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Olympic Athlete Selection*

Yoichi Hizen and Ryo Okui

Abstract

Olympic athlete selection procedures are different among countries and events, and famous athletes are often reported to have lost their selection races. This paper analyzes what kind of procedure is more likely to select high-ability athletes while preventing low-ability athletes from being selected by chance. Our game-theoretic model shows that the answer depends on how sharply high-ability athletes' race results fluctuate relative to those of low-ability athletes. Athletes' strategic choice of participation in races turns out to be crucial in addressing this question, and there are cases in which having only one race is desirable, even if the selection can involve multiple races.

KEYWORDS: Olympic, contest, tournament

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1 Introduction

To what extent can we accurately evaluate high-ability people when we cannot directly observe their abilities, or when we know their abilities but need a formal procedure to convince other people? Such evaluation is important in various types of organization such as firms, schools, bureaucratic structures and sport teams.

The selection of representatives to a national team for the Olympic Games is a typical and easily-observable example of such an evaluation procedure. In particular, national federations of individual sports, such as athletics and swimming, usually hold selection races. The goal in holding these competitions is to determine who has the highest ability in their countries and to let people, including the athletes themselves, understand who should be selected as their representatives.

However, this goal is apparently not necessarily achieved. For example, two Sydney gold medalists lost their selection races for the Athens Olympic Games although they both had been expected to achieve further success. Ian James Thorpe, the 400-meters freestyle swimming gold medalist, was disqualified from the Australian 400-meters Olympic trials due to a false start although Craig Stevens, who won the Olympic spot by coming second in the trials, later withdrew in order to give up his place to Thorpe. Thorpe won the gold medal in the 400-meters freestyle swimming in Athens. The Australian one-race selection procedure did not allow even one false start; it seems simple but risky in the sense that this procedure may fail to choose a high-ability athlete with non-negligible probability.

Naoko Takahashi, the women's marathon gold medalist in the Sydney Games, also failed in the Japanese women's marathon selection procedure for the Athens Games. The procedure is more complicated: (one plus) three selection races are held, and athletes with good results in each race are compared with each other; three of these athletes are given Olympic spots. This procedure seems less risky, but the selection committee members usually receive many complaints against their decisions. Takahashi believed herself to be selected after finishing first among the Japanese participants in the first of the three races so that she did not run in the following two races. The problem comes from the difficulty in controlling for different race conditions.

Table 1 describes how Olympic representatives in four types of individual events were selected in four of the top six countries with respect to the number of gold medals in the Athens Games.[1] For swimming, in principle, all four

[1] The remaining two countries were China and Russia.

\events nations\	Track and Field	Race Walk	Marathon	Swimming
U.S.A.	one trial	one trial	one trial	one trial
Australia	one early nomination and one trial or discretion	one early nomination and one trial or discretion	one early nomination or discretion	one trial
Japan	one early nomination and seven trials or discretion	one early nomination and three trials or discretion	one early nomination, three trials and discretion	one early nomination and one trial or discretion
Germany	any official race or discretion	any official race or discretion	any official race or discretion	one trial and discretion

Table 1: Olympic Athlete Selection Procedures in Four Countries
Note: "Or discretion" means that the selection committee selected Olympic representatives at its discretion if Olympic spots were still available after trials. "And discretion" means that the selection committee selected Olympic representatives at its discretion taking the results of trials into account.

countries had one trial only. In athletics, however, selection procedures were different among countries; the U.S.A. and Australia had only one trial whereas Japan and Germany had multiple trials. Because of this diversity of procedures across events and among countries, it seems difficult to answer, from this table, what kind of procedure is most appropriate.

To tackle this question, we construct a game-theoretic model in which two athletes, a *high-ability athlete* and a *low-ability athlete*, compete for one Olympic spot through, at most, two races. Athletes decide whether to participate in each race, and their results are influenced by uncertainties such as their physical condition on the day of the race and race conditions like weather. The high-ability athlete is expected to have a better result than the low-ability athlete. We compare three selection procedures, *one-race selection*, *best-result selection* and *average-result selection*, in terms of the probability of the high-ability athlete being selected. In the one-race selection procedure, only one race is held, and the athlete with the better result is given the Olympic spot. The best-result selection procedure involves two races. Participation in each race is voluntary, and the better result from the two races is regarded as the final result if the athlete participates in two races. Finally, the average-result selection procedure is the same as best-result selection except that the average result is taken as the final result if the athlete participates in two races. The selection procedures involving multiple races in Table 1 might be approximated by best-result selection, average-result selection or a mixture of these two.

If the selection committee could force athletes to participate in every race, the selection problem would be easily solved. That is, it is only necessary to hold as many races as needed and to compare athletes' average results; as the number of races converges to infinity, the highest-ability athlete will almost certainly attain the best average result, by the law of large numbers. One-race selection, in fact, forces athletes to participate in only one race, but under selection procedures with multiple races, athletes are usually allowed to choose in which race to participate. For example, forcing marathon athletes to run in many races would exhaust them with injury before the Olympic Games. This voluntary participation in multiple races allows the possibility for athletes to make a strategic choice of races; an athlete's decision to participate in a race may depend on whether other athletes participate in that race, and he can also choose how many races to participate in. Such strategic behavior by athletes makes the problem of designing appropriate selection procedures more complicated. In fact, holding many races in this situation does not necessarily increase the probability of high-ability athletes being selected.

We show that the probability of the high-ability athlete winning is highest under (i) best-result selection if his results fluctuate more sharply than those of the low-ability athlete, (ii) one-race selection if his results fluctuate less sharply than those of the low-ability athlete, and (iii) average-result selection if his results fluctuate similarly to those of the low-ability athlete. Situation (i) may be understood easily. Here, participation in both races is the dominant strategy for both athletes, and the maximum of two random results tends to be greater when their variances are larger. On the other hand, situations (ii) and (iii) may be non-trivial and they require a more detailed explanation. The key here is the strategic choice of races by athletes, as mentioned above, under average-result selection. Each athlete may try to attain a better average result by not participating in the second race if he finishes the first race with a good result. In addition to this individual optimization, we observe from our model that the low-ability athlete strategically avoids competing with the high-ability athlete in the same race. As a result, average-result selection becomes less attractive than one-race selection for some parameter values where average-result selection would be more attractive if each athlete did not take account of the other's participation decisions.

This paper is organized as follows. The remainder of this section discusses related work. Section 2 sets up the model and Section 3 provides a solution. Since our model is not solvable analytically, we derive the high-ability athlete's winning probabilities numerically. Section 4 contains our conclusion.

Appendix A provides more detailed information about the selection procedures stated in Table 1. Appendix B presents two different versions of the model in the main text. The first extends the model by introducing the possibility of injury, which prevents athletes from participating in particular races with a positive probability. This makes one-race selection the least attractive of the three procedures because the high-ability athlete is most likely to lose automatically due to injury. The second is a simpler version of the model, which is solvable analytically. Although the range of parameter values is restricted, the simpler model confirms the robustness of our numerical results.

1.1 Related Studies

How to design contests has been studied in tournament theory. Most articles in the literature consider the moral-hazard problem in which the designer's purpose is to extract appropriate actions from agents through a payoff scheme, as studied by Lazear and Rosen (1981). In a firm, for example, the manager designs a wage scheme to encourage his workers to work harder. An organizer of a sport event determines the amount of money in prizes for the winner and runners-up in order to stimulate players to make greater efforts so that the event will attract more spectators: see Szymanski (2003) for a survey on the application of tournament theory to sport events.

In contrast, inducing efforts from athletes is not important in Olympic trials because the chance of an Olympic spot is in itself sufficient motivation for athletes. The same view is taken by Berentsen (2002), who analyzes methods to prevent athletes from doping themselves. Hence, the only purpose of the committee is to make the probability of selecting high-ability athletes as high as possible. Our setup therefore applies to white-color workers who are sufficiently motivated by promotion, including being head-hunted, rather than typical blue-color workers who tend to provide effort in relation to their pay only.

In fact, this type of selection problem has been analyzed in the context of promotion within a firm. Meyer (1991) considers an organization maximizing the probability of promoting the abler of two workers. For each period of work, the organization records rank-order information, such as which of two workers has done a better job. It is shown to be optimal for the organization to have a final-period bias in favor of the worker with the better job history. There are two main differences between her model and ours. First, workers in her model non-strategically work in every period, whereas athletes in our model strategically choose in which race to participate. Second, the promotion decision by the organization is based on its updated belief regarding the workers' abilities whereas the final result determines the winner in our model.

Belief updating is also analyzed by Carrillo (2003). He examines how to allocate multiple tasks between two workers so that the firm can determine as precisely as possible which worker is abler. His focus is on the efficiency loss when task allocation is delegated to one of the workers. On the other hand, a tournament in which the contestant with the highest output is promoted is analyzed by Gürtler (2006) and Münster (2007). Gürtler considers how many tournaments the employer should hold when having more tournaments is accompanied by additional costs for the employer. In his model where, as in Meyer's (1991) model, employees non-strategically participate in every tournament, the winning probability of a high-ability employee increases in the number of tournaments. Münster focuses on the effect of sabotage. He shows that if the number of contestants is at least three, their winning probabilities are equalized even when their abilities are different. This is because contestants who make greater production efforts are sabotaged more heavily by others. In our model, in contrast, each athlete cannot affect the other athlete's result directly but can only control his own participation decisions.

