Discussion Paper, Series A, No. 2010-225

Taxation in the Two-Sector Neoclassical Growth Model with Sector-Specific Externalities and Endogenous Labor Supply

Daisuke Amano,

Jun-ichi Itaya

June, 2010

Graduate School of Economics & Business Administration
Hokkaido University
Kita 9 Nishi 7, Kita-Ku, Sapporo 060-0809, JAPAN

Taxation in the Two-Sector Neoclassical Growth Model with Sector-Specific Externalities and Endogenous Labor Supply*

Daisuke Amano[†] Jun-ichi Itaya[‡]

June 13, 2010

Abstract

This paper examines the long-run impacts of selective (sector-specific) commodity, payroll and profit taxes in a two-sector endogenous growth model with sector-specific production externalities, in which one sector produces consumption goods and the other produces investment goods. The novelty of the model is that it allows not only for *endogenous* labor supply (which may lead to indeterminacy) but also for the intersectional allocation of labor. We analytically show that the long-run effects of these selective taxes are closely related to the possible emergence of the indeterminacy of equilibria, which may reverse the standard results of the growth effects of distortionary taxes.

Keywords: Selective tax; Two-sector model; Endogenous growth; Production externalities; Indeterminacy; Endogenous labor supply

JEL classifications: H22, J22, O41

*We would like to thank Toshihiro Ihori, Koichi Futagami, Kazuo Mino, Noritaka Kudoh, and the other seminar participants in the Otaru University of Commerce, Kushiro Public University, and Kansai Macroeconomics Workshop. Needless to say, all remaining errors are our own.

†Graduate School of Economics and Business Administration, Hokkaido University, Sapporo 060-0809, Japan. E-mail:yogo@pop.econ.hokudai.ac.jp

‡Corresponding author: Graduate School of Economics and Business Administration, Hokkaido University, Sapporo 060-0809, Japan. Tel: +81-11-706-2858; Fax: +81-11-706-4947; E-mail: itaya@econ.hokudai.ac.jp

1 Introduction

In this paper, we examine the long-run impacts of three types of selective (sector-specific) taxes on a two-sector endogenous growth model populated by an infinitely-lived representative agent. We impose profit taxes, payroll taxes, and commodity taxes on both consumption and investment goods sectors; these are accompanied by compensating lump-sum transfers to households. We also allow for both endogenous labor supply and sector-specific production externalities that generate sustained endogenous growth. Using such a two-sector endogenous growth model, we show that the long-run effects of the changes in these tax rates are significantly affected by the emergence of equilibrium indeterminacy; most notably, the selective taxes imposed on the investment sector may lead to faster long-run growth.

Since the seminal contribution of Harberger (1962), the problem of tax incidence in a two-sector general equilibrium model has been investigated by several authors such as, Mieskowski (1967). Although these works overcome the limitation of partial equilibrium analysis, these are static and thus fail to consider the effects of taxes through capital accumulation. Friedlaender and Vanderdorp (1978) and Ballentine (1978) are the pioneers who extend the analysis to a dynamic economy, but the relationship between the structure of their models and the effects of taxes is left unexplored in their investigations. Homma (1985) examines the incidence of various selective taxes in a two-sector growth model by clarifying the relationship between tax incidence and the stability properties of the model. He shows that the incidence of any selective tax depends on the elasticity of factor substitution in the tax-imposed sector and the elasticity of demand substitution between two goods. Although his analysis has greater generality than previous studies, it fails to consider the intertemporal optimizing behavior of individuals since they are assumed to behave myopically.

In contrast, Itaya (1991) further investigates the same problem in a two-sector growth model where individuals maximize their intertemporal utility over an infinite-time horizon, and finds that the taxes on the consumption goods sector are neutral in terms of the share of labor income to capital income in a long-run equilibrium. The assumption of fixed labor supply simplifies an already complicated analysis, but it needs to be modified for the sake of reality. More importantly, labor supply is certainly flexible in the long run, and thus the responses of labor supply to tax changes clearly matter in determining the incidence of taxes.

On the other hand, the literature on endogenous growth models has reexamined the long-run effects of factor taxation in a framework where both capital and labor (human capital) are reproducible factors under constant returns-to-scale accumulation technology; see, e.g., Devereux and Love (1994), Mino (1996), Milesi-Ferretti and Roubini (1998), Hendricks (1999), and De Hek (2006). This literature, with the exception of De Hek, shows that in general both labor and capital income taxes reduce the long-run growth rate. Indeed, these distortionary taxes effectively act as a tax on human and non-human capital incomes, respectively, thereby discouraging an incentive to accumulate both types of capital. De Hek (2006) shows that when the elasticity of intertemporal substitution is relatively strong, the (negative) income effect is strong; consequently, agents tend to work more and/or invest more time in human capital accumulation in response to an increase in capital income taxation, resulting in a growth-enhanced effect in the long-run.

The impacts of taxation have been investigated using the standard version of a one- or two-sector endogenous (non-endogenous) growth model in which a dynamic equilibrium path is uniquely determined. Local stability analysis provides important information about the local uniqueness of equilibrium at the steady state. If the steady state is saddle-point stable or locally unstable, then the dynamic equilibrium is locally unique in the long run (i.e., local determinacy). In contrast, if the steady state is locally stable, then a continuum of equilibrium paths converges to the steady state (i.e., local indeterminacy). Recently, Pelloni and Waldmann (2000) show that, in view of Samuelson's (1947) correspondence principle, the stability properties of a one-sector endogenous growth model in the neighborhood of a steady state would be closely tied with the impacts of various fiscal policies. In light of this finding, the growth effects of taxes may potentially be significantly affected by the occurrence of indeterminacy.

This paper investigates how the emergence of the indeterminacy of equilibria is related to the long-run effects of the selective taxes in a two-sector endogenous growth model with endogenous labor supply. We restrict our attention to a class of two-sector neoclassical growth models in which one sector produces consumption goods and the other sector produces investment goods. This is not only because we strictly follow the tradition of Harberger, Homma, and Itaya for purposes of comparison but also because there are many recent researches on the literature on the indeterminacy of macrodynamic models, such as, Benhabib and Farmer (1996), Benhabib and Nishimura (1998), Harrison (2001), and Harrison and Weder (2000, 2002) who have investigated this type of the model to identify the role of sector-specific production externalities in generating indeterminacy.

There are four major contributions of the present paper. (i) This is the first study that investigates how indeterminacy driven by sector-specific externalities is related to the long-run impacts of the selective taxes in a two-sector endogenously growing economy. (ii) We analytically show that the growth effects of the selective taxes are closely tied with the stability properties of a balanced growth path. (iii) The tax impacts on the long-run growth rate in the endogenous growth model differ significantly from those in the non-endogenous growth model in the sense that the latter model does not exhibit any systematic relationship between the stability properties of the model and the growth effects of taxes. (iv) Under a more general constant relative risk aversion (CRRA) utility function, the externalities from the consumption sector also influence the tax effects and the stability properties of the model.

In Section 2, we first describe the behavior of households, firms, and the government in a two-sector endogenous growth model that allows for sector-specific production externalities. In Section 3, we investigate its stability properties. In Section 4, we derive the effects of the changes in selective taxes on long-run employment. In Section 5, we analyze the growth effects of the selective taxes along a balanced growth path. In Section 6, we investigate two variations of the original model, one that allows for CRRA preferences and the other that is an *exogenous* growth version of the original endogenous growth model. In Section 7, we conclude this paper. Some mathematical derivations will be given in the appendixes.

2 Model

2.1 Production

We assume that this economy is composed of two production sectors: one sector (the consumption sector or, simply, sector c) produces homogeneous consumption goods and the other sector (the investment sector or, simply, sector I) produces homogeneous investment goods, where the subscripts c and I indicate the variables pertaining to the consumption and investment sectors, respectively. It is also assumed that there is a continuum of identical competitive firms in each sector, with the total number normalized to unity. For analytical convenience, a consumption good is the numerate, so that its price at each moment in time is normalized to unity. In addition, we omit the subscript t representing time except when it is strictly necessary. The representative competitive firm of sector j $(= c, I)$ produces output y_j using a constant returns-to-scale Cobb-Douglas technology $y_j = k_j^{a_j} l_j^{b_j} X_j$ with $a_j + b_j = 1$ and $j = c, I$, where k_j and l_j represent the stock of capital and labor services employed by that firm, respectively. The term X_j stands for sector-specific externalities in production that are taken as given by each firm; i.e.,

$$X_j = \bar{k}_j^{\alpha_j - a_j} \bar{l}_j^{\beta_j - b_j}, \ \ a_j < \alpha_j \leq 1, \ b_j < \beta_j < 1 \text{ and } \alpha_j + \beta_j > 1, \ j = c, I, \quad (1)$$

where $\bar{k}_j$ and $\bar{l}_j$ represent the sector-specific average stock of capital and labor services used by the firm of sector j, respectively, both of which enhance productivity. Substituting (1) into the production function, $y_j = k_j^{a_j} l_j^{b_j} X_j$, in a symmetric equilibrium we can obtain the following *social* production function of sector j $(= c, I)$:

$$y_j = k_j^{\alpha_j} l_j^{\beta_j}. \quad (2)$$

We shall analyze the case of $\alpha_j = 1$, $j = c, I$, which corresponds to an endogenous growth model in which capital externalities are strong enough to generate perpetual growth.[1]

Given such external effects and the prices of commodities p_j (setting $p_c = 1$ throughout the paper), rental rate, wage rate, and tax parameters, the representative competitive firm of sector j maximizes its own profits:

$$\pi_j = (1 - T_{j\pi}) \left[(1 - T_{jx}) k_j^{a_j} l_j^{b_j} X_j - (1 + T_{jL}) w l_j \right] - r k_j.$$

Each firm of sector j needs to pay the selective profit tax $T_{j\pi}$, the selective payroll tax T_{jL}, and the selective commodity tax T_{jx}. Following the tradition of Harberger (1962), we assume that real investment is financed only through issuing new equities, and therefore such expenditure is not tax exempt. Then, profit maximization of the sector j's firm yields the following first-order conditions:

$$(1 - T_{j\pi})(1 - T_{jx}) p_j a_j (y_j / k_j) = r, \tag{3}$$

$$(1 - T_{jx}) p_j b_j (y_j / l_j) = (1 + T_{jL}) w, \tag{4}$$

where r and w denote the pre-tax return to capital and the pre-tax wage rate, respectively. We introduce new variables θ and ϕ that denote the fractions of the total stock of capital and total labor services devoted to the consumption sector, respectively, at each moment in time, i.e., $\theta \equiv k_c/k$ and $\phi \equiv l_c/l$. Armed with these notations, the full-employment conditions for the respective production factors are expressed as[2]

$$k = k_c + k_I = \theta k + (1 - \theta) k, \tag{5}$$

$$l = l_c + l_I = \phi l + (1 - \phi) l. \tag{6}$$

[1] When $\alpha_j > 1$, growth is explosive and thus we do not analyze this case. When $\alpha_j < 1$, decreasing returns occur and we shall investigate this case in Subsection 6.2.

