VIDEO ANALYSIS OF FISH SCHOOLING BEHAVIOR IN FINITE SPACE
USING A MATHEMATICAL MODEL

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ABSTRACT

The schooling behavior of the fish *Parapristipoma trilineatum* (chicken grunt) was examined for a range of school sizes in a water tank containing a central columnar structure using a mathematical model of fish schooling behavior that was modified from that of Sannomiya and Matuda. The two-dimensional motion of individuals was captured on digital video over about 10 minutes, and these time-series data were used to tune the model to the experimental conditions. The model parameters were then used to evaluate schooling behavior quantitatively. Comparing force magnitudes between experimental conditions revealed that propulsive force was less in a tank without the central structure than in a tank with the central structure, under all conditions tested. These results indicate that the school was less active under unobstructed conditions. Propulsive force also decreased with increasing school size. Attractive and repulsive forces of the walls were dominant for smaller school sizes; the attractive and repulsive forces of the structure were significant only for the largest school tested (25 individuals). Thus, changes in behavioral patterns caused by the number of individuals or the presence of structures can be expressed using the mathematical model.

*Keywords:* Fish behavior; mathematical model; Fractal dimension analysis
**Introduction**

To improve the design and management of fishing gear that is used in aquaculture and fisheries, fish behavior must be considered as it relates to fishing gear. Although many studies have observed the schooling behavior of fish underwater at fishing grounds, current estimates of schooling behavior characteristics are unsatisfactory. Basic measures of displacement and velocity are not sufficient to characterize schooling behavior, so a mathematical model that allows quantitative comparison between cases is needed. Matuda and Sannomiya presented a mathematical model (S&M model) that describes fish-schooling behavior in a water tank (Matuda and Sannomiya, 1980, 1985). The movement patterns of fish simulated by the S&M model are similar to the actual behavior of fish schools, and the model has been shown to be suitable for investigating fish behavior in relation to fishing gear. Takagi et al. (1993) used the S&M model to investigate the influence of tank shape and size on fish behavior: all forces except propulsive force were independent of tank size, and schooling, and the attractive force of the walls was strongest in circular tanks. Such studies are groundbreaking in their analysis of movement patterns of fish in different conditions. However, very few studies have attempted to use this model to investigate fish behavior in relation to structures and different sizes of fish schools (especially those comprising more than 10 individuals). We examined fish schooling behavior in a water tank to investigate changes in behavioral pattern in a finite space, and the effect of walls and other internal structures over a range of school sizes. We adapted parts of the S&M model to make it more suitable for the experimental conditions in this study. We examined the schooling behavior of various-sized schools of *Parapristipoma*
Analysis of fish schooling behavior
Suzuki, Takagi, Hiraishi

trilineatum, commonly known as chicken grunt or isaki, in tanks of various dimensions
with and without internal structures; each condition was modeled using changes in force
parameters.

Materials and Methods

Experimental Apparatus

Chicken grunt fish, bred in a breeding tank, were introduced to a 180
cm-diameter water tank with a concrete columnar structure at its center. The columnar
structure was 15 cm in diameter and 30 cm tall, and the water depth was 15 cm. The
school sizes examined were 1, 3, 5, 10 and 25 individuals, which were random sampled
from the breeding tank. Figure 1 shows a schematic of the experimental apparatus.
Experiments were carried out in the experiment room under daylight of uniform
intensity. The experiments were repeated 4 times under the same condition, and a
control experiment without the central columnar structure was also carried out.

Video images of schooling behavior were taken using a digital video camera set
3 m above the tank. The video was taken over a period of about ten minutes, and the x-y
position of each individual was recorded at 0.3-s intervals using a personal computer
and digital video capture card. The procedure for recording the position of individuals
and importing the data into a Microsoft Excel spreadsheet was automated by a
Microsoft Visual Basic program written specifically for this purpose. Figure 2 shows an
example of an image of fish schooling behavior obtained using this system.