Clark and Riis (2001) and Hvide and Kristiansen (2003) also consider tournaments in which the agent with the best outcome wins. The basic structure of their models, in which each agent's ability is his private information and agents play an incomplete-information game, is different from our complete-information game. In Clark and Riis's tournament, two agents make costly efforts, and their outputs are deterministic. They show that if the distributions of the two agents' abilities have different supports, the basic tournament cannot promote the higher-ability agent with certainty. However, if the designer introduces two test standards, on which agents' payoffs depend, then the higher-ability agent is promoted with certainty. Hvide and Kristiansen consider the case in which agents' choice variable is not an effort level but risk taking: each agent chooses either a safe job or a risky job, where the output yielded from the safe job fluctuates less sharply than the risky job. There is a pool of agents with two levels of abilities, high and low, from which several agents are drawn into a contest. Their main result is that the probability of the winner being a high-ability agent can decrease when the share of high-ability agents in the pool increases.

2 The Model

Two athletes, H and L, compete for one Olympic spot. In order to make a selection, the selection committee holds, at most, two races. If athlete $i = H, L$

participates in race $j = 1, 2$, his result in the race, r_i^j, is produced as the sum of an athlete-specific random variable, α_i^j, and a race-specific random variable, β^j:

$$r_i^j = \alpha_i^j + \beta^j.$$

Note that the race condition, β^j, is the same for both athletes, whereas physical condition, α_i^j, can vary between athletes and also between races. We assume that participation in each race does not incur any cost so that canceling a race stems from the strategic reason only.

Athlete H has a higher expected level of performance than athlete L. Specifically, we assume that α_H^j follows a normal distribution with mean $\mu > 0$ and variance $\nu_H^2 \geq 0$, while α_L^j follows a normal distribution with mean zero and variance $\nu_L^2 > 0$ (i.e., $\alpha_H^j \sim N(\mu, \nu_H^2)$ and $\alpha_L^j \sim N(0, \nu_L^2)$ for $j = 1, 2$). Note that only the relative relationship between α_H^j and α_L^j matters in the selection. Employing the normal distribution is for ease of calculation. We normalize β^j by assuming that it follows the standard normal distribution (i.e., $\beta^j \sim N(0, 1)$ for $j = 1, 2$). All random variables are independent of each other, and their distributions are common knowledge between athletes.

We compare the following three athlete-selection procedures in terms of the probability of athlete H being selected as the representative.[2] The athlete with no results is never selected if another athlete has at least one result. If no athlete has a result, we can assume either that one of them is chosen randomly or that no athlete can be a representative; this specification does not affect the outcomes because at least one athlete participates in at least one race in equilibrium under any procedure.

One-Race Selection: Only one race is held. Athletes simultaneously decide whether to participate in the race. Then their results are realized. The athlete with the better result is selected as the representative.

Best-Result Selection: Two races are held. At the beginning, athletes simultaneously decide whether to participate in the first race. If athlete i participates in the first race, random variables α_i^1 and β^1 are realized. After both athletes observe these variables, they simultaneously decide whether to participate in the second race. If athlete i participates in

[2] Focusing on athlete H's winning probability does not mean that selecting athlete H as the representative is desirable for the selection committee. The selection committee may seek only the gold medal and want to select athlete L if his athlete-specific condition has a larger variance. In such a case, the selection committee only has to focus on one minus athlete H's winning probability.

the second race, random variables α_i^2 and β^2 are realized. If athlete i participates in two races, the committee uses only the better result as his final result (i.e., it is $max\{r_i^1, r_i^2\}$). The athlete with the better final result is selected as the representative.

Average-Result Selection: Two races are held. The game is the same as for best-result selection except for the way of choosing the representative: if athlete i participates in two races, the committee takes his average result from the two races as his final result (i.e., it is $(r_i^1 + r_i^2)/2$).

Whether to participate in each race is determined as a Nash equilibrium between the two athletes who try to maximize their winning probabilities. When the procedure involves two races, we look for subgame perfect equilibria.

3 Analysis

In this section, we derive the formulae for athlete H's winning probabilities under the three procedures. It will turn out that analytical comparison of the probabilities is hard due to the complicated formulae. Therefore, we compare them numerically. Appendix B presents a simpler version of our model that can be solved analytically.

3.1 One-Race Selection

Suppose that the committee holds one race only and selects, as the representative, the athlete with the better result in the race. Then participation in the race is the dominant strategy for both athletes. Hence, the probability of athlete H being selected can be written as

$$\begin{aligned} Pr(\alpha_L^1 + \beta^1 < \alpha_H^1 + \beta^1) &= Pr\left(\frac{\alpha_L^1 - \alpha_H^1 + \mu}{\sqrt{\nu_L^2 + \nu_H^2}} < \frac{\mu}{\sqrt{\nu_L^2 + \nu_H^2}}\right) \\ &= \Phi\left(\frac{\mu}{\sqrt{\nu_L^2 + \nu_H^2}}\right), \end{aligned} \tag{1}$$

where $\Phi(.)$ is the cumulative distribution function of the standard normal distribution. The only source of uncertainty is the athlete-specific component. The probability of athlete H being selected increases with μ but decreases with ν_L and ν_H.

3.2 Best-Result Selection

Suppose that the committee holds two races and considers the better result from the two races for each athlete if he participates in two races. Then participation is the dominant strategy for both athletes in both races. Hence, the probability of athlete H being selected can be written as

$$Pr(max\{\alpha_L^1+\beta^1,\alpha_L^2+\beta^2\}<max\{\alpha_H^1+\beta^1,\alpha_H^2+\beta^2\}). \tag{2}$$

Notice that $max\{r_i^1, r_i^2\}$ tends to be higher when $var(r_i^j)$ is larger. Relative to one-race selection, therefore, this procedure works in favor of the athlete whose athlete-specific condition has a larger variance.

3.3 Average-Result Selection

Suppose that the committee holds two races and considers the average result of the two races for each athlete if he participates in two races. Following the backwards-induction procedure, we first examine whether athletes participate in the second race given the outcome of the first race, and then derive the participation decisions in the first race. Hereinafter, we abbreviate "participation" as "P" and "nonparticipation" as "N."

3.3.1 Subgame after (N, N)

If an athlete does not participate in the first race, participation in the second race is the dominant strategy for him. Hence, in the subgame after no one participates in the first race, both athletes participate in the second race, which is equivalent to the case of one-race selection; athlete H is selected with probability $\Phi(\mu/\sqrt{\nu_L^2+\nu_H^2})$.

3.3.2 Subgame after (P, N)

Suppose that athlete H participates in the first race while athlete L does not. Then, in the second race, participation is the dominant strategy for athlete L. Whether athlete H also participates in the second race depends on the realized value of r_H^1. If athlete H does not participate in the second race, he is selected as the representative with probability

$$Pr(\alpha_L^2+\beta^2<\alpha_H^1+\beta^1)=Pr\left(\frac{\alpha_L^2+\beta^2}{\sqrt{\nu_L^2+1}}<\frac{\alpha_H^1+\beta^1}{\sqrt{\nu_L^2+1}}\right)=\Phi\left(\frac{\alpha_H^1+\beta^1}{\sqrt{\nu_L^2+1}}\right). \tag{3}$$

If athlete H participates in the second race, on the other hand, he is selected with probability

$$\begin{aligned} & Pr\left(\alpha_L^2+\beta^2<\frac{1}{2}(\alpha_H^1+\beta^1+\alpha_H^2+\beta^2)\right) \\ = \quad & Pr\left(\frac{2\alpha_L^2-\alpha_H^2+\beta^2+\mu}{\sqrt{4\nu_L^2+\nu_H^2+1}}<\frac{\alpha_H^1+\beta^1+\mu}{\sqrt{4\nu_L^2+\nu_H^2+1}}\right) \\ = \quad & \Phi\left(\frac{\alpha_H^1+\beta^1+\mu}{\sqrt{4\nu_L^2+\nu_H^2+1}}\right). \end{aligned} \tag{4}$$

Hence, athlete H participates in the second race only if

$$\frac{\alpha_H^1+\beta^1}{\sqrt{\nu_L^2+1}}\leq\frac{\alpha_H^1+\beta^1+\mu}{\sqrt{4\nu_L^2+\nu_H^2+1}},$$

which can be rewritten as

$$\frac{\alpha_H^1+\beta^1-\mu}{\sqrt{\nu_H^2+1}}\leq\frac{2\sqrt{\nu_L^2+1}-\sqrt{4\nu_L^2+\nu_H^2+1}}{\sqrt{4\nu_L^2+\nu_H^2+1}-\sqrt{\nu_L^2+1}}\frac{\mu}{\sqrt{\nu_H^2+1}}\equiv x. \tag{5}$$

This condition implies that athlete H participates in the second race if his result in the first race is bad.

Using (3), (4) and (5), we can express the probability of athlete H being selected in this subgame, denoted as w_{PN}, as

$$\int_{-\infty}^{x}\Phi\left(\frac{z\sqrt{\nu_H^2+1}+2\mu}{\sqrt{4\nu_L^2+\nu_H^2+1}}\right)\phi(z)dz+\int_{x}^{\infty}\Phi\left(\frac{z\sqrt{\nu_H^2+1}+\mu}{\sqrt{\nu_L^2+1}}\right)\phi(z)dz, \tag{6}$$

where $\phi(.)$ is the probability density function of the standard normal distribution.

