting, conceding, and withdrawing but also acts of commanding, promising, etc. are involved along with non-verbal actions.

Then the dynamic logics of speech acts can be used to reason about the changes and non-changes brought about by speech acts.

The effects of illocutionary acts of asserting and conceding as well as illocutionary acts of commanding and promising can be captured in logical terms. Their effects are public and involve more than mere securing of the uptake.

The workings of acts of withdrawing can also be modeled, but the axiomatizability is still open.

The dynamic logics can be seen as partially characterizing the scorekeeping function for a complex social interaction.