Measurement-Theoretic Foundation of Threshold Utility Maximiser's Preference Logic

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Structure of This Talk

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Counterexample to Transitivity of Indifference (1)

- The economist Armstrong ([Armstrong 1939]) was one of the first to argue that indifference is not always transitive.

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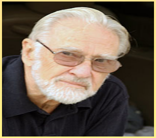
- Luce ([[Luce 1956]: 179]) gave the following counterexample to the transitivity of indifference:



Luce, D.:
Semiorders and a Theory of Utility Discrimination.
Econometrica **24** (1956) 178–191.

Counterexample to Transitivity of Indifference (2)

Example (Avoidance of Sorites Paradox in Preference)

- If indifference were transitive, then he would be unable to detect any weight differences, however great, which is patently false. . . .

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- If indifference were transitive, then he would be unable to detect any weight differences, however great, which is patently false....
- Find a subject who prefers a cup of coffee with one cube of sugar to one with five cubes....
- Now prepare 401 cups of coffee with $(1 + \frac{i}{100})x$ grams of sugar, $i = 0, 1, \ldots, 400$, where x is the weight of one cube of sugar.

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- It is evident that he will be indifferent between cup i and cup $i + 1$, for any i, but by choice he is not indifferent between $i = 0$ and $i = 400$.

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This example shows a situation where we would face the Sorites Paradox in preference if indifference were transitive.

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What kind of preference logic can formalise inferences in which indifference is not transitive? ■

We call it the Nontransitivity Problem.

The Aim of This Talk

The aim of this talk is to propose a new version of complete and decidable extrinsic preference logic–threshold utility maximiser's preference logic (TUMPL) that can solve the Nontransitivity Problem.

Fundamental Problem of Intrinsic Preference (1)

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- Von Wright divided preferences into two categories: extrinsic and intrinsic preference.

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The Logic of Preference.
Edinburgh UP, Edinburgh (1963).

Fundamental Problem of Intrinsic Preference (2)

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- If we cannot explain preference from any explicit point of view, we call it intrinsic.

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- We call it the Fundamental Problem of Intrinsic Preference.

Fundamental Problem of Intrinsic Preference (4)

Example

	von Wright	Martin	Chisholm and Sosa
Transitivity	+	+	+
Contraposition	−	+	−
Conjunctive Expansion	+	−	−
Disjunctive Distribution	−	−	−
Conjunctive Distribution	+	−	−

Chisholm, R. M. and Sosa, E.:
On the Logic of "Intrinsically Better".
American Philosophical Quarterly **3** (1966) 244–249.

Martin, R. M.:
Intension and Decision.
Prentice-Hall, Inc., Englewood Cliffs (1963).

Mullen's Analysis of Cause of Fundamental Problem (1)

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- The adequacy criteria for intrinsic preference principles considered by preference logicians have been whether the principles are consistent with their intuitions of reasonableness.
- But each intuitions often disagrees even on the fundamental properties.

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- Different theories, such as ethics, welfare economics, consumer demand theory, game theory and decision theory make different demands upon the fundamental properties of preference.

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- Different theories, such as ethics, welfare economics, consumer demand theory, game theory and decision theory make different demands upon the fundamental properties of preference.
- So if we would like to propose preference logic that can avoid the Fundamental Problem of Intrinsic Preference, it should be constructed not from intuition but from a theory or a rule in a theory, that is, it should be extrinsic.

Measurement Theory (1)

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- There are two main problems with measurement theory:
  1. the representation problem–justifying the assignment of numbers to objects or propositions,
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- A solution to the first problem can be furnished by a representation theorem, which establishes that the chosen numerical system preserves the relations of the relational system.

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- When we provide TUMPL with a model based on semiorders, by virtue of a corollary of the Scott-Suppes representation theorem, we can adopt threshold utility maximisation as a rule in utility theory that makes demands upon the fundamental properties of preference, which enables TUMPL to avoid the Fundamental Problem of Intrinsic Preference.

Structure of This Talk

Global Rationality

- The standard model of economics is based on global rationality that requires an optimising behavior.

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- Utility maximisation is a typical example of an optimising behavior.

Representation Theorem for Utility Maximisation

- Cantor ([Cantor 1895]) proved the representation theorem for utility maximisation.

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Theorem (Representation for Utility Maximisation, Cantor ([Cantor 1895]))

Suppose $\mathbf{A}$ *is a countable set and* $\succeq$ *is a binary relation on* $\mathbf{A}$*. Then* $\succeq$ *is a weak order (transitive and connected) iff there is a function* $u : \mathbf{A} \to \mathbb{R}$ *such that for any* $x, y \in \mathbf{A}$*,*

$$x \succeq y \text{ iff } u(x) \geq u(y).$$

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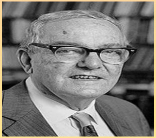
Bounded Rationality

- But according to Simon ([Simon 1982]), cognitive and information-processing constrains on the capabilities of agents, together with the complexity of their environment, render an optimising behavior an unattainable ideal.