Methods of Analysis

The position of each individual was used as time-series data in the model of
fish-schooling behavior. Each individual was considered a point of mass in the model, and the motion of each fish, assumed to be limited by the shallow water depth, was modeled in two-dimensional space. The model, modified from that developed by Sannomiya and Matuda (1987), is as follows:

\[
m\ddot{x}_i = F_i(t, x_i, \dot{x}_i, \ddot{x}_j)
\]

\[
F_i = F_{di} + F_{ai} + F_{bi} + F_{wi}^+ + F_{wi}^- + F_{gi}^+ + F_{gi}^- + F_{ri}
\]

\[
= \nu f_{di} + a f_{ai} + k_b f_{bi} + k_w^+ f_{wi}^+ + k_w^- f_{wi}^- + k_g^+ f_{gi}^+ + k_g^- f_{gi}^- + F_{ri} \cdots \cdots (1)
\]

where \( m \) is the mass of an individual and \( \ddot{x}_i \) is the acceleration of an individual. Table 1 shows each function on the right-hand side of equation 1 and its parameters. Each function includes unknown force-magnitude parameters and distance parameters. The distance parameters were obtained from observations of fish-schooling behavior in the water tank. The functions \( F_{di}, F_{ai}, F_{bi}, F_{wi}^+, F_{wi}^-, \) and \( F_{ri} \) are identical to those in the original S&M model. In this study, we considered the influence of structures in the water tank, just as the force of a wall was considered in the original model. However, \( F_{gi}^+ \) and \( F_{gi}^- \) were newly defined in this study.

**Repulsive Force from the Structure \( F_{gi}^+ \)**

When an individual approaches the structure, it avoids striking against the structure in an influence zone. \( F_{gi}^+ \) express such behavior, and is given by

\[
F_{gi}^+ = k_g^+ f_{gi}^+
\]

\[
f_{gi}^+ = \begin{cases} v_{il} \frac{g^+ - g_{di}}{g^+} e_l \nu_{il} < 0 \text{ and } g_{di} < g^+ \\ 0 \text{ otherwise} \end{cases} \cdots \cdots (2)
\]
where \( g^+ \) is the width of the influence zone of the structure, \( v_{il} \) is the velocity component normal to the surface of the structure, \( e_i \) is the unit vector normal to the surface of the structure, and \( g_{il} \) is the distance from the individual \( i \) to the structure.

**Attractive Force toward the Structure \( F_{gi}^- \)**

The structure in the water tank has an attractive effect in the same manner as the wall of the water tank. \( F_{gi}^- \) expresses such an effect, and is given by

\[
F_{gi}^- = k_g f_{gil}^-
\]

\[
f_{gil}^- = \begin{cases} 
  v_{il} \frac{g^- - g_{il}}{g^-} e_i & v_{il} > 0 \text{ and } g_{il} < g^- \\
  0 & \text{otherwise}
\end{cases}
\]

where \( g^- \) is the width of the influence zone of the force.

By estimating the coefficients, which includes the terms on the right-hand side of equation (1), the forces can be compared under different experimental conditions. When \( m\ddot{x}_i, f_{di}, f_{ai}, f_{bi}, f_{wi}^+, f_{wi}^-, f_{gi}^+ \) and \( f_{gi}^- \) are obtained for an individual position, a linear equation that includes the unknown parameters which defined the magnitude of the force can be derived for the individual. The least squares method can then be applied to the linear equations derived observed data in each time step, allowing the unknown parameters to be estimated for an individual under each experimental condition. The parameter \( \nu \) can be estimated using measurable data and the drag coefficient \( C_D = 0.1 \), and the random force can be evaluated as the residual of the difference between the inertia force of an individual and six forces, i.e., \( F_{di}, F_{ai}, F_{bi}, F_{wi}^+, F_{wi}^-, F_{gi}^+ \) and \( F_{gi}^- \). The unknown parameters in each case can be calculated by substituting the 1000 time series coordinates for an individual in the same way as in the previous paper (Takagi et al., 1993).
Results and discussion