[2] Recently, Herrendorf and Valentinyi (2006) have investigated the two-sector endogenous growth model with capital adjustment costs that entail imperfect substitutability between the investments allocated to the two sectors, and show that local indeterminacy is easier to obtain in this model than in the model where the two investments are perfect substitutes.

Dividing (3) by (4) yields the common wage/rental ratio:

$$\frac{1}{(1-T_{j\pi})(1+T_{jL})}\frac{b_j}{a_j}\frac{k_j}{l_j}=\frac{w}{r},\ j=c,I. \tag{7}$$

Equilibrating the left-hand side of (7) between the two sectors and rearranging, we can get the following relationship:

$$\frac{k_I}{k_c}=\frac{\tau_{IL}}{\tau_{cL}}\frac{b_c a_I}{a_c b_I}\frac{l_I}{l_c}, \tag{8}$$

where $\tau_{jL}\equiv(1-T_{j\pi})(1+T_{jL})$. By making use of the fractions θ and ϕ defined in (5) and (6), we can rewrite (8) as

$$\frac{1-\theta}{\theta}=\frac{\tau_{IL}}{\tau_{cL}}\frac{b_c a_I}{a_c b_I}\frac{1-\phi}{\phi}. \tag{9}$$

We further solve (9) for θ to get

$$\theta=\frac{\tau_{cL}a_c b_I\phi}{\tau_{cL}a_c b_I\phi+\tau_{IL}b_c a_I(1-\phi)}. \tag{10}$$

In addition, dividing (3) in sector I by that in sector c, using the definitions of ϕ and θ, and noting $p_c=1$, we can derive the relative price of investment:

$$p_I=\frac{\tau_{cx}a_c}{\tau_{Ix}a_I}\frac{\phi^{\beta_c}}{(1-\phi)^{\beta_I}}l^{\beta_c-\beta_I}, \tag{11}$$

where $\tau_{jx}\equiv(1-T_{j\pi})(1-T_{jx})$. Making use of the definitions of ϕ and θ, finally, the market equilibrium conditions for consumption and investment goods, respectively, are given by

$$c=y_c=k_c\,(l_c)^{\beta_c}=\theta k\,(\phi l)^{\beta_c}, \tag{12}$$

$$\dot{k}=y_I-\delta k=k_I\,(l_I)^{\beta_I}-\delta k=(1-\theta)\,k\,[(1-\phi)\,l]^{\beta_I}-\delta k. \tag{13}$$

2.2 Households

There is a unit measure of identical infinitely-lived households, each of whom maximizes its lifetime utility:

$$\int_0^\infty \left[\ln c - \frac{l^{1+\chi}}{1+\chi}\right] e^{-\rho t} dt, \tag{14}$$

where c and l are the household's consumption and hours worked, respectively. The constant parameter χ (> 0) denotes the inverse of the wage elasticity of labor supply. ρ (> 0) is the subjective rate of time preference. The assumed functional form for utility is not only analytically the simplest, but also most commonly used in the literature on economic growth and indeterminacy; see, e.g., Benhabib and Farmer (1994, 1996).

The flow budget constraint faced by the representative household can be expressed as

$$c + p_I \dot{k} = wl + \left(\frac{r}{p_I} - \delta\right) p_I k + z, \tag{15}$$

where z denotes the transfer payments that are rebated to the households in a lump-sum fashion.[3] We now set up the current-value Hamiltonian function as follows:

$$H(c, l, k, q) \equiv \ln c - \frac{l^{1+\chi}}{1+\chi} + \frac{q}{p_I}\left[wl + \left(\frac{r}{p_I} - \delta\right) p_I k + z - c\right],$$

where q represents the shadow price of capital holdings. The first-order

[3] More precisely, the household's flow budget constraint can be expressed as

$$c + \frac{d(p_I k)}{dt} = wl + z + (R - \delta) p_I k, \ k_0 \ given,$$

where R denotes the interest rate for safety assets. By making use of the arbitrage condition between the rate of interest and the sum of the rate of return to capital and capital gains, i.e., $R = (r + \dot{p}_I)/p_I$, the above budget constraint can be reduced to (15) in the text.

conditions for this problem are given by

$$\frac{1}{c} = \frac{q}{p_I}, \tag{16}$$

$$l^{\chi} = \frac{q}{p_I} w, \tag{17}$$

$$\frac{\dot{q}}{q} = \rho + \delta - \frac{r}{p_I}, \tag{18}$$

the given initial capital stock k_0, and the transversality condition $\lim_{t\to\infty} e^{-\rho t} qk = \lim_{t\to\infty} e^{-\rho t} (p_I k/c) = 0$.

We focus on the effects of the changes in the respective selective tax rates on long-run growth. To focus on the problem at hand, we rule out a market for government bonds and public expenditure. Hence, the government's flow budget should be balanced by adjusting the size of lump-sum transfers at each moment in time when the government changes each of the tax parameters. Its flow budget constraint is thus expressed by

$$z = \sum_{j=c,I} (T_{jx} p_j y_j + T_{jL} w l_j) + \sum_{j=c,I} T_{j\pi} \left[(1 - T_{jx}) p_j y_j - (1 + T_{jL}) w l_j \right]. \tag{19}$$

3 Local dynamics

Since we focus on a symmetric perfect-foresight equilibrium, we suppose that the households know the future paths of the relative price of investment, factor prices, tax instruments, and transfer payments when they decide how much to consume, work, and invest over their lifetime.

Combining (16) with (17) and substituting (4) in sector c into the newly found expression results in

$$c l^{\chi} = \frac{1 - T_{cx}}{1 + T_{cL}} b_c k_c (l_c)^{\beta_c - 1},$$

which, by making use of the definitions of θ, ϕ, τ_{cx}, and τ_{cL}, can be rewritten as

$$c l^{\chi} = \frac{\tau_{cx}}{\tau_{cL}} b_c \theta k (\phi l)^{\beta_c - 1}. \tag{20}$$

This condition requires that the marginal rate of substitution (MRS) between consumption and labor supply should be equated to the real wage rate adjusted for commodity and payroll taxes in sector c at each moment in time.

Substituting (12) into c in (20) and rearranging gives

$$\phi = \frac{\tau_{cx}}{\tau_{cL}} b_c l^{-(1+\chi)} \equiv \phi(l). \tag{21}$$

Substituting (21) into ϕ in (10) gives

$$\theta = \frac{\tau_{cL} a_c b_I \phi(l)}{\tau_{cL} a_c b_I \phi(l) + \tau_{IL} b_c a_I \left[1 - \phi(l)\right]} \equiv \theta(l), \tag{22}$$

It follows from (21) and (22) that not only the fractions θ and ϕ are functions of labor supply but also $\phi'(l) < 0$ and $\theta'(l) < 0$ [see (A.1) and (A.2) in Appendix A]. Moreover, by substituting (A.1) into (11) and differentiating, it can be verified that $p_I'(l) < 0$. To understand the intuition behind this, note first from (16) and (17) that a larger labor supply implies a lower demand for leisure, thereby reducing the demand for consumption due to the normality assumption. This reduction shrinks the output of the consumption sector, thus shifting more resources (i.e., labor and capital) to the investment sector and expanding the output of the investment sector. As stated in Benhabib and Farmer (1996) and Harrison (2001), since the marginal product of each factor used in the production of the investment good increases due to the increasing returns, the *social* production possibilities frontier (SPPF) is convex to the origin. In this case, the relative price of investment, which corresponds to the (negative) slope of the SPPF, will fall.

Defining $\kappa \equiv qk = p_I(k/c)$ and then taking the logarithmic time-derivative of both sides of this expression, we substitute (13) and (18) into the newly found expression to give:

$$\frac{\dot{\kappa}}{\kappa} = \frac{\dot{q}}{q} + \frac{\dot{k}}{k} = \rho + \delta - \frac{r}{p_I} + \left[1 - \theta(l)\right] \left[\{1 - \phi(l)\} l\right]^{\beta_I} - \delta.$$

We further substitute (3) in sector I into r in the above expression to get

$$\frac{\dot{\kappa}}{\kappa} = \rho - [\tau_{Ix} a_I - \{1 - \theta(l)\}] [\{1 - \phi(l)\} l]^{\beta_I} . \tag{23}$$

To complete the dynamics of the present model, we substitute (4) in sector I into w in (17), yielding

$$l^{\chi} = q \frac{1 - T_{Ix}}{1 + T_{IL}} b_I k_I l_I^{\beta_I - 1},$$

which, using the definitions of $\kappa \equiv qk$, $\theta(l)$, $\phi(l)$, τ_{Ix}, and τ_{IL}, can be further rewritten as

$$l^{1+\chi-\beta_I} = \kappa \frac{\tau_{Ix}}{\tau_{IL}} b_I [1 - \theta(l)] [1 - \phi(l)]^{\beta_I - 1} . \tag{24}$$

The dynamic evolution of this economy is completely characterized by the two-dimensional system given by (23) and (24) in the variables l and κ, the initial capital stock, and the transversality condition.

The balanced growth path (BGP) of this model is characterized by a situation where both c and k grow at the same rate, while leaving l constant (consequently, the variables p_I, θ, and ϕ remain constant along the BGP). As a result, κ also remains constant along the BGP owing to the definition of $\kappa \equiv qk = p_I(k/c)$. Setting $\dot{\kappa} = 0$ in (23) results in the following BGP condition:

$$\rho = [\tau_{Ix} a_I - \{1 - \theta(\hat{l})\}][\{1 - \phi(\hat{l})\}\hat{l}]^{\beta_I}, \tag{25}$$

where the notation $\hat{}$ stands for a steady-state variable. It is seen that (25) solely determines the BGP level of employment $\hat{l}$. In what follows, we require that $\tau_{Ix} a_I > 1 - \hat{\theta}$.[4] Note also that the constant BGP level of employment, $\hat{l}$, renders $p_I(\hat{l})$ invariant along the BGP, i.e., $\dot{p}_I / p_I = 0$.