3.3.3 Subgame after (N, P)

Suppose that athlete L participates in the first race while athlete H does not. The similar logic to the subgame after (P, N) applies to this subgame. If athlete L does not participate in the second race, athlete H is selected with probability

$$\begin{aligned} Pr(\alpha_L^1 + \beta^1 < \alpha_H^2 + \beta^2) &= Pr\left(\frac{\alpha_L^1 + \beta^1 - \mu}{\sqrt{\nu_H^2 + 1}} < \frac{\alpha_H^2 + \beta^2 - \mu}{\sqrt{\nu_H^2 + 1}} \right) \\ &= \Phi\left(\frac{\mu - \alpha_L^1 - \beta^1}{\sqrt{\nu_H^2 + 1}} \right). \end{aligned} \tag{7}$$

If athlete L participates in the second race, on the other hand, athlete H is selected with probability

$$\begin{aligned} & Pr\left(\frac{1}{2}(\alpha_L^1 + \beta^1 + \alpha_L^2 + \beta^2) < \alpha_H^2 + \beta^2 \right) \\ = \; & Pr\left(\frac{\alpha_L^1 + \beta^1 - 2\mu}{\sqrt{4\nu_H^2 + \nu_L^2 + 1}} < \frac{2\alpha_H^2 - \alpha_L^2 + \beta^2 - 2\mu}{\sqrt{4\nu_H^2 + \nu_L^2 + 1}} \right) \\ = \; & \Phi\left(\frac{2\mu - \alpha_L^1 - \beta^1}{\sqrt{4\nu_H^2 + \nu_L^2 + 1}} \right). \end{aligned} \tag{8}$$

Hence, athlete L participates in the second race only if

$$\frac{2\mu - \alpha_L^1 - \beta^1}{\sqrt{4\nu_H^2 + \nu_L^2 + 1}} \leq \frac{\mu - \alpha_L^1 - \beta^1}{\sqrt{\nu_H^2 + 1}},$$

which can be rewritten as

$$\frac{\alpha_L^1 + \beta^1}{\sqrt{\nu_L^2 + 1}} \leq \frac{2\sqrt{\nu_H^2 + 1} - \sqrt{4\nu_H^2 + \nu_L^2 + 1}}{\sqrt{\nu_H^2 + 1} - \sqrt{4\nu_H^2 + \nu_L^2 + 1}} \frac{\mu}{\sqrt{\nu_L^2 + 1}} \equiv y. \tag{9}$$

Using (7), (8) and (9), we can express the probability of athlete H being selected in this subgame, denoted as w_{NP}, as

$$\int_{-\infty}^{y} \Phi\left(\frac{2\mu - z\sqrt{\nu_L^2 + 1}}{\sqrt{4\nu_H^2 + \nu_L^2 + 1}} \right) \phi(z) dz + \int_{y}^{\infty} \Phi\left(\frac{\mu - z\sqrt{\nu_L^2 + 1}}{\sqrt{\nu_H^2 + 1}} \right) \phi(z) dz. \tag{10}$$

3.3.4 Subgame after (P, P)

Suppose that both athletes participate in the first race. There are four pure-action profiles in the second race, that is, (N, N), (P, N), (N, P) and (P, P). The probability of athlete H being selected under each profile is summarized in Table 2.

$H\backslash L$	P	N
P	$\Phi\left(\frac{\alpha_H^1-\alpha_L^1+\mu}{\sqrt{\nu_L^2+\nu_H^2}}\right)$	$\Phi\left(\frac{\alpha_H^1-2\alpha_L^1-\beta^1+\mu}{\sqrt{\nu_H^2+1}}\right)$
N	$\Phi\left(\frac{2\alpha_H^1-\alpha_L^1+\beta^1}{\sqrt{\nu_L^2+1}}\right)$	1 if $\alpha_L^1 < \alpha_H^1$, 0 otherwise

Table 2: The Payoff Matrix after (P, P) under Average-Result Selection
Note: The row and column labels represent athletes H and L's actions respectively in the second race.

Note that if no athlete participates in the second race (i.e., if (N, N)), the athlete with the better result in the first race is selected with certainty, which induces the loser to participate. Hence, the second-race action profile (N, N) is never chosen with certainty in equilibrium. Which of the other profiles is chosen depends on the realized values of α_H^1, α_L^1 and β^1. We can express the probability of athlete H being selected in the subgame after (P, P), denoted as w_{PP}, as

$$\sum_{i\in\{PN,NP,PP\}}\iiint_i \Phi\left(V_i\right)\phi(z_H)\phi(z_L)\phi(\beta^1)dz_H dz_L d\beta^1$$
$$+\sum_{i=1}^{2}\iiint_{M_i} p_i(z_H, z_L, \beta^1)\phi(z_H)\phi(z_L)\phi(\beta^1)dz_H dz_L d\beta^1, \tag{11}$$

where

$$\begin{aligned}
V_{PN} &= \frac{\nu_H z_H - 2\nu_L z_L - \beta^1 + 2\mu}{\sqrt{\nu_H^2+1}}, \\
V_{NP} &= \frac{2\nu_H z_H - \nu_L z_L + \beta^1 + 2\mu}{\sqrt{\nu_L^2+1}}, \\
V_{PP} &= \frac{\nu_H z_H - \nu_L z_L + 2\mu}{\sqrt{\nu_L^2+\nu_H^2}}, \\
p_1(z_H, z_L, \beta^1) &= \frac{-\Phi\left(V_{PN}\right)\Phi\left(V_{NP}\right)}{\Phi\left(V_{PP}\right)-\Phi\left(V_{PN}\right)-\Phi\left(V_{NP}\right)}, \\
p_2(z_H, z_L, \beta^1) &= \frac{\Phi\left(V_{PP}\right)-\Phi\left(V_{PN}\right)\Phi\left(V_{NP}\right)}{1+\Phi\left(V_{PP}\right)-\Phi\left(V_{PN}\right)-\Phi\left(V_{NP}\right)}, \\
PN &= \left\{z_H, z_L, \beta^1 \;\middle|\; V_{PN} < V_{PP},\ \nu_L z_L > \nu_H z_H + \mu\right\},
\end{aligned}$$

$$
\begin{aligned}
NP &= \left\{ z_H, z_L, \beta^1 \;\middle|\; V_{PP} < V_{NP},\ \nu_L z_L < \nu_H z_H + \mu \right\}, \\
PP &= \left\{ z_H, z_L, \beta^1 \;\middle|\; V_{NP} < V_{PP} < V_{PN} \right\}, \\
M_1 &= \left\{ z_H, z_L, \beta^1 \;\middle|\; V_{PP} < min\{V_{PN}, V_{NP}\},\ \nu_L z_L > \nu_H z_H + \mu \right\}, \\
M_2 &= \left\{ z_H, z_L, \beta^1 \;\middle|\; V_{PP} > max\{V_{PN}, V_{NP}\},\ \nu_L z_L < \nu_H z_H + \mu \right\}.
\end{aligned}
$$

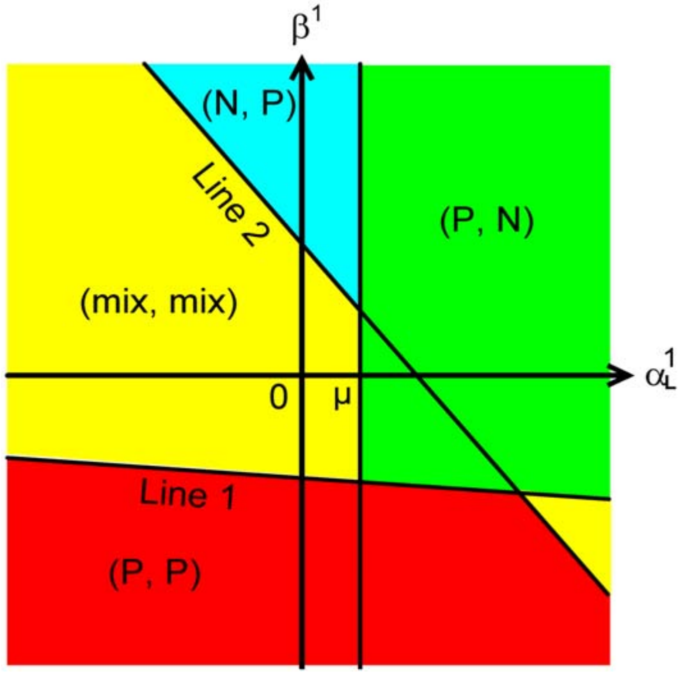
Figure 1(a) (1(b), respectively) illustrates Nash-equilibrium action profiles in the second race after (P, P) according to the realized values of α_L^1 and β^1, where we fix $\nu_H = 0$ and $\nu_L < 0.75$ ($\nu_L > 0.75$). In the figures, lines 1 and 2 respectively represent

$$
\begin{aligned}
\beta^1 &= 2\left(1 - \frac{1}{\nu_L}\right)\alpha_H - \left(2 - \frac{1}{\nu_L}\right)\alpha_L^1, \\
\beta^1 &= 2\left(\sqrt{1 + \frac{1}{\nu_L^2}} - 1\right)\alpha_H - \left(\sqrt{1 + \frac{1}{\nu_L^2}} - 1\right)\alpha_L^1.
\end{aligned}
$$

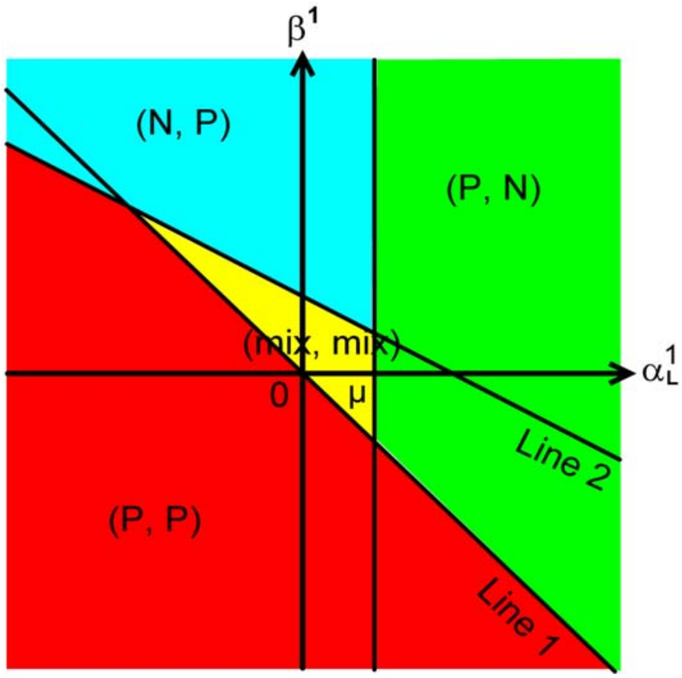
When β^1 is sufficiently small, both athletes participate in the second race (i.e., (P, P)) to improve their average results. When α_L^1 is greater than μ, athlete L returns a better result than athlete H in the first race. Hence, if β^1 is also large, athlete L refrains from participating in the second race whereas athlete H participates to overtake athlete L's first-race result (i.e., (P, N)). The opposite is also true; when α_L^1 is smaller than μ and β^1 is sufficiently large, athlete H refrains from participating in the second race while athlete L participates (i.e., (N, P)). When α_L^1 is smaller than μ and β^1 is realized around its expected value, athlete L's incentive to participate in the second race depends on athlete H's decisions (i.e., (mix, mix)). That is, if athlete H does not participate, athlete L participates to overtake athlete H's first-race result. If athlete H participates, on the other hand, athlete L avoids competing with athlete H again in the second race so that the variance of the difference between the two athletes' final results becomes larger, which increases the probability of athlete H's average result falling below athlete L's average (i.e., first-race) result. Athlete H, on the other hand, tries to prevent this, which results in the mixed-strategy equilibrium.