**Trajectory of Fish Swimming Behavior**

Figure 3 shows examples of the trajectory of individuals for each experimental case over five minutes. The fish school swam along the wall for school sizes of 1, 3 and 5 individuals, and for 10 and 25 individuals, the school swam in a complicated manner in a limited area, not along the wall. The distance between individuals and the wall for each school size is shown in Fig. 4. The distance is the average for the school size and observation time. The average distance from the wall increased exponentially with increasing school size, indicating that as school sizes get larger the fish gather closer to the central structure. It is considered that fish schooling behavior itself was not affected by the structure according to the observed trajectories.

\[ F_{bi}, F_{wi}^+, F_{wi}^-, F_{gi}^+ \text{ and } F_{gi}^- \] include unknown parameters \( \alpha, \beta, d^+, d^-, g^+ \text{ and } g^- \), which need to be determined before estimating \( a, k_b, k_w^+, k_w^-, k_g^+ \text{ and } k_g^- \). These parameters were determined by fitting the relevant equation to the observed data by trial and error because the equation is non-linear. The estimated parameters are given below.

\[ \alpha = 1 \text{ cm}, \beta = 3 \text{ cm}, d^+ = 10 \text{ cm}, d^- = 30 \text{ cm}, g^+ = 20 \text{ cm}, g^- = 30 \text{ cm} \]

The parameter \( \nu \) can be determined from seawater density \( \rho = 1.02 \text{ g/cm}^3 \), the drag coefficient \( C_D = 0.1 \), and the average cross-sectional area of fish \( S = 7.85 \text{ cm}^2 \), giving \( \nu = 0.402 \text{ g/cm} \). The mass \( m_i \) is assumed to the average of all fish, in this case \( m = 20.0 \text{ g} \).

**Validity of Fish Behavior Model**

The validity of this mathematical model was verified by analyzing time-series...
data using two methods: power spectrum analysis and fractal dimension analysis (Ueno et al., 2000). Defined as a random force, $F_{ri}$ is the residual force that cannot be attributed to any other forces; its power spectrum should lack periodicity. Figure 5a shows an example of the power spectrum analysis; in all cases, the power spectrum did not exhibit a clear peak for periodicity, indicating that $F_{ri}$ is indeed random.

We carried out fractal dimension analysis of time-series data using Higuchi’s method, which characterizes the randomness of the time series in terms of fractal dimensions (Higuchi, 1988, 1989). This method has been widely adopted in many fields as a tool to measure the randomness of time series that exhibit complicated behavior over time. We applied the function $L_d(\tau)$, which explains the length of the trajectory in the time interval $\tau$, according to Higuchi’s method:

$$L_d(\tau) = \frac{1}{\tau} \cdot \frac{N-1}{S_\tau} \cdot \sum_{i=1}^{S_\tau/\tau} |x(k + i\tau) - x(k + (i-1)\tau)| \ldots \ldots (4)$$

where $x(t)$ is the time-series data set and $S_\tau$ is $\left\lceil (N-k)/\tau \right\rceil$. If the following relation is confirmed, $d$ is the fractal dimension of the time-series data:

$$L(\tau) \propto \tau^{-d} \ldots \ldots (5)$$

where $L(\tau)$ is the average of $L_d(\tau)$ ($k = 1, 2, \ldots$). When the time-series data are completely random, the fractal dimension $d$ should be 2.0; when $d$ is closer to 1, the data are somewhat periodic. Therefore, the randomness of $F_{ri}$ can be verified. Figure 5b shows an example of the results of fractal dimension analysis. The results are plotted as $\log(L(\tau))$ against $\log(\tau)$. The slope of the plots approaches 2.0, indicating that the data are random.