To identify the stability properties of this model, we combine (23) and (24) to eliminate the variable κ and then take a linear approximation of the newly found equation in terms of l around the BGP (see Appendix A for

[4] Under this condition, the transversality condition $\lim_{t\to\infty} e^{-\rho t} qk = \lim_{t\to\infty} e^{-\rho t} (p_I k/c) = 0$ also holds.

derivation):

$$\frac{\dot{l}}{l} = \Omega(\hat{l}) \frac{[(1+\chi)(1-\hat{\theta}) - \beta_I(1+\chi\hat{\phi})\Delta(\hat{l})]}{[(1+\chi)(1-\hat{\theta}) - \beta_I(1+\chi\hat{\phi})]}(l-\hat{l}), \tag{26}$$

where $\Omega(\hat{l}) \equiv [(1-\hat{\phi})\hat{l}]^{\beta_I - 1}\hat{\theta}(1-\hat{\phi}) > 0$, $\hat{\theta} \equiv \theta(\hat{l})$, $\hat{\phi} \equiv \phi(\hat{l})$, and $\Delta(\hat{l}) \equiv [\tau_{Ix}a_I - (1-\hat{\theta})]/\hat{\theta}$. Note that $0 < \Delta(\hat{l}) < 1$ owing to condition $\tau_{Ix}a_I > 1-\hat{\theta}$. Nevertheless, the sign of the coefficient of the term $(l-\hat{l})$ on the right-hand side of (26) is still ambiguous. This makes it possible that indeterminacy arises in the present model. More precisely, we can distinguish three cases as follows: (i) When $(1+\chi)(1-\hat{\theta}) < \beta_I(1+\chi\hat{\phi})\Delta(\hat{l})$, the numerator and denominator on the right-hand side of (26) are negative and thus the coefficient is positive, implying that the BGP is locally unstable and the equilibrium path is locally determinate. (ii) When $\beta_I(1+\chi\hat{\phi})\Delta(\hat{l}) < (1+\chi)(1-\hat{\theta}) < \beta_I(1+\chi\hat{\phi})$, the numerator and denominator have opposite signs, and therefore the BGP is locally unstable and the equilibrium path is locally indeterminate. (iii) When $\beta_I(1+\chi\hat{\phi}) < (1+\chi)(1-\hat{\theta})$, the numerator and denominator are positive; hence, the equilibrium path toward the BGP is locally determinate.

The following gives a summary of the above:

Proposition 1 *(i) When $(1+\chi)(1-\hat{\theta}) < \beta_I(1+\chi\hat{\phi})\Delta(\hat{l})$, the equilibrium is locally determinate;*

(ii) when $\beta_I(1+\chi\hat{\phi})\Delta(\hat{l}) < (1+\chi)(1-\hat{\theta}) < \beta_I(1+\chi\hat{\phi})$, the equilibrium is locally indeterminate; and

(iii) when $\beta_I(1+\chi\hat{\phi}) < (1+\chi)(1-\hat{\theta})$, the equilibrium is locally determinate, where $0 < \Delta(\hat{l}) \equiv [\tau_{Ix}a_I - (1-\hat{\theta})]\hat{\theta}^{-1} < 1$.

To understand the intuition behind Proposition 1, we first compute the difference between the slopes of the $\dot{k}/k$ and $\dot{c}/c$ curves at the BGP:

$$\left.\frac{d(\dot{k}/k)}{dl}\right|_{l=\hat{l}} - \left.\frac{d(\dot{c}/c)}{dl}\right|_{l=\hat{l}} = [(1-\hat{\phi})\hat{l}]^{\beta_I-1}\hat{\theta}\left[(1+\chi)(1-\hat{\theta}) - \beta_I(1+\chi\hat{\phi})\Delta(\hat{l})\right], \tag{27}$$

where it follows from (16), (18), (3), and $\dot{p}_I/p_I|_{l=\hat{l}} = 0$ that

$$\left.\frac{\dot{c}}{c}\right|_{l=\hat{l}} = -\left.\frac{\dot{q}}{q}\right|_{l=\hat{l}} = \tau_{Ix}a_I[(1-\hat{\phi})\hat{l}]^{\beta_I} - (\rho+\delta), \tag{28}$$

Note, moreover, that the right-hand side of (27) coincides with the numerator of (26). In other words, when $(1+\chi)(1-\hat{\theta}) > \beta_I(1+\chi\hat{\phi})\Delta(\hat{l})$, i.e., the sign of (27) is positive, the $\dot{k}/k$ curve cuts the $\dot{c}/c$ curve from below, as illustrated in Figs. 3, 4, 6, and 8. When $(1+\chi)(1-\hat{\theta}) < \beta_I(1+\chi\hat{\phi})\Delta(\hat{l})$, i.e., the numerator in (26) is negative, the $\dot{k}/k$ curve cuts the $\dot{c}/c$ curve from above, as illustrated in Figs. 5, 7, and 9.[5]

On the other hand, we can interpret the sign of the denominator in (26) using the labor market equilibrium condition. We compute the elasticity of the Frisch labor supply curve implicitly defined by (17), evaluated at the BGP, to get[6]

$$\left(\left.\frac{d\ln w}{d\ln l}\right|_{l=\hat{l}}\right)^S - \left.\frac{d\ln p_I}{d\ln l}\right|_{l=\hat{l}} = \chi, \tag{29}$$

given the fixed level of q. The Frisch labor supply curve is positively sloping. Using (4), (2), and the definitions of θ and ϕ, we obtain the *reduced-from* labor demand curve in the investment sector given by $w = [(1-T_{Ix})/(1+T_{IL})]p_I b_I(1-\theta)k[(1-\phi)l]^{\beta_I-1}$. We take the logarithm of both sides of this expression to get

$$\ln w = \mathit{const} + \ln(1-\theta) + (\beta_I - 1)\left[\ln(1-\phi) + \ln l\right] + \ln p_I + \ln k,$$

where the term *const* represents the constant terms. The real wage elasticity, evaluated at the BGP, is given by the substitution of (A.1) and (A.2), and

[5] Since there may be multiple BGPs, these figures depict these curves in the neighborhood of the respective BGPs.

[6] According to Bennett and Farmer (2000), the Frisch labor supply curve implies labor supply as a function of the real wage, holding constant the marginal utility of consumption. In contrast, in our two-sector model, we take the shadow price of capital q rather than the marginal utility of consumption $1/c$ as given since the relative price of investment p_I is a function only of labor supply.

yields

$$\left(\left.\frac{d\ln w}{d\ln l}\right|_{l=\hat{l}}\right)^{D} - \left.\frac{d\ln p_I}{d\ln l}\right|_{l=\hat{l}} = \frac{(1+\chi)\hat{\theta} - (1-\beta_I)(1+\chi\hat{\phi})}{1-\hat{\phi}}, \tag{30}$$

given the fixed level of k. The difference between (29) and (30) yields

$$\begin{aligned}\left(\left.\frac{d\ln w}{d\ln l}\right|_{l=\hat{l}}\right)^{S} - \left(\left.\frac{d\ln w}{d\ln l}\right|_{l=\hat{l}}\right)^{D} &= \frac{(1+\chi)(1-\hat{\theta}) - \beta_I(1+\chi\hat{\phi})}{1-\hat{\phi}}, \\ &= \frac{1-\hat{\theta}}{1-\hat{\phi}}\left[\chi - \left\{\frac{\beta_I(1+\chi\hat{\phi})}{1-\hat{\theta}} - 1\right\}\right],\end{aligned}$$

the second expression of which coincides with the denominator in (26). Since the term $\{\beta_I(1+\chi\hat{\phi})/(1-\hat{\theta})\}-1$ in the third expression stands for the slope of the *reduced* labor demand curve of sector I, the relative slopes of the Frisch labor supply and labor demand curves are determined according to the sign of the term $\chi-[\{\beta_I(1+\chi\hat{\phi})/(1-\hat{\theta})\}-1]$; i.e., if $\chi < \{\beta_I(1+\chi\hat{\phi})/(1-\hat{\theta})\}-1$, the slope of the labor demand curve exceeds that of the Frisch labor supply curve as illustrated in Fig. 1 (which we call the *wrong* sloping case), and vice versa (which we call the *normal* sloping case). When $\hat{\theta} = \hat{\phi} = 0$, this condition reduces to $\chi + 1 < \beta_I$, which corresponds to a necessary condition for indeterminacy found by Benhabib and Farmer (1994) where they allow β_I to be greater than unity in their one-sector model.

Further insight can be gained by examining the following discrete time version of the Euler equation (28):

$$\frac{c_{t+1}}{c_t} = \frac{1}{1+\rho}\left[\frac{r(l_{t+1})}{p_I(l_t)} + (1-\delta)\frac{p_I(l_{t+1})}{p_I(l_t)}\right]. \tag{31}$$

For expositional purposes, we focus on case (ii) in Proposition 1 where indeterminacy arises due to the *wrong* sloping of the labor demand and supply curves. We suppose that agents optimistically expect a sudden rise in the future return to capital (including capital gains). The resulting increase in savings immediately curtails current consumption, c_t, which in turn raises its marginal utility, thus making the left-hand side of (31) larger. The decrease in c_t shifts the Frisch labor supply curve to the right due to the normality assumption. Hence, the equilibrium level of employment l_t falls as shown in Fig. 1.

In period $t+1$, tomorrow's capital stock k_{t+1} increases as a result of the increased savings, which ends up expanding the production of the consumption good c_{t+1}, thus further increasing the left-hand side of (31). The increase in c_{t+1}, on the other hand, shifts the labor supply curve to the left due to the normality assumption, while the increased k_{t+1} shifts the labor demand curve to the right, thus leading to a large rise in employment, l_{t+1}, as illustrated in Fig. 2. Although $p_I(l_{t+1})$ falls, it follows from (28) that $r(l_{t+1})/p_I(l_t)$ may rise enough to outweigh the reduction in $p_I(l_{t+1})/p_I(l_t)$, which makes the equality of (31) possible.

Accordingly, the $\dot{c}/c$ curve shifts upwards in response to a sudden rise in the future return to capital due to optimistic beliefs; this upward shift brings about more employment when the slope of the $\dot{k}/k$ curve is greater than that of the $\dot{c}/c$ curve, as illustrated in Fig. 3. The higher level of employment is consistent with the increased amount of savings and thus we obtain a higher BGP growth rate at point E_2, because larger employment will expand the production of the investment sector.