3.3.5 The First Race

Using (1), (6), (10) and (11), we can write the probability of athlete H being selected according to the pair of actions of the two athletes in the first race as in Table 3. Which cell is chosen by the two athletes depends on the relative



(a) $\nu_H = 0$ and $\nu_L < 0.75$



(b) $\nu_H = 0$ and $\nu_L \geq 0.75$

Figure 1: Nash Equilibria in the Second Race after $(P,\ P)$ under Average-Result Selection

relationship among the probabilities in the four cells.

$H\backslash L$	P	N
P	w_{PP}	w_{PN}
N	w_{NP}	$\Phi\left(\frac{\mu}{\sqrt{\nu_L^2+\nu_H^2}}\right)$

Table 3: The Payoff Matrix in the First Race under Average-Result Selection

3.4 Comparing the Three Procedures

Although athlete H's winning probability is easily calculated analytically under one-race selection (i.e., (1)), this is not the case under best-result selection (i.e., (2)) and average-result selection (i.e., (6), (10) and (11)). Hence, we derive it numerically. Taking each triplet of the values of $\mu \in [0.2, 3.8]$, $\nu_H \in [0, 3.8]$ and $\nu_L \in [0.2, 4]$ by 0.2 units, we generate random variables α_H^j, α_L^j and β^j $(j = 1, 2)$ fifty thousand times to obtain the average winning probability for athlete H under each selection procedure.[3] Note that under average-result selection, the values of μ, ν_H and ν_L affect the realization of each cell in Table 3 (i.e., (1), (6), (10) and (11)), according to which the two athletes' equilibrium first-race actions are determined. The results are generated by Ox version 3.40 for Linux (see Doornik, 2002).

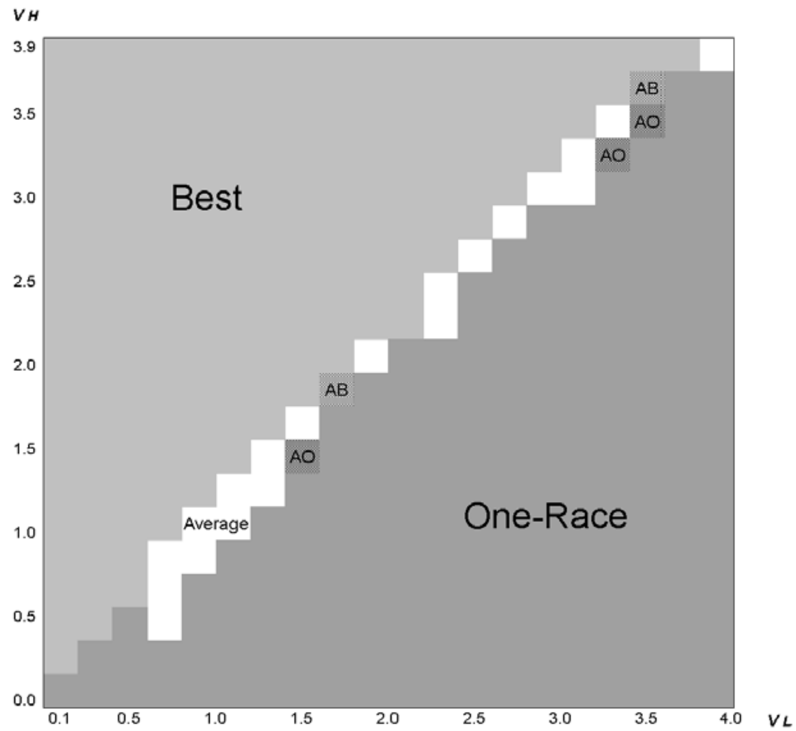
Table 4 describes athlete H's winning probabilities under the three procedures for a subset of parameter values. Figure 2 expresses which selection procedure yields the highest winning probability for athlete H for each pair of values ν_L (horizontal axis) and ν_H (vertical axis), where (a), (b) and (c) are for $\mu = 0.2, 2.0, 3.8$ respectively. We obtain:

Findings: *When μ is sufficiently small, athlete H's winning probability is the highest*
(i) under best-result selection, if athlete H's athlete-specific condition has a larger variance than athlete L's;
(ii) under one-race selection, if athlete H's athlete-specific condition has a smaller variance than athlete L's; and
(iii) under average-result selection, if athlete H's athlete-specific condition has a similar variance to athlete L's.

[3] When we calculate athlete H's winning probability under one-race selection and average-result selection, we have only to generate the first-race random variables α_H^1, α_L^1 and β^1.

Table 4: Athlete H's Winning Probabilities

Parameter Values			Selection Procedures														
μ	ν_H	ν_L	One Race	Best	Average	μ	ν_H	ν_L	One Race	Best	Average	μ	ν_H	ν_L	One Race	Best	Average
0.2	0.2	0.2	0.760	0.768	0.664	2.0	0.2	0.2	1.000	1.000	0.996	3.8	0.2	0.2	1.000	1.000	1.000
		1.0	0.578	0.496	0.561			1.0	0.975	0.971	0.950			1.0	1.000	1.000	0.999
		2.0	0.540	0.394	0.486			2.0	0.840	0.782	0.834			2.0	0.971	0.960	0.964
		3.0	0.527	0.354	0.455			3.0	0.747	0.628	0.726			3.0	0.897	0.845	0.896
		3.8	0.521	0.331	0.438			3.8	0.700	0.556	0.658			3.8	0.841	0.756	0.832
	1.0	0.2	0.578	0.675	0.594		1.0	0.2	0.975	0.993	0.975		1.0	0.2	1.000	1.000	1.000
		1.0	0.556	0.563	0.569			1.0	0.921	0.944	0.923			1.0	0.996	0.998	0.996
		2.0	0.536	0.452	0.504			2.0	0.814	0.793	0.826			2.0	0.955	0.957	0.957
		3.0	0.525	0.388	0.472			3.0	0.736	0.657	0.730			3.0	0.885	0.856	0.893
		3.8	0.520	0.366	0.451			3.8	0.695	0.578	0.671			3.8	0.833	0.768	0.833
	2.0	0.2	0.540	0.690	0.600		2.0	0.2	0.840	0.944	0.900		2.0	0.2	0.971	0.996	0.991
		1.0	0.536	0.632	0.580			1.0	0.814	0.903	0.868			1.0	0.955	0.988	0.979
		2.0	0.528	0.533	0.535			2.0	0.760	0.799	0.804			2.0	0.910	0.943	0.938
		3.0	0.522	0.461	0.499			3.0	0.710	0.692	0.726			3.0	0.854	0.860	0.881
		3.8	0.519	0.421	0.478			3.8	0.679	0.617	0.676			3.8	0.812	0.789	0.830
	3.0	0.2	0.527	0.704	0.605		3.0	0.2	0.747	0.900	0.838		3.0	0.2	0.897	0.980	0.959
		1.0	0.525	0.669	0.592			1.0	0.736	0.872	0.820			1.0	0.855	0.967	0.945
		2.0	0.522	0.591	0.556			2.0	0.710	0.796	0.775			2.0	0.854	0.926	0.909
		3.0	0.519	0.520	0.522			3.0	0.681	0.716	0.717			3.0	0.815	0.857	0.861
		3.8	0.516	0.475	0.504			3.8	0.660	0.653	0.677			3.8	0.784	0.801	0.818
	3.8	0.2	0.521	0.713	0.610		3.8	0.2	0.700	0.875	0.803		3.8	0.2	0.841	0.959	0.927
		1.0	0.520	0.682	0.599			1.0	0.695	0.854	0.787			1.0	0.833	0.948	0.915
		2.0	0.519	0.623	0.566			2.0	0.679	0.796	0.754			2.0	0.812	0.910	0.885
		3.0	0.516	0.562	0.537			3.0	0.660	0.725	0.710			3.0	0.784	0.854	0.843
		3.8	0.515	0.519	0.518			3.8	0.645	0.672	0.677			3.8	0.760	0.803	0.806