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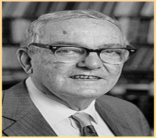

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- He dismissed the idea that agents should exhibit global rationality and suggested that they in fact exhibit bounded rationality that allows a satisficing behavior.

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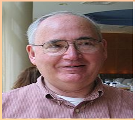
- The concept of a semiorder was introduced by Luce ([Luce 1956]) to construct a model to interpret situations like Example 1 of nontransitive indifference with a threshold of discrimination.

Semiorders (2)

- Scott and Suppes defined ([[Scott and Suppes 1958]: 117]) a semiorder as follows:

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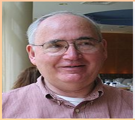


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3. If $w \succ x$ and $x \succ y$, then $w \succ z$ or $z \succ y$. (Semitransitivity).

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Theorem (Representation for Threshold Utility Maximisation, Scott and Suppes ([Scott and Suppes 1958]))

Suppose that $\succ$ *is a binary relation on a finite set* **A** *and* δ *is a positive number. Then* $\succ$ *is a semiorder iff there is a function* $u : \mathbf{A} \to \mathbb{R}$ *such that for any* $x, y \in \mathbf{A}$,

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- The set of all well-formed formulae of $\mathcal{L}_{\text{TUMPL}}$ will be denoted by $\Phi_{\mathcal{L}_{\text{TUMPL}}}$. ■

The Point of Introducing $\Box$

Remark

We introduce $\Box$ to construct a Boolean algebra of subsets of $\mathbf{W}$ that is accessible from $w \in \mathbf{W}$. ■

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- V is a truth assignment to each $s \in \mathbf{S}$ for each $w \in \mathbf{W}$,
- ρ is a preference space assignment that assigns to each $w \in \mathbf{W}$ a preference space $(\mathcal{F}_w, \succ_w)$ such that $\mathcal{F}_w$ is a Boolean algebra of subsets of $\{w' \in \mathbf{W} : R(w, w')\}$ and $\succ_w$ on $\mathcal{F}_w$ is a semiorder.

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Suppose that $\mathbf{W}$ *is a finite set of possible worlds and* $\mathcal{F}$ *is a finite Boolean algebra of subsets of* $\mathbf{W}$ *and* $\succ$ *is a binary relation on* $\mathcal{F}$*, and* δ *is a positive number. Then* $\succ$ *is a semiorder iff there is a function* $u : \mathcal{F} \to \mathbb{R}$ *such that for any* $\alpha, \beta \in \mathcal{F}$*,*

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Remark

This corollary, by virtue of filtration theory, can guarantee that $\succ_w$ *on* $\mathcal{F}_w$ *is a threshold utility maximiser's preference relation.* ■

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- $(\mathcal{M}, w) \models_{\mathsf{TUMPL}} s$ iff $V(w)(s) = \textbf{true}$,
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- $(\mathcal{M}, w) \models_{\mathsf{TUMPL}} \varphi_1 \& \varphi_2$
iff $(\mathcal{M}, w) \models_{\mathsf{TUMPL}} \varphi_1$ and $(\mathcal{M}, w) \models_{\mathsf{TUMPL}} \varphi_2$,
- $(\mathcal{M}, w) \models_{\mathsf{TUMPL}} \neg\varphi$ iff $(\mathcal{M}, w) \not\models_{\mathsf{TUMPL}} \varphi$,
- $(\mathcal{M}, w) \models_{\mathsf{TUMPL}} \Box\varphi$
iff, for any w' such that $R(w, w')$, $(\mathcal{M}, w') \models_{\mathsf{TUMPL}} \varphi$,

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- $(\mathcal{M}, w) \models_{\text{TUMPL}} \neg\varphi$ iff $(\mathcal{M}, w) \not\models_{\text{TUMPL}} \varphi$,
- $(\mathcal{M}, w) \models_{\text{TUMPL}} \Box\varphi$ iff, for any w' such that $R(w, w')$, $(\mathcal{M}, w') \models_{\text{TUMPL}} \varphi$,
- $(\mathcal{M}, w) \models_{\text{TUMPL}} \mathbf{SPR}(\varphi_1, \varphi_2)$ iff $[\![\varphi_1]\!]_w^{\mathcal{M}} \succ_w [\![\varphi_2]\!]_w^{\mathcal{M}}$,

where $[\![\varphi]\!]_w^{\mathcal{M}} := \{w' \in \mathbf{W} : R(w, w') \text{ and } (\mathcal{M}, w') \models_{\text{TUMPL}} \varphi\}$. ■

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- If $(\mathcal{M}, w) \models_{\mathsf{TUMPL}} \varphi$ for all $w \in \mathbf{W}$, we write $\mathcal{M} \models_{\mathsf{TUMPL}} \varphi$ and say that φ is valid in $\mathcal{M}$.