Harrison (2001) and Harrison and Weder (2002) pointed out that externalities in the consumption sector do not affect the stability properties when utility is logarithmic. This observation holds true to our model as well, because we have also assumed the same logarithmic utility function. According to their exposition, as returns to scale in the consumption sector rise, households want to smooth consumption due to risk aversion, while they might prefer volatile consumption to take advantage of the increasing returns to scale. With logarithmic utility, these opposing effects exactly cancel out each other.

Furthermore, we conduct a numerical analysis to examine the empirical plausibility of indeterminacy for our model economy. We first adopt the following parameter values suggested by Harrison (2001).

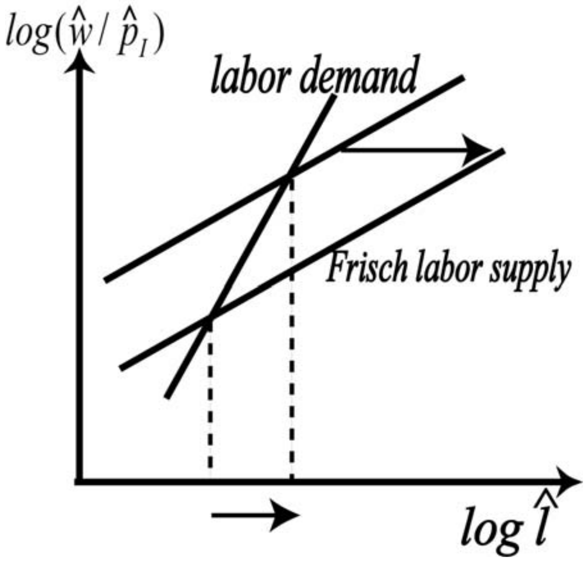

Figure 1: Wrong sloping in period t

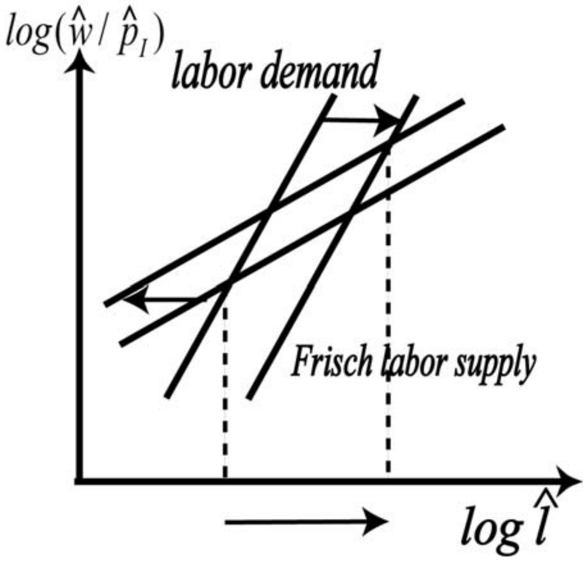

Figure 2: Wrong sloping in period $t + 1$

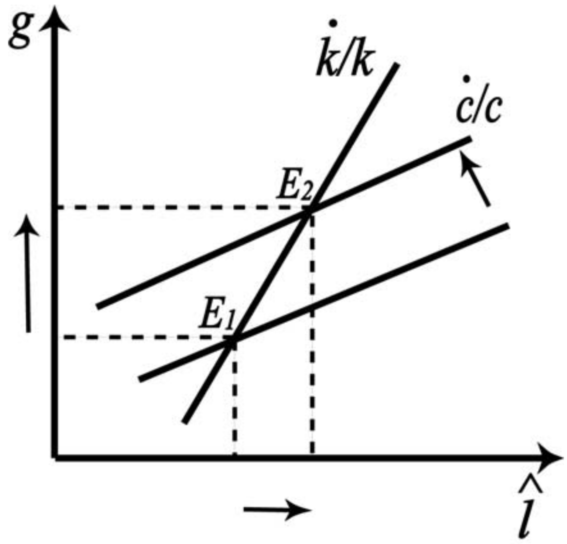

Figure 3: Sudden rise in the future return to capital due to optimistic beliefs

Variable	δ	ρ	χ	$\beta_I = \beta_c$	$a_I = a_c$	$b_I = b_c$
Calibrated value	0.025	0.01	0	0.754	0.3	0.7

Table 1: Harrison's (2001) model

Since there are many combinations for tax parameter values, we will focus on several typical combinations of taxes such as in Table 2. Under these calibrated values and given $\chi = 0$, Table 3 shows the values of $\hat{l}$; $\hat{g}$ (the growth rate of $\dot{k}/k$ evaluated along the BGP); the numerator and denominator of the right-hand side of (26), evaluated at the BGP; and whether indeterminacy arises or not:

	T_{Ix}	T_{IL}	$T_{I\pi}$	T_{cx}	T_{cL}	$T_{c\pi}$
Case 1	0	0	0	0	0	0
Case 2	0.10	0.28	0.28	0.10	0.28	0.28
Case 3	0.10	0.28	0.28	0	0	0
Case 4	0	0	0	0.10	0.28	0.28
Case 5	0.10	0	0	0.10	0	0
Case 6	0	0	0.28	0	0	0.28
Case 7	0	0.28	0	0	0.28	0

Table 2: Tax rates

Tax rate	$\hat{l}$	$\hat{g}$	numerator	denominator	result
Case 1	0.962	0.074	0.244	-0.481	Inder.
	0.712	-0.024	-0.201	-0.737	Deter.
Case 2	0.556	-0.011	0.047	-0.639	Inder.
	0.525	-0.020	-0.045	-0.692	Deder.
Case 3	0.832	0.007	0.108	-0.606	Inder.
	0.725	-0.023	-0.095	-0.723	Deter.
Case 4	0.652	0.040	0.219	-0.494	Inder.
	0.505	-0.024	-0.185	-0.727	Deter.
Case 5	0.824	0.043	0.202	-0.518	Inder.
	0.644	-0.024	-0.169	-0.732	Deter.
Case 6	0.847	0.016	0.135	-0.580	Inder.
	0.720	-0.023	-0.117	-0.726	Deter.
Case 7	0.745	0.054	0.231	-0.488	Inder.
	0.559	-0.024	-0.193	-0.732	Deter.

Table 3: $\chi = 0$

Note that the "numerator" and "denominator" in Tables 3 and 4 represent the numerator and denominator of the second term on the right-hand side of

(26), respectively.

Next, we use the same parameter values as those listed in Tables 1 and 2 except for $\chi = 1$. The result is summarized in Table 4. In light of Tables 3 and 4, the present calibration analysis shows that indeterminacy is likely to arise under plausible empirical parameter values.

Tax rate	$\hat{l}$	$\hat{g}$	numerator	denominator	result
Case 1	0.981	0.076	0.498	−0.756	Inder.
	0.843	−0.025	−0.407	−1.470	Deter.
Case 2	0.757	0.001	0.192	−1.122	Inder.
	0.714	−0.023	−0.172	−1.409	Deder.
Case 3	0.915	0.001	0.236	−1.080	Inder.
	0.849	−0.023	−0.206	−1.436	Deter.
Case 4	0.811	0.055	0.474	−0.785	Inder.
	0.708	−0.024	−0.392	−1.459	Deter.
Case 5	0.909	0.055	0.421	−0.852	Inder.
	0.801	−0.024	−0.383	−1.482	Deter.
Case 6	0.922	0.020	0.288	−1.021	Inder.
	0.847	−0.024	−0.247	−1.444	Deter.
Case 7	0.866	0.065	0.487	−0.764	Inder.
	0.745	−0.024	−0.400	−1.464	Deter.

Table 4: $\chi = 1$

4 Tax effects on employment

In this section, we first examine the long-run impacts of the changes in the respective selective tax rates on employment along the BGP of the endogenous growth model presented in the previous section. Since, in the endogenous growth models, consumption and the stock of capital both grow indefinitely, we cannot analyze the impacts of tax changes on these economic aggregates. Instead, we first identify the tax effects on employment, and then on the growth rate of the BGP in the next section.

Substituting (21) and (22) into (25), we totally differentiate the newly found expression with respect to each tax rate to get[7]

$$\frac{d\hat{l}}{dT_{Ix}} = -\frac{\hat{l}}{1-T_{Ix}}\frac{\tau_{Ix}a_I(1-\hat{\phi})}{\hat{\theta}N(\hat{l})}, \tag{32}$$

$$\frac{d\hat{l}}{dT_{IL}} = -\frac{\hat{l}}{1+T_{IL}}\frac{(1-\hat{\theta})(1-\hat{\phi})}{N(\hat{l})}, \tag{33}$$

$$\frac{d\hat{l}}{dT_{I\pi}} = -\frac{\hat{l}}{1-T_{I\pi}}\frac{[\tau_{Ix}a_I-\hat{\theta}(1-\hat{\theta})](1-\hat{\phi})}{\hat{\theta}N(\hat{l})}, \tag{34}$$

$$\frac{d\hat{l}}{dT_{cx}} = -\frac{\hat{l}}{1-T_{cx}}\frac{(1+\chi)(1-\hat{\theta})-\beta_I(1+\chi)\hat{\phi}\Delta(\hat{l})}{(1+\chi)\,N(\hat{l})}, \tag{35}$$

$$\frac{d\hat{l}}{dT_{cL}} = -\frac{\hat{l}\hat{\phi}}{1+T_{cL}}\frac{(1+\chi)\,(1-\hat{\theta})-\beta_I\,(1+\chi)\,\Delta(\hat{l})}{(1+\chi)\,N(\hat{l})}, \tag{36}$$

$$\frac{d\hat{l}}{dT_{c\pi}} = -\frac{\hat{l}}{1-T_{c\pi}}\frac{(1-\hat{\theta})(1-\hat{\phi})}{N(\hat{l})}, \tag{37}$$

where $N(\hat{l}) \equiv (1+\chi)(1-\hat{\theta})-\beta_I(1+\chi\hat{\phi})\Delta(\hat{l}) \gtreqless 0$ (which corresponds to the numerator of (26) or the relative slopes of the $\dot{c}/c$ and $\dot{k}/k$ curves given by (27)) and $0 < \Delta(\hat{l}) \equiv [\tau_{Ix}a_I-(1-\hat{\theta})]/\hat{\theta} < 1$. It can be seen that the effects of the changes in T_{Ix}, T_{IL}, $T_{I\pi}$ and $T_{c\pi}$ on the BGP level of employment depend critically on the sign of $N(\hat{l})$, whereas those of T_{cx} and T_{cL} do not.