VH
3.9
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Best
AB
AO
AB
AO
AO
Average
One-Race
0.1
0.5
1.0
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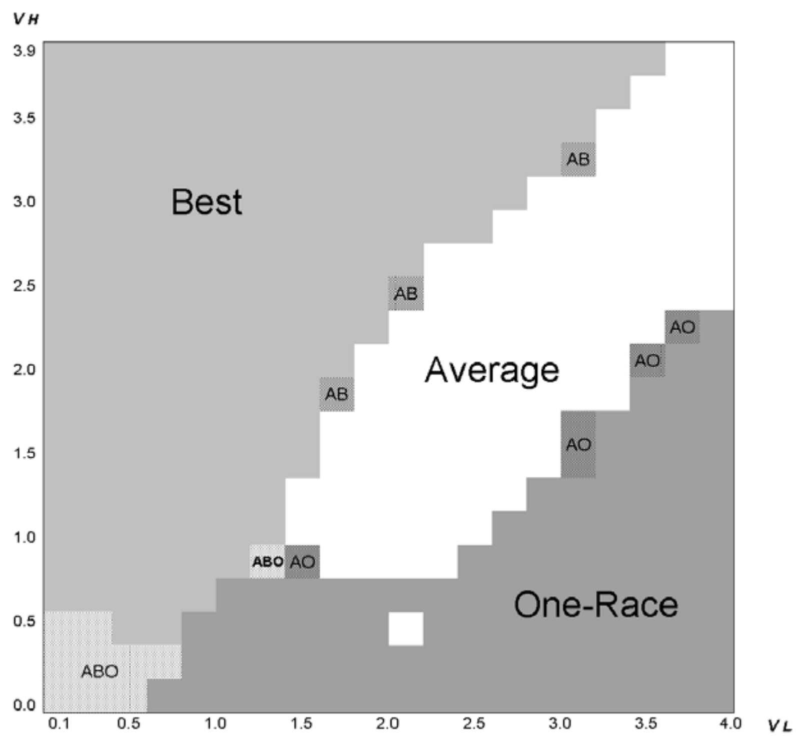
2.0
2.5
3.0
3.5
4.0
VL

(a) $\mu = 0.2$


VH
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0.5
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Best
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AB
AB
Average
AO
AO
AO
ABO
AO
ABO
One-Race
0.1
0.5
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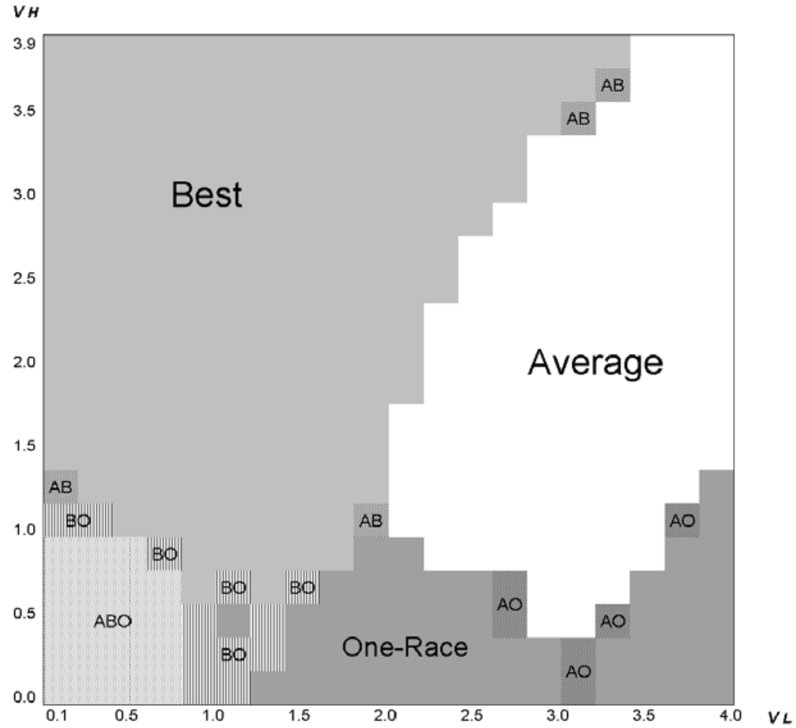
1.5
2.0
2.5
3.0
3.5
4.0
VL

(b) $\mu = 2.0$



(c) $\mu = 3.8$

Figure 2: Comparison of the Three Procedures
Note: "A", "B" and "O" represent average-result selection, best-result selection and one-race selection respectively. In the area designated "AB," for example, average-result selection and best-result selection produce the same highest probability of athlete H being selected, to three decimal places.

As μ increases, one-race selection is replaced with average-result selection for a larger set of values ν_H and ν_L.

These findings tell the selection committee what to take into account when it designs a selection procedure. Even though the selection committee employs average-result selection with the intention to select the athlete who has the better expected result, this purpose may not be achieved because of the strategic choice of races by athletes if the variances of athlete-specific conditions are sufficiently different for the two athletes. The selection committee must take into account not only each athlete's expected result but also the variance of his result when the participation in races is voluntary.

Although each athlete's previous results obtained before the trials reveal how sharply his result tends to fluctuate, it might be difficult for the selection committee to obtain the precise information about each athlete's variance of athlete-specific condition. In this case, the committee cannot identify which of the three procedures is the most appropriate to select the athlete who has the better expected result. Instead, the selection committee must specify what type of athlete it would like to select, including not only the expected result but also the variance of athlete-specific condition. An athlete who has a large variance of athlete-specific condition is more likely to be selected under best-result selection, whereas an athlete who has a small variance is more likely to be selected under one-race selection.

Let us examine the mechanism working behind the above findings. Finding (i) is intuitively plausible. Under best-result selection, the larger variance of r_i^j locates the probability distribution of $max\{r_i^1, r_i^2\}$ farther right to that of r_i^j. Therefore, best-result selection favors the athlete whose results fluctuate more sharply. Note that because participation in both races is the dominant strategy for each athlete under best-result selection, there is no strategic component, and we have only to consider statistical properties.

Findings (ii) and (iii) are non-trivial. We examine these results in detail below.

Strategic Properties of Average-Result Selection

If both athletes were forced to participate in every race under average-result selection, having two races would reduce the variance of the final results, which would, in turn, yield a higher probability of the high-ability athlete being selected than under one-race selection. However, if participation is voluntary, the possibility of strategic choice by athletes to participate in races complicates the characterization of average-result selection. Two kinds of strategic behavior mentioned below are important in our case.

Own-Result Maximization

Under average-result selection, the strategy for attaining the best final result is to participate in the first race with certainty and also to participate in the second race if the first-race result is worse than the expected result (i.e., if $r_i^1 < E(r_i^1)$) but to refrain otherwise; the athlete's expected final result under this strategy is $0.75E(r_i^1) + 0.25E(r_i^1|r_i^1 \geq E(r_i^1))$, which is greater as r_i^1 has a larger variance.[4] Therefore, an athlete whose athlete-specific condition has a

[4] This expected final result is derived as follows:
$$Pr(r_i^1 \geq E(r_i^1))E(r_i^1|r_i^1 \geq E(r_i^1)) + Pr(r_i^1 < E(r_i^1))[0.5E(r_i^1|r_i^1 < E(r_i^1)) + 0.5E(r_i^2)]$$
$$= 0.5E(r_i^1|r_i^1 \geq E(r_i^1)) + 0.5\left[0.5\left(2E(r_i^1) - E(r_i^1|r_i^1 \geq E(r_i^1))\right) + 0.5E(r_i^1)\right]$$

larger variance has a stronger incentive to participate in the first race to make use of his advantage to improve his final result. The outcome is that average-result selection yields a higher winning probability for athlete H than one-race selection when his athlete-specific condition has a larger variance than athlete L's (i.e., $\nu_H > \nu_L$) (see Table 4).

Variance Maximization

Average-result selection involves another strategic property. An athlete whose athlete-specific condition has a small variance is at a disadvantage in own-result maximization. Hence, such an athlete tries to increase, for himself, the variance of the difference of final results between the two athletes so that he obtains a bigger chance to finish with a better final result than the opponent.

There are two ways to achieve this variance maximization. The first way stems from the race-specific random variable. That is, the athlete with a disadvantage avoids competing with the opponent under the same race-specific condition, by participating in a different race from the opponent.

The second way stems from the athlete-specific random variable. That is, the athlete with a disadvantage participates in one race only. If an athlete uses this second way, he participates in the second race, rather than the first race. This is because if he participates in the first race, even though his first-race result turns out good, the opponent, who observes the athlete's first-race result, tries to achieve the better final result by participating in the second race as well as the first race. The opponent cannot behave in such a way if the athlete participates in the second race only.

First-Race Action Profiles

Figure 3 expresses equilibrium first-race action profiles under average-result selection, where (a), (b) and (c) are for $\mu = 0.2, 2.0, 3.8$, respectively. Let us begin with Figure 3(a) to understand the strategic interaction between athletes.

First, suppose that the variances of athlete-specific conditions are sufficiently close to each other for the two athletes. When they are sufficiently large, both athletes try to make use of the large variances by participating in the first race (i.e., (P, P)), which is the best way to maximize their own final results. When their variances are sufficiently small, on the other hand, it is hard for athlete L to overcome his disadvantage of a poorer expected

$= 0.75E(r_i^1) + 0.25E(r_i^1 | r_i^1 \geq E(r_i^1))$,
where we use $E(r_i^1) = 0.5E(r_i^1 | r_i^1 \geq E(r_i^1)) + 0.5E(r_i^1 | r_i^1 < E(r_i^1))$ and $E(r_i^1) = E(r_i^2)$.

result (i.e., $E(\alpha_L^j) = 0 < \mu = E(\alpha_H^j)$). Hence, athlete L needs to increase, for himself, the variance of the difference between the two athletes' final results. For this purpose, athlete L avoids competing with athlete H in the same race. Athlete H, on the other hand, tries to preserve his advantage by competing with athlete L under the same race-specific condition. As a result, mixed strategies are chosen in the south-west region of the figure (i.e., (mix, mix)).[5]