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- If φ is valid in all structured Kripke models for TUMPL, we write $\models_{\mathsf{TUMPL}} \varphi$ and say that φ is valid.

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Counter-Model of Transitivity of Indifference (1)

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  for $[\![\varphi_j]\!]^{\mathcal{U}}_{w_i} = \{w_j\}$ holds for any j $(0 \leq j \leq 400)$.
- It must be noted that, for any i $(0 \leq i \leq 400)$, because $\succ_{w_i}$ on $\mathcal{F}_{w_i}$ is a semiorder,
  $[\![\varphi_j]\!]^{\mathcal{U}}_{w_i} \sim_{w_i} [\![\varphi_{j+1}]\!]^{\mathcal{U}}_{w_i}$ for any j $(0 \leq j \leq 400)$ does not imply
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- Therefore we obtain the following proposition.

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- Therefore we obtain the following proposition.

Proposition (Nontransitivity of Indifference)

$$\not\models_{\text{TUMPL}} (\mathbf{IND}(\varphi_0, \varphi_1) \& \cdots \& \mathbf{IND}(\varphi_{399}, \varphi_{400})) \rightarrow \mathbf{IND}(\varphi_0, \varphi_{400}).$$

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We provide TUMPL with a proof system.

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- A proof of $\varphi \in \Phi_{\mathsf{TUMPL}}$ is a finite sequence of $\mathcal{L}_{\mathsf{TUMPL}}$-formulae having φ as the last formula such that either each formula is an instance of an axiom, or it can be obtained from formulae that appear earlier in the sequence by applying an inference rule.

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- If there is a proof of φ, we write $\vdash_{\mathsf{TUMPL}} \varphi$. ∎

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Theorem (Soundness)

For any $\varphi \in \Phi_{\mathcal{L}_{\mathsf{TUMPL}}}$*, if* $\vdash_{\mathsf{TUMPL}} \varphi$*, then* $\models_{\mathsf{TUMPL}} \varphi$*.* ■

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Segerberg, K.:
Qualitative Probability in a Modal Setting. in Fenstad, J. E. (ed.):
Proceedings of the Second Scandinavian Logic Symposium. North-Holland, Amsterdam (1971) 341–352.

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Theorem (Completeness)

For any $\varphi \in \Phi_{\mathcal{L}_{\text{TUMPL}}}$, *if* $\models_{\text{TUMPL}} \varphi$, *then* $\vdash_{\text{TUMPL}} \varphi$. ■

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Theorem (Decidability)

TUMPL *is decidable.* ■

Structure of This Talk

Summary and Further Investigation

- SUMMARY: In this talk we have proposed a new version of complete and decidable extrinsic preference logic–threshold utility maximiser's preference logic (TUMPL) that can solve the Nontransitivity Problem and avoid the Fundamental Problem of Intrinsic Preference.

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- FURTHER INVESTIGATION: This talk is only a part of a larger measurement-theoretic study. We are now trying to construct such logics as dyadic deontic logic, logic for goodness and badness, and logic of questions and answers by means of measurement theory.

Our Related Work

Satoru Suzuki:
Preference Logic and Its Measurement-Theoretic Semantics.
In: Accepted Papers of 8th Conference on Logic and the Foundations of Game and Decision Theory (LOFT 2008), Universiteit van Amsterdam (2008).

Satoru Suzuki:
Prolegomena to Dynamic Epistemic Preference Logic.
In: Hattori, H. et al. (eds.), New Frontiers in Artificial Intelligence, LNAI 5447, Springer-Verlag, Berlin (2009) 177–192.

Satoru Suzuki:
Measurement-Theoretic Foundation of Preference-Based Dyadic Deontic Logic.
In: He, X. et al. (eds.), Proceedings of the Second International Workshop on Logic, Rationality, and Interaction (LORI-II), LNAI 5834, Springer-Verlag, Berlin, (2009) 278–291.

Satoru Suzuki:
Measurement-Theoretic Foundation of Logic for Goodness and Badness.
In: Bekki, D. (ed.), Proceedings of the Sixth Workshop on Logic and Engineering of Natural Language Semantics (LENLS 2009), JSAI, (2009) 1–14.

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Thank You for Your Attention!