On the basis of these two analyses, the time-series data for $F_{ri}$ are considered random, without systematic variation. As such, this fish behavior model is valid for analyzing fish schooling behavior.
Parameter Estimation

The estimated parameters are plotted in Fig. 6 against school size for each experimental condition. Although $k_b$, $k_g^+$ and $k_g^-$ do not exhibit any variation that can be considered a trend, the other parameters exhibit clear trends with increasing school size; parameters $a$ and $k_w^-$ decrease with increasing school size, and $k_w^+$ exhibits a slight increase. These results describe the tendency of fish to become less active and remain further away from the wall as school sizes increase. The parameter $a$ for the control experiment was smaller than in any of the other cases, suggesting that the fish school was not as active in the tank without the structure. There is no significant variation in the attractive or repulsive force of the structure except for the school size of 25 individuals, in which the parameter values varied widely and no firm conclusion can be drawn.

It is convenient to normalize these parameters to allow the magnitudes of the parameters to be compared without processing the estimated parameters. The parameters $\nu$, $a$, $k_b$, $k_w^+$, $k_w^-$, $k_g^+$ and $k_g^-$ are then normalized, taking the average as zero and the standard deviation as 1, giving the new normalized parameters $\nu^*$, $a^*$, $k_b^*$, $k_w^{+*}$, $k_w^{-*}$, $k_g^{+*}$ and $k_g^{-*}$. The process for this is simple; for example, $a^*$ is given as $a\sqrt{S_a / S_F}$, where $S_a$ is the variance of $f_{ai}$ and $S_F$ is the variance of $F_i$ in equation (1). Normalized values for each parameter are shown in Fig. 7. Overall, the magnitude of parameters for school sizes of 1, 3 and 5 differs from that for 10 and 25 individuals. This may indicate that the fish schooling behavior in this finite space are essentially different at these school sizes. The magnitudes of the attractive and repulsive forces of the wall for 1, 3 and 5 individuals are larger than for 10 and 25 individuals, and the magnitudes of the
propulsive force parameters are smaller. It is interesting that the interactive force is not larger for larger school sizes, whereas propulsive force is larger. At school sizes of 10 and 25 individuals, the propulsive force dominates all other forces. Therefore, fish schooling behavior can be characterized by propulsive force as the number of individuals increases. The parameter ratio distribution in the experiments was consistent with the control in all cases except for the school of 25 individuals. In the case of 25 individuals, the attractive and repulsive forces of the structure were significant, whereas these forces were zero in the other cases. Therefore, it is thought that the schooling behavior of larger schools is affected by the presence of a structure in a finite space, while the behavior of smaller schools remains virtually unchanged from that in the case without a structure.

In this study, the presence of the central structure was not found to have a clear effect on fish schooling behavior. However, these results confirmed that the mathematical model presented here is useful for evaluating the characteristics of fish behavior quantitatively. In order to develop a comprehensive understanding of fish schooling behavior in response to structures and special limitations, it is necessary to collect basic information useful for predicting fish behavior in various cases by carrying out further experiments for different fish species and patterns of space.

Acknowledgements

We wish to thank Dr. Kimihiko Ueno for helpful discussion regarding time series data analysis. This research was supported in part by a grant from the Toyo Environmental Protection Fund of the Osaka Community Foundation.
References

CAPTION

Fig. 1. Schematic diagram of experimental equipment. A: digital video camera, B: experimental fish, C: columnar structure.

Fig. 2. An example of an image of fish schooling behavior obtained using this system. The front points of each individual are used as x-y position of each fish.

Fig. 3. Examples of the trajectory of individuals for each experimental case over five minutes.

Fig. 4. The distance between individuals and the wall for each school size. N: Number of individuals.

Fig. 5. Analyses of residual values of $F_{ri}$ by two methods for a control case (5 individuals). (a) Sample power spectrums. (b) Sample results of the fractal dimension analysis. $d = $ fractal dimension.

Fig. 6. Estimated parameters against school size for each experimental condition. N: Number of individuals.

Fig. 7. Normalized values for each parameter.

Table 1. Each function of forces in the external force $F_{i}$ and unknown parameters which are included in each function.