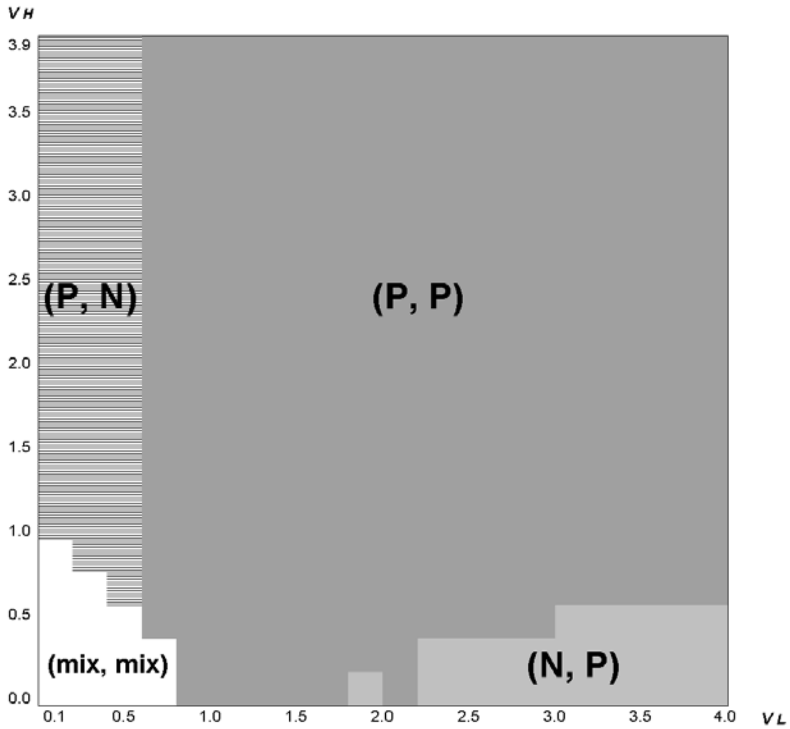
Next, suppose that the variances of athlete-specific conditions are sufficiently different for the two athletes. The athlete with the larger variance participates in the first race to optimize his own result. Since the athlete with the smaller variance is at a disadvantage in improving his final result, he instead tries to increase the variance of the difference between the two athletes' final results by abstaining from the first race (i.e., (P, N) and (N, P)). As the expected result for athlete H improves (i.e., Figures 3(b) for $\mu = 2.0$ and (c) for $\mu = 3.8$), his incentive to preserve this advantage becomes stronger. Hence, even when the variance of his athlete-specific condition is much smaller than for athlete L, athlete H participates in the first race if athlete L does. As a result, the region (N, P) is replaced with (P, P) in the figures. At the same time, the larger part of region (P, P) is replaced with (P, N). This is because athlete L, whose disadvantage in terms of a poorer expected result becomes more severe, tries to increase the variance of the difference between the two athletes' final results.

In this way, the opportunity for variance maximization under average-result selection enables the athlete with a disadvantage to mitigate it in part. This opportunity does not exist under the other procedures. This only matters in the comparison between average-result selection and one-race selection.

Average versus One Race

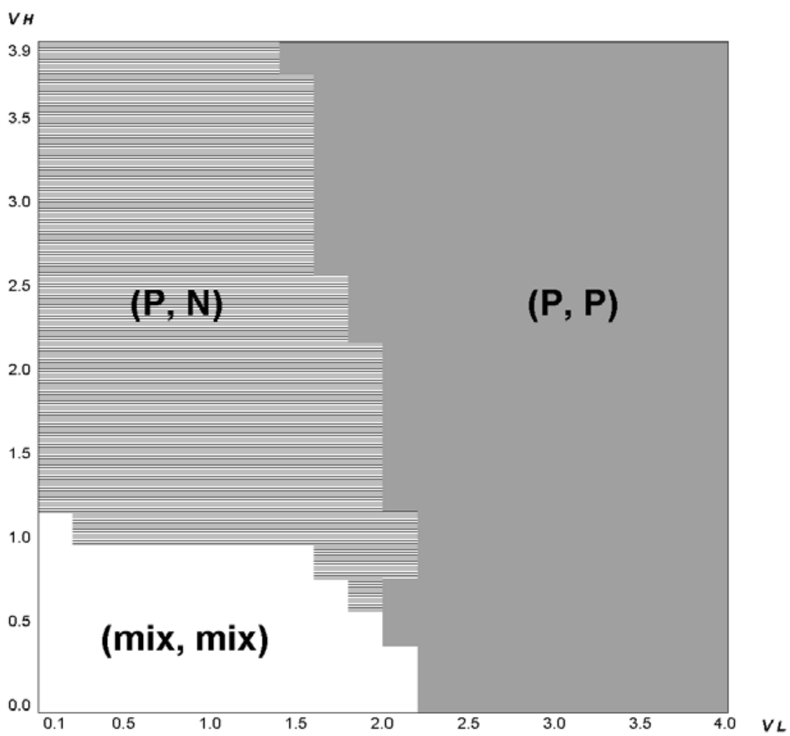
Now we compare average-result selection with one-race selection in the region where $\nu_H < \nu_L$. From Figures 2 and 3, we can ascertain that athlete H's winning probability is higher under average-result selection than under one-race selection if the first-race action profile under average-result selection is (P, N) or if it is (P, P) and ν_H is sufficiently large. The opposite is true if

[5] This mixed-strategy equilibrium action profile is replaced with (P, N) if the race-specific random variable, β^j, is eliminated from the model. This is because athlete H loses the incentive to compete with athlete L under the same race-specific condition, whereas athlete L still has the incentive to participate in one race only. Since the incentive of variance maximization which stems from the athlete-specific random variable remains even without the race-specific random variable, the comparison of the three selection procedures is not qualitatively affected by the elimination of β^j.



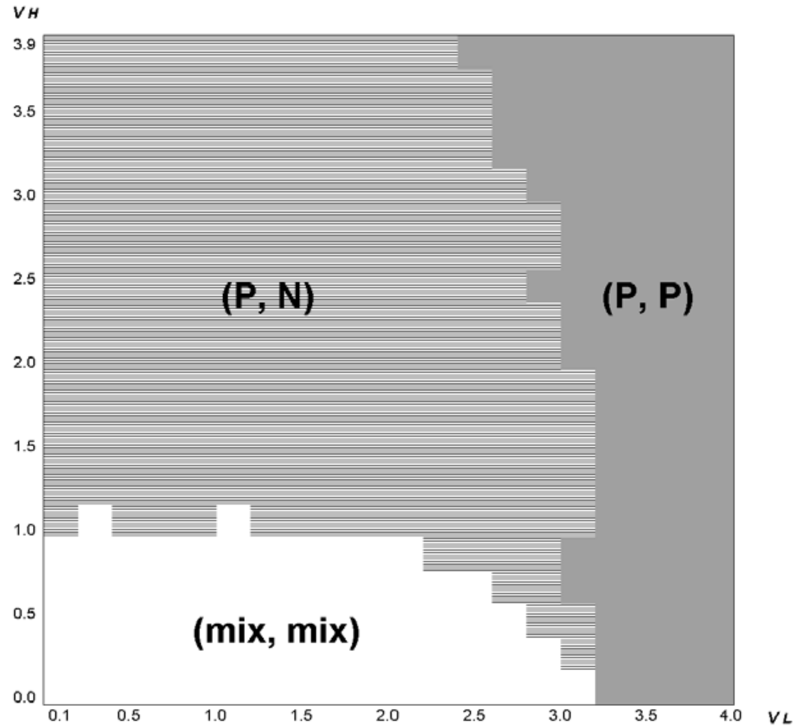
VH
3.9
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
(P, N)
(P, P)
(mix, mix)
(N, P)
0.1
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
VL

(a) $\mu = 0.2$



VH
3.9
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
(P, N)
(P, P)
(mix, mix)
0.1
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
VL

(b) $\mu = 2.0$



(c) $\mu = 3.8$

Figure 3: Equilibrium First-Race Action Profiles under Average-Result Selection

the first-race action profile is (N, P) or (mix, mix) or if it is (P, P) and ν_H is sufficiently small.

Let us examine what is happening behind each of the above first-race action profiles under average-result selection. (P, N) implies that athlete L successfully avoids competing with athlete H in the same race and also participates in one race only (i.e., variance maximization), whereas only athlete H uses the own-result maximization strategy (i.e., own-result maximization). Because ν_H is sufficiently large while ν_L is sufficiently small in the region (P, N), the effect of own-result maximization overcomes the effect of variance maximization.

(P, P) implies that both athletes use the own-result maximization strategy and that athlete H successfully competes with athlete L in the same race. These two effects favor athlete H if ν_H is sufficiently large, but the effect of own-result maximization favors athlete L enough to overcome the effect of variance maximization if ν_H is small.

In the region (N, P), both μ and ν_H are small while ν_L is sufficiently large, and so the expected final result under the own-result maximization strategy is greater for athlete L than for athlete H. This effect is severe, although athlete H can mitigate it in part by not participating in the first race.

Finally, (mix, mix) implies that athlete H can neither employ his own-result maximization strategy nor compete with athlete L in the same race with certainty. These two disadvantages make average-result selection less attractive for athlete H.

4 Conclusion

We have compared three selection procedures in the context of Olympic trials. While one-race selection and best-result selection do not create any opportunity for strategic choice of races, average-result selection does, which makes the comparison non-trivial and requires a game-theoretic analysis. Our model reveals athletes' incentives and shows which procedure is most likely to select a high-ability athlete as the representative for each set of parameter values.

There may exist more complicated procedures which yield higher winning probabilities for the high-ability athlete than the three procedures we have dealt with. In finding the optimal selection procedure in our context, we may have to consider whether athletes' participation decisions themselves bring any information about their abilities. In addition, if there are two or more Olympic spots, as in the actual trials, the combination of which athletes to select becomes another issue. Designing these types of procedures is a topic for further study.

Appendix

A Olympic Athlete Selection in Practice

This appendix provides more detailed information about how athletes in the U.S.A., Australia, Japan and Germany were selected for the Beijing Olympic Games in 2008 (i.e., Table 1). In every country, athletes must meet or better some Olympic qualification standard.

A.1 U.S.A.