The following gives a summary of the above:

Proposition 2 *(i) When* $(1+\chi)(1-\hat{\theta}) < \beta_I(1+\chi\hat{\phi})\Delta(\hat{l})$, *the equilibrium is locally determinate and*

$$\frac{d\hat{l}}{dT_{Ix}}, \frac{d\hat{l}}{dT_{IL}}, \frac{d\hat{l}}{dT_{I\pi}}, \frac{d\hat{l}}{dT_{c\pi}} > 0, \ \frac{d\hat{l}}{dT_{cL}} < 0, \ \frac{d\hat{l}}{dT_{cx}} \gtreqless 0;$$

(ii) when $\beta_I(1+\chi\hat{\phi})\Delta(\hat{l}) < (1+\chi)(1-\hat{\theta}) < \beta_I(1+\chi\hat{\phi})$, *the equilibrium*

[7] Detailed derivations are available from the corresponding author upon request.

is locally indeterminate and

$$\frac{d\hat{l}}{dT_{Ix}}, \frac{d\hat{l}}{dT_{IL}}, \frac{d\hat{l}}{dT_{I\pi}}, \frac{d\hat{l}}{dT_{cx}}, \frac{d\hat{l}}{dT_{c\pi}} < 0, \frac{d\hat{l}}{dT_{cL}} \gtreqless 0; \text{ and}$$

(iii) when $\beta_I(1+\chi\hat{\phi}) < (1+\chi)(1-\hat{\theta})$, *the equilibrium is locally determinate and*

$$\frac{d\hat{l}}{dT_{Ix}}, \frac{d\hat{l}}{dT_{IL}}, \frac{d\hat{l}}{dT_{I\pi}}, \frac{d\hat{l}}{dT_{cx}}, \frac{d\hat{l}}{dT_{c\pi}} < 0, \frac{d\hat{l}}{dT_{cL}} \gtreqless 0.$$

5 Growth effects of taxation

In this section, we investigate the impacts of the changes in the various selective taxes on the long-run growth rate. Substituting (21) and (22) into $\hat{\phi}$ and $\hat{\theta}$ in (13) along the BGP, setting $\hat{g} \equiv \dot{k}/k$ in (13), and rearranging, we can arrive at the following expression:

$$\hat{g} = \left[1 - \tau_{cL}a_c b_I \left\{\tau_{cL}a_c b_I + \tau_{IL}b_c a_I \left(\frac{1}{b_c}\frac{\tau_{cL}}{\tau_{cx}}\hat{l}^{1+\chi} - 1\right)\right\}^{-1}\right] \times \left(\hat{l} - \frac{\tau_{cx}}{\tau_{cL}}b_c\hat{l}^{-\chi}\right)^{\beta_I} - \delta. \tag{38}$$

Differentiating (38) with respect to each tax rate, (32) – (37), and manipulating yields[8]

$$\frac{d\hat{g}}{dT_{Ix}} = -\frac{\hat{g}+\delta}{1-T_{Ix}}\frac{\tau_{Ix}a_I[(1+\chi)\hat{\theta} + \beta_I(1+\chi\hat{\phi})]}{\hat{\theta}N(\hat{l})},$$

$$\frac{d\hat{g}}{dT_{IL}} = -\frac{\hat{g}+\delta}{1+T_{IL}}\frac{\tau_{Ix}a_I\beta_I(1+\chi\hat{\phi})}{N(\hat{l})},$$

$$\frac{d\hat{g}}{dT_{I\pi}} = -\frac{\hat{g}+\delta}{1-T_{I\pi}}\frac{\tau_{Ix}a_I[(1+\chi)\hat{\theta} + \beta_I(1+\chi\hat{\phi})(1-\hat{\theta})]}{\hat{\theta}N(\hat{l})},$$

$$\frac{d\hat{g}}{dT_{cx}} = -\frac{\hat{g}+\delta}{1-T_{cx}}\frac{\tau_{Ix}a_I\beta_I}{N(\hat{l})},$$

[8] Detailed derivations are available from the corresponding author upon request.

$$\frac{d\hat{g}}{dT_{cL}} = \frac{\hat{g}+\delta}{1+T_{cL}} \frac{\tau_{Ix} a_I \beta_I \chi \hat{\phi}}{N(\hat{l})},$$

$$\frac{d\hat{g}}{dT_{c\pi}} = -\frac{\hat{g}+\delta}{1-T_{c\pi}} \frac{\tau_{Ix} a_I \beta_I (1+\chi \hat{\phi})}{N(\hat{l})},$$

noting from (13) that $\hat{g}+\delta = y_I/k = (1-\hat{\theta})[(1-\hat{\phi})\hat{l}]^{\beta_I}$.

These results show that the growth effects of the respective taxes hinge solely on the sign of $N(\hat{l})$, unlike the effects of the taxes on employment discussed in the previous section. Combined with the results of Proposition 2, we can derive the following results:

Proposition 3 *(i) When* $(1+\chi)(1-\hat{\theta}) < \beta_I(1+\chi\hat{\phi})\Delta(\hat{l})$, *the equilibrium is locally determinate and*

$$\frac{d\hat{g}}{dT_{Ix}}, \frac{d\hat{g}}{dT_{IL}}, \frac{d\hat{g}}{dT_{I\pi}}, \frac{d\hat{g}}{dT_{cx}}, \frac{d\hat{g}}{dT_{c\pi}} > 0, \; \frac{d\hat{g}}{dT_{cL}} < 0;$$

(ii) when $\beta_I(1+\chi\hat{\phi})\Delta(\hat{l}) < (1+\chi)(1-\hat{\theta}) < \beta_I(1+\chi\hat{\phi})$, *the equilibrium is locally indeterminate and*

$$\frac{d\hat{g}}{dT_{Ix}}, \frac{d\hat{g}}{dT_{IL}}, \frac{d\hat{g}}{dT_{I\pi}}, \frac{d\hat{g}}{dT_{cx}}, \frac{d\hat{g}}{dT_{c\pi}} < 0, \; \frac{d\hat{g}}{dT_{cL}} > 0; \textit{ and}$$

(iii) when $\beta_I(1+\chi\hat{\phi}) < (1+\chi)(1-\hat{\theta})$, *the equilibrium is locally determinate, and*

$$\frac{d\hat{g}}{dT_{Ix}}, \frac{d\hat{g}}{dT_{IL}}, \frac{d\hat{g}}{dT_{I\pi}}, \frac{d\hat{g}}{dT_{cx}}, \frac{d\hat{g}}{dT_{c\pi}} < 0, \; \frac{d\hat{g}}{dT_{cL}} > 0.$$

In short, in the determinate equilibrium, higher rates of the selective taxes may or may not harm long-run growth ((i) and (iii) in Proposition 3), whereas in the indeterminate equilibrium, higher rates of the taxes unambiguously depress economic growth except for the selective payroll tax imposed on the consumption sector ((ii) in Proposition 3). Most notably, the selective taxes imposed on the investment sector may **not** harm long-run growth, as shown in (i) in Proposition 3, which stands in sharp contrast with the results of Devereux and Love (1994), and Milesi-Ferretti and Roubini (1998).

If T_{Ix} is increased, then the after-tax marginal product of capital in sector I instantaneously falls, which in turn depresses the after-tax rate of real return to capital, r/p_I, in (3). As seen from (28) and (13), this reduction in r/p_I immediately shifts the $\dot{c}/c$ curve downwards, while leaving the $\dot{k}/k$ curve unchanged. The latter property stems from the fact that there are no instantaneous intersectoral reallocations of labor and capital, since these changes do not alter the after-tax relative costs of production (the factor substitution effect does not work);[9] consequently, θ and ϕ both remain unchanged at the moment when T_{Ix} is increased. As a result, when the $\dot{k}/k$ curve cuts the $\dot{c}/c$ curve from below (i.e., $\beta_I(1+\chi\hat{\phi})\Delta(\hat{l}) < (1+\chi)(1-\hat{\theta})$), as illustrated in Fig. 4, the growth rate $\dot{k}/k$ will be greater than $\dot{c}/c$ at the initial BGP level of employment at point E_1. To achieve the new BGP (point E_2), the level of employment has to fall, and along with it the growth rate. Although the growth rate $\dot{c}/c$ is further reduced by the decrease in the return to capital (3) in response to the lower l, $\dot{k}/k$ will fall much as compared to the reduction in the growth rate $\dot{c}/c$ since the output elasticity of sector I with respect to l is larger than the elasticity of the real interest rate with respect to l. This corresponds to (ii) and (iii) in Proposition 3.

By contrast, when the $\dot{k}/k$ curve cuts the curve $\dot{c}/c$ from above (i.e., $(1+\chi)(1-\hat{\theta}) < \beta_I(1+\chi\hat{\phi})\Delta(\hat{l})$), the downward shift of the curve $\dot{c}/c$ entails higher employment and a larger growth rate to achieve the new BGP (point E_2) in Fig. 5. Since the output elasticity of sector I with respect to l is less than the elasticity of the real interest rate with respect to l, employment has to rise. This corresponds to (i) in Proposition 3.[10]

If $T_{I\pi}$ increases, the after-tax marginal product of capital in sector I falls, and so does r/p_I in (18). This reduction causes the $\dot{c}/c$ curve to shift

[9] The concepts of *output substitution* and *factor substitution* effects have been frequently used in the literature on tax incidence (see, e.g., Mieszkowski (1967), Homma (1986)). The output substitution effect is the impact of tax changes on the demand function for consumption goods (investment goods) through changes in the relative price, while the factor substitution effect is the impact of tax changes through alternations in the after-tax relative cost between capital and labor, thereby affecting the demands for capital and labor in the respective sectors.

[10] This case allows only for the wrong sloping case because $(1+\chi)(1-\hat{\theta}) < \beta_I(1+\chi\hat{\phi})$ is automatically satisfied, and therefore the equilibrium path must be determinate.