Track and field including 20 km race walk: In order to qualify for nomination, athletes must have competed in the 2008 U.S. Olympic Team

Trials - Track & Field on June 27-July 6, 2008 in Eugene, OR. The criterion used to nominate athletes in individual events was each athlete's rank order of finish in a designated event at the Trials.[6]

Men's 50 km race walk: The procedure was the same as track and field except that the trial was held on February 9, 2008 in Miami, FL.

Marathon: Up to three athletes per gender were nominated based on rank order of finish at the U.S. Olympic Team Trials - Men's marathon on November 3, 2007, in New York City, NY, and Women's marathon on April 20, 2008, in Boston, MA.[7]

Swimming: Athletes were selected as nominees to the Olympic Team based on their performance in the final of each individual event at the 2008 U.S. Olympic Team Trials - Swimming on June 29-July 6, 2008 in Omaha, NE. Nominees were selected from the order of finish priorities.[8]

A.2 Australia

Track and field including 20 km race walk: Athletes were to be entitled to Early Nomination as a result of their performance at the 2007 IAAF World Championships in Osaka, Japan on August 25-September 2, 2007, as set out as follows. An athlete who won a medal in each individual event was entitled to Early Nomination. If more than two Australian athletes won medals in the same event, only the gold and silver medalists in that event were entitled to Early Nomination. At most, two athletes were to be entitled to, or be chosen for, Early Nomination in each individual event.

In addition to the athletes who were entitled to Early Nomination as set out immediately above, the Selectors, at their discretion, chose any athletes for Early Nomination who, in their opinion, had by virtue of their past international performances and their current form, indicated that they were medal

[6]Source: USA Track & Field Athlete Selection Procedures Olympic Games - 2008, Revised March 13, 2008 (`http://www.usatf.org/events/2008/OlympicGames/entry/selectionProcedures_TF.asp`).

[7]Source: USA Track & Field: Selection Procedures - Marathon (`http://www.usatf.org/events/2008/OlympicGames/entry/selectionProcedures_marathon.asp`).

[8]Source: USA Swimming Athlete Selection Procedures, 2008 Olympic Games, Beijing, China; February 23, 2007, Amended: February 22, 2008 (`http://usaswimming.org/USASWeb/_Rainbow/Documents/8af2b987-6b50-4b82-92de-322f657afb7f/08%20SOG%20SWI%20ATH%20(Pool)%20AMENDED%202-22-0820signed.pdf`).

prospects at the 2008 Beijing Olympic Games. This discretion was absolute and need not be exercised.

Besides the Early Nomination, athletes were to be entitled to Nomination by right of performance at the Nomination Trials in Brisbane on February 29-March 2, 2008 (including 20 km race walk). The first-placed eligible athlete was entitled to Nomination. The Selectors, at their discretion, chose additional athletes up to the limit of three per individual event for Nomination following the Nomination Trials. This discretion was absolute and need not be exercised.[9]

Men's 50 km race walk: The procedure was the same as track and field except that the trial was the 2008 Australian 50 km Walk Championships.

Marathon: The early nomination procedure was the same as track and field. However, there was no nomination trial. Athletes were chosen for Nomination at the discretion of the Selectors at a meeting of the Selectors that was held on or before May 19, 2008. This discretion was absolute and need not be exercised.

Swimming: Swimming Australia Ltd only nominated athletes who had competed in the Telstra Swimming Selection Trials for the Beijing 2008 Australian Olympic Team on March 22-29, 2008 in Sydney, and only nominated the first two placed finishers in the final of all individual Olympic events.[10]

A.3 Japan

Track and field:

1. The highest-ranked athlete among the Japanese participants who got a place in each individual event at the 2007 IAAF World Championships was selected as a representative.

2. Except for clause (1) above, the athlete who won the first place in each individual event at the 92nd Japan Track and Field National Championships on June 26-29, 2008 in Kawasaki was selected as a representative.

[9]Source: 2008 Australian Olympic Team, Athlete Australia, Nomination Criteria (http://corporate.olympics.com.au/files/13/Athletics_Australia_Nomination_Criteria_FINAL_Amended_14_January_2008.pdf).

[10]Source: 2008 Australian Olympic Team, Swimming Australia Ltd, Nomination Criteria (http://www.wasa.asn.au/documents/Public/Competitors/2007/2008%20australian%20olympic%20team %20nomination%20&%20criteria.pdf).

3. Except for clauses (1) and (2) above, hopeful athletes at the 2008 Olympic Games were selected as representatives, out of athletes who got the first place, or highly-ranked athletes who got a place at the series of trials, including the 92nd Japan Track and Field National Championships.[11]

Race walk:

1. The highest-ranked athlete among the Japanese participants who got a place in each event at the 2007 IAAF World Championships was selected as a representative.

2. Except for clause (1) above, athletes who won the first place in each event at the 91st Japan National Race Walk Championships (20 km) on February 27, 2008 in Kobe, and the 92nd Japan National Race Walk Championships (50 km) on April 12-13, 2008 in Wajima, were selected as representatives.

3. Except for clauses (1) and (2) above, hopeful athletes at the 2008 Olympic Games were selected as representatives out of athletes who got the first place, or highly-ranked athletes who got a place at two trials per event, including the 91st Japan National Race Walk Championships (20 km) and the 92nd Japan National Race Walk Championships (50 km).

Marathon: Up to three athletes per gender were selected as representatives.

1. The highest-ranked Japanese medalist at the 2007 IAAF World Championships was selected as a representative.

2. Except for clause (1) above, athletes who were expected to win a medal or a place at the 2008 Olympic Games, were selected as representatives out of highly-ranked athletes at each of three trial races.

Swimming:

1. Athletes who established world records at the International Swim Meet 2007 in Japan, on August 21-24, 2007, were selected as representatives.

[11]Source: Track and Field, the 29th Olympic Games (2008/Beijing) Qualification (`http://www.joc.or.jp/beijing/sports/atheletics.html`).

2. Except for clause (1) above, the first two placed athletes in the finals of individual events at the Japan Swim 2008 on April 15-20, 2008 in Tokyo were selected as representatives.[12]

A.4 Germany

Track and field: Athletes had to fulfill both two (1st and 2nd) criteria in at least one race from several alternatives such as (a) European Cup (Annecy/France, June 21-22, 2008), (b) German Championships (Nuremberg/Germany, July 5-6, 2008), (c) other official games qualified by international authorities, and (d) regional games held by local authorities. As an exception, athletes who finished in the top 8 in each event at the 2007 IAAF World Championships were required only to satisfy the 1st criterion. In case more than three athletes had passed these criteria, the authority had the final say on the selection.[13]

Race walk and marathon: At most, three athletes were selected in each event according to the following criteria: (a) Athletes who finished in the top 10 at the 2007 IAAF World Championships; (b) Athletes who fulfilled the Olympic criterion in any official games; and (c) Athletes who won 11th to 20th places in the 2007 IAAF World Championships. In case more than three athletes passed these criteria, the authority had the final say on the selection.

Swimming: Athletes had to win the first or second place with Olympic criteria in the German Championships 2008 in Berlin. In all cases, the authority had the final say on the selection.[14]

B Two Variations of the Model

We consider two different versions of the model in the main text. The first introduces the possibility of injury. The second is specific but analytically solvable.

[12]Source: Japan Swimming Federation (`http://www.swim.or.jp/11_committee/01_swim/0711271.html`).

[13]Source: German Athletics (`http://www.deutscher-leichtathletik-verband.de/image.php?AID=13735&VID=0`).

[14]Source: German Swimming (`http://schwimmen.dsv.de/Files/MeetInfos/ol2008.pdf`).

B.1 The Effect of Injury

Intuitively, if there is a possibility of injury, which prevents athletes from participating in particular races with a positive probability, then the high-ability athlete may not prefer one-race selection because he loses as soon as he misses that one race. Let us confirm this intuition by extending our model.

Athletes decide simultaneously and independently whether they are going to participate in each race. After both athletes make their participation decisions, each athlete may be injured with probability q so that he is unable to participate in the race. This probability is independent between athletes and between races, and it is also independent of any random variables such as athlete-specific conditions and race-specific conditions.

We assume that an athlete is not selected if he does not participate in any race (even if it is because of injury). When neither athlete participates in any race, both of them are excluded from the selection process.

B.1.1 One-Race Selection

The probability of athlete H being selected is

$$(1-q)^2\Phi\left(\frac{\mu}{\sqrt{\nu_L^2+\nu_H^2}}\right)+q(1-q).$$

B.1.2 Best-Result Selection

The dominant strategy for both athletes is to decide to participate in both races. The probability of athlete H being selected is

$$\begin{aligned}
& \Pr(H\ wins) \\
= \; & (1-q)^4 \Pr(\max\{\alpha_L^1+\beta^1, \alpha_L^2+\beta^2\} < \max\{\alpha_H^1+\beta^1, \alpha_H^2+\beta^2\}) \\
& +(1-q)^3 q \Pr(\max\{\alpha_L^1+\beta^1, \alpha_L^2+\beta^2\} < \alpha_H^2+\beta^2) \\
& +(1-q)^3 q \Pr(\max\{\alpha_L^1+\beta^1, \alpha_L^2+\beta^2\} < \alpha_H^1+\beta^1) \\
& +(1-q)^3 q \Pr(\alpha_L^1+\beta^1 < \max\{\alpha_H^1+\beta^1, \alpha_H^2+\beta^2\}) \\
& +(1-q)^3 q \Pr(\alpha_L^2+\beta^2 < \max\{\alpha_H^1+\beta^1, \alpha_H^2+\beta^2\}) \\
& +(1-q)^2 q^2 \Pr(\alpha_L^1+\beta^1 < \alpha_H^1+\beta^1) \\
& +(1-q)^2 q^2 \Pr(\alpha_L^1+\beta^1 < \alpha_H^2+\beta^2) \\
& +(1-q)^2 q^2 \Pr(\alpha_L^2+\beta^2 < \alpha_H^1+\beta^1) \\
& +(1-q)^2 q^2 \Pr(\alpha_L^2+\beta^2 < \alpha_H^2+\beta^2) \\
& +(1-q)^2 q^2 + 2(1-q) q^3
\end{aligned}$$

$$
\begin{aligned}
= \quad & (1-q)^4 \Pr(\max\{\alpha_L^1+\beta^1, \alpha_L^2+\beta^2\} < \max\{\alpha_H^1+\beta^1, \alpha_H^2+\beta^2\}) \\
& +2(1-q)^3 q \Pr(\max\{\alpha_L^1+\beta^1, \alpha_L^2+\beta^2\} < \alpha_H^2+\beta^2) \\
& +2(1-q)^3 q \Pr(\alpha_L^1+\beta^1 < \max\{\alpha_H^1+\beta^1, \alpha_H^2+\beta^2\}) \\
& +2(1-q)^2 q^2 \Phi\left(\frac{\mu}{\sqrt{\nu_L^2+\nu_H^2}}\right) + 2(1-q)^2 q^2 \Phi\left(\frac{\mu}{\sqrt{\nu_L^2+\nu_H^2+2}}\right) \\
& +(1-q)^2 q^2 + 2(1-q) q^3.
\end{aligned}
$$