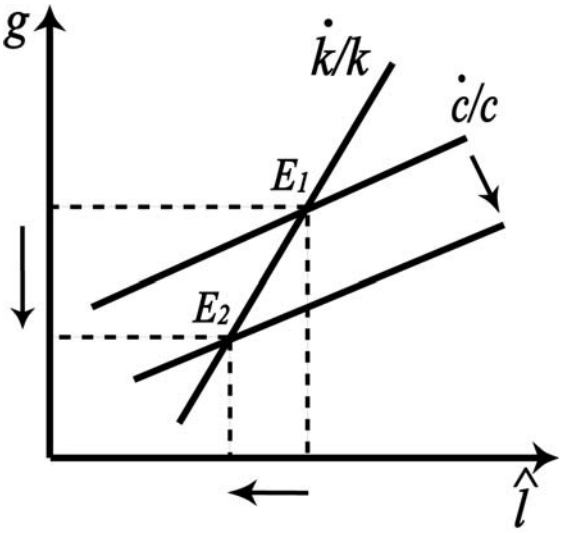

Figure 4: Effect of an increase in T_{Ix} if the $\dot{k}/k$ curve cuts the $\dot{c}/c$ curve from below.

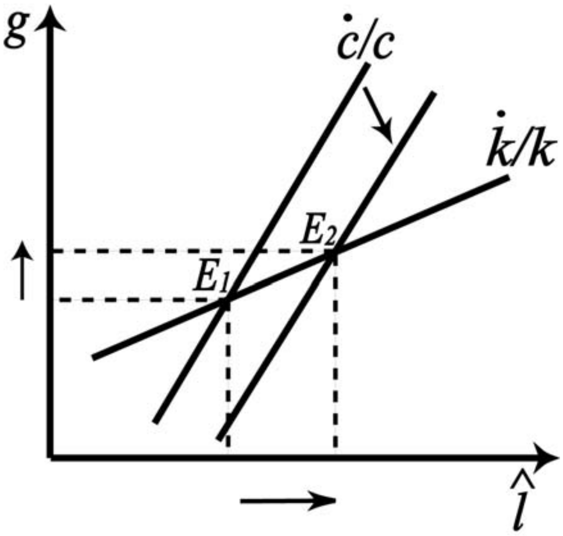

Figure 5: Effect of an increase in T_{Ix} if the $\dot{k}/k$ curve cuts the $\dot{c}/c$ curve from above.

downwards as in the increase in T_{Ix}. At the same time, since the after-tax marginal product of capital in sector I falls short of that in sector c, p_I has to rise immediately to resort to the equality between these returns in (3). In addition, the increase in p_I drives up the after-tax marginal product of labor in sector I. Nevertheless, since it follows from (21) that the reallocation of labor between the two sectors never arises ($\hat{\phi}$ remains invariant) as long as l is fixed, k_I has to decrease to resort to the equality of the after-tax marginal products of labor between the two sectors in (4); consequently, $\hat{\theta}$ will increase. These instantaneous impacts unambiguously shrink the output of sector I, which causes the curve $\dot{k}/k$ to shift downwards. Hence, both $\dot{k}/k$ and $\dot{c}/c$ curves end up moving downwards, as shown in Figs. 6 and 7.

When the $\dot{k}/k$ curve cuts the $\dot{c}/c$ curve from below (i.e., $(1+\chi)(1-\hat{\theta}) > \beta_I(1+\chi\hat{\phi})\Delta(\hat{l})$) as shown in Fig. 6, the BGP level of employment and thus the growth rate must fall to achieve the new BGP. Note also that under condition $\tau_{Ix}a_I > 1-\hat{\theta}$, the downward shift of the $\dot{c}/c$ curve is larger than that of the $\dot{k}/k$ curve (see (13) and (28)). This corresponds to (ii) and (iii) in Proposition 3. In contrast, when the $\dot{k}/k$ curve cuts the $\dot{c}/c$ curve from above (i.e., $(1+\chi)(1-\hat{\theta}) < \beta_I(1+\chi\hat{\phi})\Delta(\hat{l})$) in Fig. 7, the BGP level of employment must rise, which corresponds to (i) in Proposition 3.

If T_{cL} increases, the after-tax cost of a unit of labor employed in sector c also increases; consequently, labor immediately moves from sector c to sector I (the factor substitution effect comes into play) and therefore $\hat{\phi}$ falls. Moreover, since the reduction in l_c lowers the marginal product of capital in sector c, p_I has to fall to resort to the equality of (3), given the instantaneously fixed l. As a result, the marginal product of labor in sector c is larger than that in sector I, and k_I has to rise to achieve the equality of (4), which expands the production of sector I. These impacts cause both the $\dot{k}/k$ and $\dot{c}/c$ curves to shift upwards (recall (13) and (28)). When the $\dot{k}/k$ curve cuts the $\dot{c}/c$ curve from below (i.e., $(1+\chi)(1-\hat{\theta}) > \beta_I(1+\chi\hat{\phi})\Delta(\hat{l})$), the level of employment may fall or rise at the new BGP but the growth rate unambiguously rises, although the upward shift of the $\dot{c}/c$ curve is larger than that of the $\dot{k}/k$ curve because of condition $\tau_{Ix}a_I > 1-\hat{\theta}$. This is illustrated in Fig. 8, and corresponds to (ii) and (iii) in Proposition 3. When the $\dot{k}/k$ curve cuts the

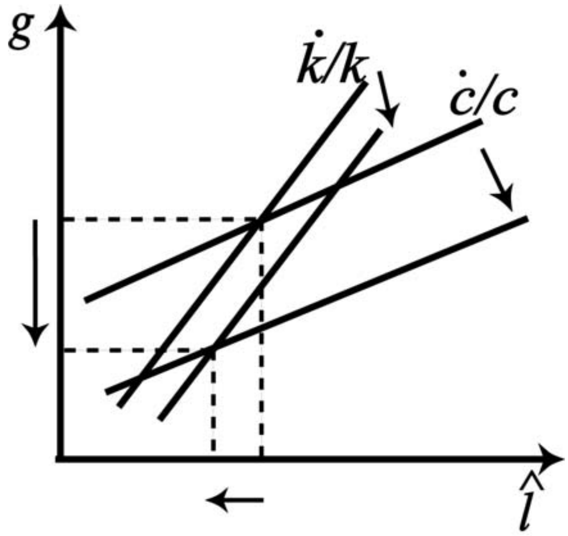

Figure 6: Effect of an increase in $T_{I\pi}$ if the $\dot{k}/k$ curve cuts the $\dot{c}/c$ curve from below.

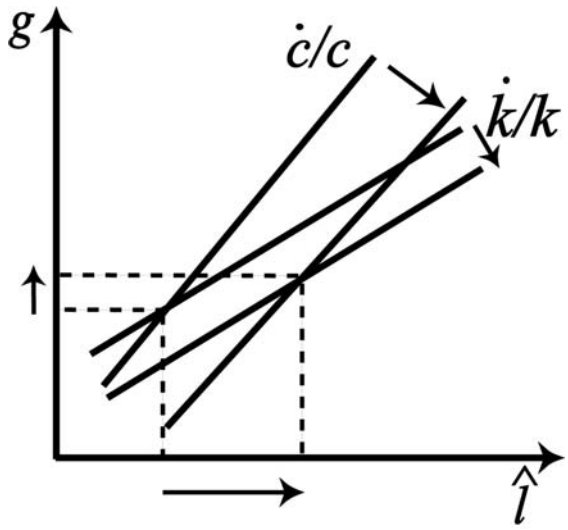

Figure 7: Effect of an increase in $T_{I\pi}$ if the $\dot{k}/k$ curve cuts the $\dot{c}/c$ curve from above.

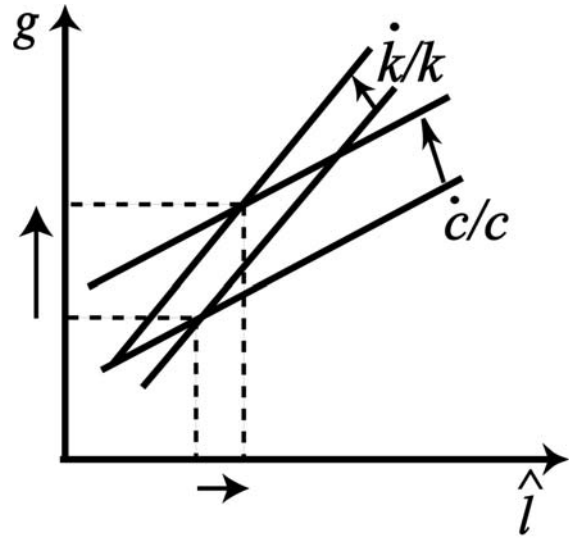

Figure 8: Effect of an increase in T_{cL} if the $\dot{k}/k$ curve cuts the $\dot{c}/c$ curve from below.

$\dot{c}/c$ curve from above (i.e., $(1+\chi)(1-\hat{\theta}) < \beta_I(1+\chi\hat{\phi})\Delta(\hat{l})$), the opposite occurs and the level of employment (as well as the growth rate) unambiguously falls as illustrated in Fig.9. This corresponds to (i) in Proposition 3.

6 Two variations

6.1 CRRA preferences

We have found that when the instantaneous utility function is logarithmic in consumption, the local stability properties are completely independent of the size of the externalities in sector c. In this subsection, we explore the robustness of this finding under more general preferences. Following King, Plosser and Rebelo (1988), we use a more general CRRA utility function:

$$\int_0^\infty \frac{\left[c\exp\left\{-l^{1+\chi}/(1+\chi)\right\}\right]^{1-\sigma}-1}{1-\sigma}e^{-\rho t}dt, \tag{39}$$

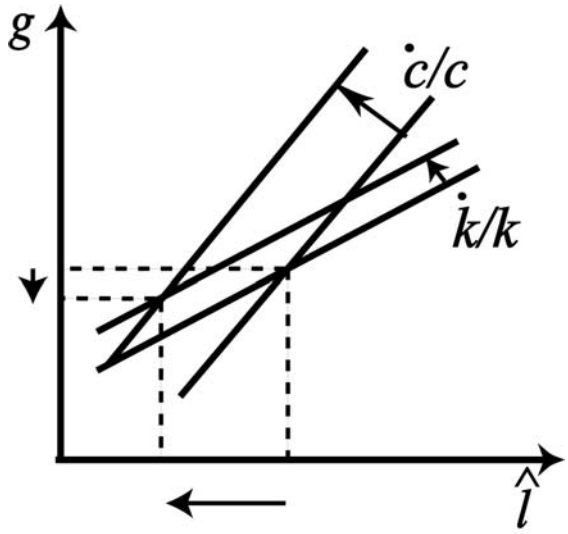

Figure 9: Effect of an increase in T_{cL} if the $\dot{k}/k$ curve cuts the $\dot{c}/c$ curve from above.

where σ^{-1} stands for the intertemporal elasticity of substitution in consumption. Note that (39) reduces to (14) by setting $\sigma = 1$.

In an analogous manner, we can derive the following one-dimensional linearized dynamic system in l:

$$\frac{\dot{l}}{l} = \Omega(\hat{l})(l - \hat{l})\Gamma(\hat{l}), \tag{40}$$

where $\Gamma(\hat{l}) \equiv$

$$\frac{\sigma(1+\chi)(1-\hat{\theta}) - \beta_I(1+\chi\hat{\phi})[\tau_{Ix}a_I - \sigma(1-\hat{\theta})]\hat{\theta}^{-1}}{(1+\chi)(1-\hat{\theta}) - \beta_I(1+\chi\hat{\phi}) - (1-\sigma)[(1+\chi)(1-\hat{\theta}) + (1-\hat{\phi})(\chi\beta_c + \hat{l}^{1+\chi})]}.$$

When $\sigma = 1$, (40) simplifies to (26). In this case, the numerator of the term $\Gamma(\hat{l})$ is the same as that of (26), while the denominator of (26) does not contain β_c. This comparison reveals that indeterminacy is caused by the externalities in sector c as well as those in sector I, unlike in the previous model with $\sigma = 1$.

Furthermore, we can interpret the sign of the denominator within the term $\Gamma(\hat{l})$ using the labor market equilibrium condition as before. The Frisch labor supply curve is modified as follows:

$$w = \left[\theta k \left(\phi l\right)^{\beta_c} \exp\left(\frac{-l^{1+\chi}}{1+\chi}\right)\right]^{1-\sigma} \frac{p_I}{q} l^{\chi},$$

where the above expression follows from the substitution of the last expression in (12) into c. Its elasticity, evaluated at the BGP, is

$$\left(\left.\frac{d\ln w}{d\ln l}\right|_{l=\hat{l}}\right)^S - \left.\frac{d\ln p_I}{d\ln l}\right|_{l=\hat{l}} = \chi - (1-\sigma)\left[\frac{(1+\chi)(1-\hat{\theta})}{1-\hat{\phi}} + \beta_c\chi + \hat{l}^{1+\chi}\right], \tag{41}$$

given a fixed value of q. Taking the difference between the elasticity of the labor demand curve in sector I, i.e., (30) and (41), yields

$$\left(\left.\frac{d\ln w}{d\ln l}\right|_{l=\hat{l}}\right)^S - \left(\left.\frac{d\ln w}{d\ln l}\right|_{l=\hat{l}}\right)^D = (1-\hat{\phi})^{-1}\Big[(1+\chi)(1-\hat{\theta})$$
$$-\beta_I(1+\chi\hat{\phi}) - (1-\sigma)[(1+\chi)(1-\hat{\theta}) + (1-\hat{\phi})(\beta_c\chi + \hat{l}^{1+\chi})\Big],$$

whose square-bracketed term on the right-hand side coincides with the denominator of $\Gamma(\hat{l})$ in (40). As before, this denominator represents the relative slopes of the Frisch labor supply curve and labor demand curve in sector I. Unlike in the previous model, since the presence of externalities in sector c alters not only the elasticity of the labor supply curve (41) through consumption but also the stability properties, the inverse of the intertemporal elasticity of substitution in consumption, $1/\sigma$, is equal to unity.

As in the derivation of (25), we can obtain the following BGP condition:

$$\rho + (1-\sigma)\delta = [\tau_{Ix}a_I - \sigma\{1-\theta(\hat{l})\}][\{1-\phi(\hat{l})\}\hat{l}]^{\beta_I}. \tag{42}$$

After substituting (21) and (22) into (42), we totally differentiate the newly found expression with respect to the respective selective taxes to get their impacts on employment in the BGP. Using these results, we further differentiate

(38) with respect to each tax rate to get

$$\frac{d\hat{g}}{dT_{Ix}} = -\frac{\hat{g}+\delta}{1-T_{Ix}} \frac{\tau_{Ix} a_I [(1+\chi)\hat{\theta} + \beta_I (1+\chi\hat{\phi})]}{\hat{\theta} M(\hat{l})},$$

$$\frac{d\hat{g}}{dT_{IL}} = -\frac{\hat{g}+\delta}{1+T_{IL}} \frac{\tau_{Ix} a_I \beta_I (1+\chi\hat{\phi})}{M(\hat{l})},$$

$$\frac{d\hat{g}}{dT_{I\pi}} = -\frac{\hat{g}+\delta}{1-T_{I\pi}} \frac{\tau_{Ix} a_I [(1+\chi)\hat{\theta} + \beta_I (1+\chi\hat{\phi})(1-\hat{\theta})]}{\hat{\theta} M(\hat{l})},$$

$$\frac{d\hat{g}}{dT_{cx}} = -\frac{\hat{g}+\delta}{1-T_{cx}} \frac{\tau_{Ix} a_I \beta_I}{M(\hat{l})},$$

$$\frac{d\hat{g}}{dT_{cL}} = \frac{\hat{g}+\delta}{1+T_{cL}} \frac{\tau_{Ix} a_I \beta_I \chi\hat{\phi}}{M(\hat{l})},$$

$$\frac{d\hat{g}}{dT_{c\pi}} = -\frac{\hat{g}+\delta}{1-T_{c\pi}} \frac{\tau_{Ix} a_I \beta_I (1+\chi\hat{\phi})}{M(\hat{l})},$$

where $M(\hat{l}) \equiv \sigma(1+\chi)(1-\hat{\theta}) - \beta_I(1+\chi\hat{\phi})[\tau_{Ix}a_I - \sigma(1-\hat{\theta})]/\hat{\theta} \gtreqless 0$, which corresponds to the numerator of $\Gamma(\hat{l})$. Note first that when $\sigma = 1$, the above results simply reduce to their corresponding counterparts in the previous model since $M(\hat{l}) = N(\hat{l})$. Moreover, the effects of the respective taxes on the long-run growth rate is *qualitatively* the same as those in Proposition 3, except that the threshold values determining the comparative statics results are replaced by the conditions determining the signs of $M(\hat{l})$ instead of $N(\hat{l})$ in Section 5. This result guarantees the robustness of Proposition 3 with respect to the various non-unity values of the intertemporal consumption elasticity.

6.2 Non-endogenous growth model

In this subsection, we explore how the stability properties of the model are related to the growth effects of the selective taxes in an *exogenous* growth model. In this model, the social production function in each sector displays decreasing returns-to-scale technology $y_j = k_j^{\alpha_j} l_j^{\beta_j}$ with $\alpha_j, \beta_j < 1$, $j = c, I$.

Since the steady-state growth rate is exogenously fixed (i.e., equal to zero) in this model, we focus on the tax effects on the steady-state capital stock rather than on the growth rate.

Setting $\dot{q} = 0$ and $\dot{k} = 0$ in (18) and (13); $y_I = k_I^{\alpha_I} l_I^{\beta_I}$; and (3) in sector I, give the following steady-state conditions:

$$\rho + \delta = \tau_{Ix} a_I [\{1 - \theta(l(\hat{k}, \hat{q}))\}\hat{k}]^{\alpha_I - 1}[\{1 - \phi(l(\hat{k}, \hat{q}))\}l(\hat{k}, \hat{q})]^{\beta_I}, \tag{43}$$

$$\delta\hat{k} = [\{1 - \theta(l(\hat{k}, \hat{q}))\}\hat{k}]^{\alpha_I}[\{1 - \phi(l(\hat{k}, \hat{q}))\}l(\hat{k}, \hat{q})]^{\beta_I}. \tag{44}$$

Totally differentiating (43) and (44) with respect to the respective selective taxes and then solving for the tax effects on the steady-state capital stock, we obtain

$$\frac{1 - T_{Ix}}{\hat{k}} \frac{d\hat{k}}{dT_{Ix}} = \frac{1 - T_{I\pi}}{\hat{k}} \frac{d\hat{k}}{dT_{I\pi}} = -\frac{\beta_I(1 + \chi\hat{\phi}) + (1 + \chi)\alpha_I\hat{\theta}}{(1 - \alpha_I)(1 + \chi)\hat{\theta}} < 0, \tag{45}$$

$$\frac{d\hat{k}}{dT_{IL}} = \frac{d\hat{k}}{dT_{cx}} = \frac{d\hat{k}}{dT_{cL}} = \frac{d\hat{k}}{dT_{c\pi}} = 0. \tag{46}$$

These seemingly *dichotomized* comparative statics results stem from the fact that the taxes, except for T_{Ix} and $T_{I\pi}$, cannot affect the net return to capital at the steady state, which is implied by (43). Moreover, as shown in Appendix B, we were unable to find any tied or systematic relationship between the effects of the taxes and the local stability properties of the steady-state equilibrium in the *exogenous* growth model, unlike in the *endogenous* growth model in the previous sections.

7 Concluding remarks

In this paper, we have explored how the emergence of indeterminacy is related to the growth effects of the selective commodity, payroll, and profit taxes in a class of two-sector neoclassical growth models augmented with both sector-specific externalities in production and endogenous labor supply. Our main findings are summarized as follows: (i) when the equilibrium path is indeterminate, the only selective payroll tax imposed on the consumption sector stimulates long-run economic growth, while the other selective taxes depress economic growth; (ii) when the equilibrium path is determinate, all selective taxes except for the selective payroll tax imposed on the consumption sector may or may not promote long-term economic growth; and (iii) unlike the conventional results of distortionary taxes in endogenous or exogenous growth models, such as those in Itaya (1991), Milesi-Ferretti and Roubini (1998), higher distortionary taxes may stimulate economic growth.

Our model can be extended in several ways. The most interesting extension is the allowing of *aggregate* production externalities in addition to the sector-specific externalities postulated in the present model. Although we have investigated the model with aggregate production externalities, this exercise delivers qualitatively the same results as those in the present model, except that the threshold values determining whether the equilibrium path is determinate or indeterminate are *quantitatively* different from those in Proposition 1 of the present model. Second, we can allow for both or one of the sectors to produce consumption as well as investment goods as in Herrendorf and Valentinyi (2006), which would provide a more realistic analysis for the effects of distortionary taxes in a two-sector growing economy.

Appendix A: Linear approximation for the dynamic system (23) and (24)

We first differentiate (21) with respect to l at the BGP, to obtain

$$\phi'(\hat{l}) = -(1+\chi)\frac{\tau_{cx}}{\tau_{cL}} b_c \hat{l}^{-(1+\chi)-1} = -\frac{(1+\chi)\phi(\hat{l})}{\hat{l}} < 0, \tag{A.1}$$

where the last equality follows from the substitution of (21).

On the other hand, differentiating (22) with respect to l at the BGP, yields

$$\theta'(\hat{l}) = \frac{\hat{\theta}(1-\hat{\theta})}{\hat{\phi}(1-\hat{\phi})}\phi'(\hat{l}) = \frac{-(1+\chi)\hat{\theta}(1-\hat{\theta})}{(1-\hat{\phi})\hat{l}} < 0, \tag{A.2}$$

where the last equality follows from substitution of (A.1).

Solving (24) for κ yields

$$\kappa = \frac{1}{b_I}\frac{\tau_{IL}}{\tau_{Ix}}\frac{l^{\chi+1-\beta_I}}{[1-\theta(l)]\{1-\phi(l)\}^{\beta_I-1}} \equiv \kappa(l). \tag{A.3}$$

Taking the logarithmic-time differentiation of (A.3) yields $\dot{\kappa}/\kappa = [\kappa'(l)/\kappa(l)]\dot{l}$. Solving this expression for $\dot{l}$ and then substituting (23) into $\dot{\kappa}/\kappa$ in the newly found expression results in

$$\dot{l} = \frac{\kappa(l)}{\kappa'(l)}\left[\rho - [\tau_{Ix}a_I - \{1-\theta(l)\}][\{1-\phi(l)\}l]^{\beta_I}\right]. \tag{A.4}$$

To identify the stability properties of (A4), we take its linear approximation around the BGP to get

$$\begin{aligned}\dot{l} &= [-\hat{\theta}'(1-\hat{\phi})\hat{l} - \{\tau_{Ix}a_I - (1-\hat{\theta})\}\beta_I\{-\hat{\phi}'\hat{l} + 1 - \hat{\phi}\}] \\ &\times \frac{\kappa(\hat{l})}{\kappa'(\hat{l})}[(1-\hat{\phi})\hat{l}]^{\beta_I-1}(l-\hat{l}).\end{aligned} \tag{A.5}$$

Taking logs and differentiating (A.3) with respect to $\hat{l}$ yields

$$\frac{\kappa'(\hat{l})}{\kappa(\hat{l})} = \frac{(1+\chi)(1-\hat{\theta}) - \beta_I(1+\chi\hat{\phi})}{(1-\hat{\phi})\hat{l}}. \tag{A.6}$$

Substituting (A.1), (A.2), and (A.6) into $\hat{\phi}'$, $\hat{\theta}'$, and $\kappa(\hat{l})/\kappa'(\hat{l})$ in (A.5), respectively, and rearranging, we obtain (26) in the text.

Appendix B: Steady-state effects in the non-endogenous growth model

Dividing (3) in sector c by that in sector I, and using (21), (22), and the production function $y_I = k_I^{\alpha_I} l_I^{\beta_I}$, we can express the relative price of investment

p_I as a function of l and k:

$$p_I = \frac{\tau_{cx} a_c}{\tau_{Ix} a_I} \frac{\theta(l)^{\alpha_c - 1} \phi(l)^{\beta_c}}{[1-\theta(l)]^{\alpha_I - 1} [1-\phi(l)]^{\beta_I}} k^{\alpha_c - \alpha_I} l^{\beta_c - \beta_I} \equiv p_I(l,k). \tag{B.1}$$

Substituting (B.1) into (16), we obtain $c = p_I(l,k)/q$. Then, substituting (16) and (4) in sector c into the variables p_I/q and w in (17) and using the definitions of θ, ϕ, τ_{cx}, and τ_{cL}, we get

$$cl^{\chi} = \frac{\tau_{cx}}{\tau_{cL}} b_c [\theta(l)k]^{\alpha_c} [\phi(l)l]^{\beta_c - 1}. \tag{B.2}$$

Applying the implicit function theorem to (B.1), (B.2), and (16), we can get the C^1 function $l(k,q)$. After the substitution of the function $l(k,q)$, we further substitute the resulting functions $\theta(l(k,q))$ and $\phi(l(k,q))$ into (13) and (18). This substitution along with (3) in sector I gives

$$\dot{q}/q = \rho + \delta - \tau_{Ix} a_I [\{1 - \theta(l(k,q))\} k]^{\alpha_I - 1} [\{1 - \phi(l(k,q))\} l(k,q)]^{\beta_I}, \tag{B.3}$$

$$\dot{k} = [\{1 - \theta(l(k,q))\} k]^{\alpha_I} [\{1 - \phi(l(k,q))\} l(k,q)]^{\beta_I} - \delta k. \tag{B.4}$$

The determinant of the Jacobian matrix in the linearized system of (B.3) and (B.4) around the steady state is given by

$$\begin{vmatrix} (\alpha_I - 1)\delta + l_k A \delta \hat{k} & l_q A \delta \hat{k} \\ -(\rho+\delta)\left\{(\alpha_I - 1)\hat{k}^{-1} + l_k B\right\}\hat{q} & -(\rho+\delta) B l_q \hat{q} \end{vmatrix}$$
$$= \frac{-(\rho+\delta)\delta(1-\alpha_I)(1+\chi)\hat{\theta}}{(1+\chi)(1-\alpha_I\hat{\theta}) - \beta_I(1+\chi\hat{\phi})}, \tag{B.5}$$

where

$$l_k \equiv \frac{d\hat{l}}{d\hat{k}} = \frac{\alpha_I}{\hat{c}\hat{k}D}, \quad l_q \equiv \frac{d\hat{l}}{d\hat{q}} = \frac{1}{\hat{c}\hat{q}D}, \ D \equiv \frac{(1+\chi)(1-\alpha_I\hat{\theta}) - \beta_I(1+\chi\hat{\phi})}{\hat{c}(1-\hat{\phi})\hat{l}},$$
$$A \equiv \frac{\beta_I(1+\chi\hat{\phi}) + (1+\chi)\alpha_I\hat{\theta}}{(1-\hat{\phi})\hat{l}} \text{ and } B \equiv \frac{\beta_I(1+\chi\hat{\phi}) - (1+\chi)(1-\alpha_I)\hat{\theta}}{(1-\hat{\phi})\hat{l}}.$$

The inspection of (B5) reveals that this system displays saddle-point stability if $(1+\chi)(1-\alpha_I\hat{\theta}) > \beta_I(1+\chi\hat{\phi})$. On the other hand, if $(1+\chi)(1-\alpha_I\hat{\theta}) < \beta_I(1+\chi\hat{\phi})$, then indeterminacy arises provided that the trace, $(\rho+\delta)(1+\chi)\hat{\theta} - [(1-\alpha_I)\delta+\rho]\,[\beta_I(1+\chi\hat{\phi}) + (1+\chi)\alpha_I\hat{\theta}]$, is negative.[11]

References

Ballentine, J.G., 1978, The incidence of a corporation income tax in a growing economy, *Journal of Political Economy* 86, 863-875.

Benhabib, J., Farmer, R.E.A., 1994, Indeterminacy and increasing returns, *Journal of Economic Theory* 63, 19-41.

Benhabib, J., Farmer, R.E.A., 1996, Indeterminacy and sector-specific externalities, *Journal of Monetary Economics* 37, 421-443.

Bennett, R. L. and Farmer, R.E., 2000, Indeterminacy with non-separable utility, *Journal of Economic Theory* 93, 118–143.

Benhabib, J., Nishimura K., 1998, Indeterminacy and sunspots with constant returns, *Journal of Economic Theory* 81, 58-96.

De Hek, P.A., 2006, On taxation in a two-sector endogenous growth model with endogenous labor supply, *Journal of Economic Dynamics & Control* 30, 655-685.

Devereux, M. B., Love D.R.F., 1994, The effects of factor taxation in a two-sector model of endogenous growth, *Canadian Journal of Economics* 27, 509-536.

Friedlaender, A. F., Vandendorpe, A,F., 1978, Capital taxation in a dynamic general equilibrium setting, *Journal of Public Economics* 10, 1-24.

[11] It is also possible to interpret the sign of the determinant of the Jacobian matrix using the elasticities of the Frisch labor supply and the labor demand curves in sector I as in the endogenous growth model in the previous sections; namely, if $(1+\chi)(1-\alpha_I\hat{\theta}) < \beta_I(1+\chi\hat{\phi})$, the slope of the Frisch labor supply curve is less than that of the labor demand curve, and vice versa. Detailed derivations are available from the corresponding author upon request.

Harberger, A.C., 1962, The incidence of the corporation income tax, *Journal of Political Economy* 70, 215-240.

Harrison, S.G., 2001, Indeterminacy in a model with sector-specific externalities, *Journal of Economic Dynamics & Control* 25, 747-764.

Harrison, S.G., Weder, M., 2000, Indeterminacy in a model with aggregate and sector-specific externalities, *Economics Letters* 69, 173-179.

Harrison, S.G., Weder, M., 2002, Tracing externalities as sources of indeterminacy, *Journal of Economic Dynamics & Control* 26, 851-867.

Hendricks, L., 1999, Taxation and long-run growth, *Journal of Monetary Economics* 43, 411-434.

Herrendorf, B., Valentinyi, A., 2006, On the stability of the two-sector neoclassical growth model with externalities, *Journal of Economic Dynamics & Control* 30, 1339-1361.

Homma, M., 1985, Dynamic incidence in a two-sector growing economy, *Environment and Planning C: Government and Policy* 3, 285-297.

Itaya, J., 1991, Tax incidence in a two-sector growing economy with perfect foresight, *Journal of Public Economics* 44, 95-118.

King, R.G., Plosser, C.I., Rebelo, S.T., 1988, Production, growth and business cycles: 1. the basic neoclassical model, *Journal of Monetary Economics* 21, 195-232.

Mieszkowski, P.M., 1967, On the theory of tax incidence, *Journal of Political Economy* 75, 250-262.

Milesi-Ferretti, G.M., Roubini, N., 1998, On the taxation of human and physical capital in models of endogenous growth, *Journal of Public Economics* 70, 237-254.

Mino, K., 1996, Analysis of a two-sector model of endogenous growth with capital income taxation, *International Economic Review* 37, 227-251.

Pelloni, A., Waldmann, R., 2000, Can waste improve welfare?, *Journal of Public Economics* 77, 45-79.

Samueslon, P.A., 1947, *The Foundation of Economic Analysis*, Harvard University Press, Cambridge, Mass.