B.1.3 Average-Result Selection

Subgame after (N, N)**:** The probability of athlete H being selected in this subgame, denoted as P_{nn}, is the same as that for one-race selection. Note that the probability of athlete L being selected is $1-P_{nn}-q^2$, and neither of them is selected with probability q^2.

Subgame after (P, N)**:** Athlete L decides to participate in the second race. If athlete H does not participate in the second race, his winning probability is

$$
(1-q)\Pr(\alpha_L^2+\beta^2 < \alpha_H^1+\beta^1) + q = (1-q)\Phi\left(\frac{\alpha_H^1+\beta^1}{\sqrt{\nu_L^2+1}}\right) + q.
$$

If athlete H decides to participate in the second race, his winning probability is

$$
\begin{aligned}
& (1-q)^2 \Pr\left(\alpha_L^2+\beta^2 < \frac{1}{2}(\alpha_H^1+\beta^1+\alpha_H^2+\beta^2)\right) \\
& +q(1-q)\Pr(\alpha_L^2+\beta^2 < \alpha_H^1+\beta^1) + q \\
& = (1-q)^2\Phi\left(\frac{\alpha_H^1+\beta^1+\mu}{\sqrt{4\nu_L^2+\nu_H^2+1}}\right) + q(1-q)\Phi\left(\frac{\alpha_H^1+\beta^1}{\sqrt{\nu_L^2+1}}\right) + q.
\end{aligned}
$$

Hence, athlete H's winning probability, denoted as P_{pn}, is

$$
\begin{aligned}
E\Big[\max\Big\{ & (1-q)\Phi\left(\frac{\alpha_H^1+\beta^1}{\sqrt{\nu_L^2+1}}\right), \\
& (1-q)^2\Phi\left(\frac{\alpha_H^1+\beta^1+\mu}{\sqrt{4\nu_L^2+\nu_H^2+1}}\right) + q(1-q)\Phi\left(\frac{\alpha_H^1+\beta^1}{\sqrt{\nu_L^2+1}}\right) \Big\}\Big] + q.
\end{aligned}
$$

Subgame after (N, P)**:** Athlete H decides to participate in the second race. If athlete L does not participate in the second race, athlete H's winning probability is

$$(1-q)\Pr(\alpha_L^1 + \beta^1 < \alpha_H^2 + \beta^2) = (1-q)\Phi\left(\frac{\mu - \alpha_L^1 - \beta^1}{\sqrt{\nu_H^2 + 1}}\right).$$

If athlete L decides to participate in the second race, athlete H's winning probability is

$$\begin{aligned}
&(1-q)^2 \Pr\left(\frac{1}{2}(\alpha_L^1 + \beta^1 + \alpha_L^2 + \beta^2) < \alpha_H^2 + \beta^2\right) \\
&+q(1-q)\Pr(\alpha_L^1 + \beta^1 < \alpha_H^2 + \beta^2) \\
&= (1-q)^2\Phi\left(\frac{2\mu - \alpha_L^1 - \beta^1}{\sqrt{4\nu_H^2 + \nu_L^2 + 1}}\right) + q(1-q)\Phi\left(\frac{\mu - \alpha_L^1 - \beta^1}{\sqrt{\nu_H^2 + 1}}\right).
\end{aligned}$$

Hence, athlete H's winning probability, denoted as P_{np}, is

$$\begin{aligned}
&E\Big[\min\Big\{(1-q)\Phi\left(\frac{\mu - \alpha_L^1 - \beta^1}{\sqrt{\nu_H^2 + 1}}\right), \\
&\quad (1-q)^2\Phi\left(\frac{2\mu - \alpha_L^1 - \beta^1}{\sqrt{4\nu_H^2 + \nu_L^2 + 1}}\right) + q(1-q)\Phi\left(\frac{\mu - \alpha_L^1 - \beta^1}{\sqrt{\nu_H^2 + 1}}\right)\Big\}\Big].
\end{aligned}$$

Subgame after (P, P)**:** The probability of athlete H being selected is summarized in Table B1, where

$$\begin{aligned}
A &= (1-q)^2\Phi\left(\frac{\alpha_H^1 - \alpha_L^1 + \mu}{\sqrt{\nu_L^2 + \nu_H^2}}\right) + q(1-q)\Phi\left(\frac{\alpha_H^1 - 2\alpha_L^1 - \beta^1 + \mu}{\sqrt{\nu_H^2 + 1}}\right) \\
&\quad +(1-q)q\Phi\left(\frac{2\alpha_H^1 - \alpha_L^1 + \beta^1}{\sqrt{\nu_L^2 + 1}}\right) + q^2\mathbf{1}\{\alpha_L^1 < \alpha_H^1\}, \\
B &= (1-q)\Phi\left(\frac{\alpha_H^1 - 2\alpha_L^1 - \beta^1 + \mu}{\sqrt{\nu_H^2 + 1}}\right) + q\mathbf{1}\{\alpha_L^1 < \alpha_H^1\}, \\
C &= (1-q)\Phi\left(\frac{2\alpha_H^1 - \alpha_L^1 + \beta^1}{\sqrt{\nu_L^2 + 1}}\right) + q\mathbf{1}\{\alpha_L^1 < \alpha_H^1\}.
\end{aligned}$$

Note that $\mathbf{1}\{\alpha_L^1 < \alpha_H^1\} = 1$ if $\alpha_L^1 < \alpha_H^1$ while $\mathbf{1}\{\alpha_L^1 < \alpha_H^1\} = 0$ otherwise. Let

$H \backslash L$	P	N
P	A	B
N	C	$\mathbf{1}\{\alpha_L^1 < \alpha_H^1\}$

Table B1: The Payoff Matrix in the Second Race after (P, P) under Average-Result Selection with Injury

P_{pp} denote the probability of athlete H being selected in this subgame. It is difficult to compute P_{pp}, and we rely on numerical techniques to compute it.

The First Race: The payoff matrix in the first race is described in Table B2, where

$$\begin{aligned} D &= (1-q)^2 P_{pp} + q(1-q)P_{pn} + (1-q)qP_{np} + q^2 P_{nn}, \\ E &= (1-q)P_{pn} + qP_{nn}, \\ F &= (1-q)P_{np} + qP_{nn}. \end{aligned}$$

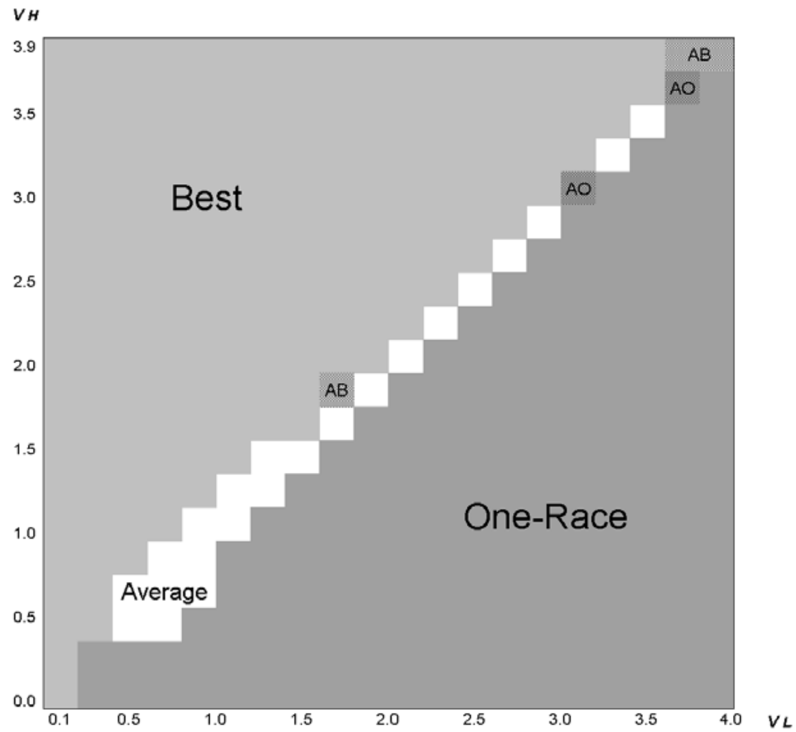
As for the model in the main text, we compute the equilibrium probabilities numerically.

$H \backslash L$	P	N
P	$D, 1-D-q^4$	$E, 1-E-q^3$
N	$F, 1-F-q^3$	$P_{nn}, 1-P_{nn}-q^2$

Table B2: The Payoff Matrix in the First Race under Average-Result Selection with Injury

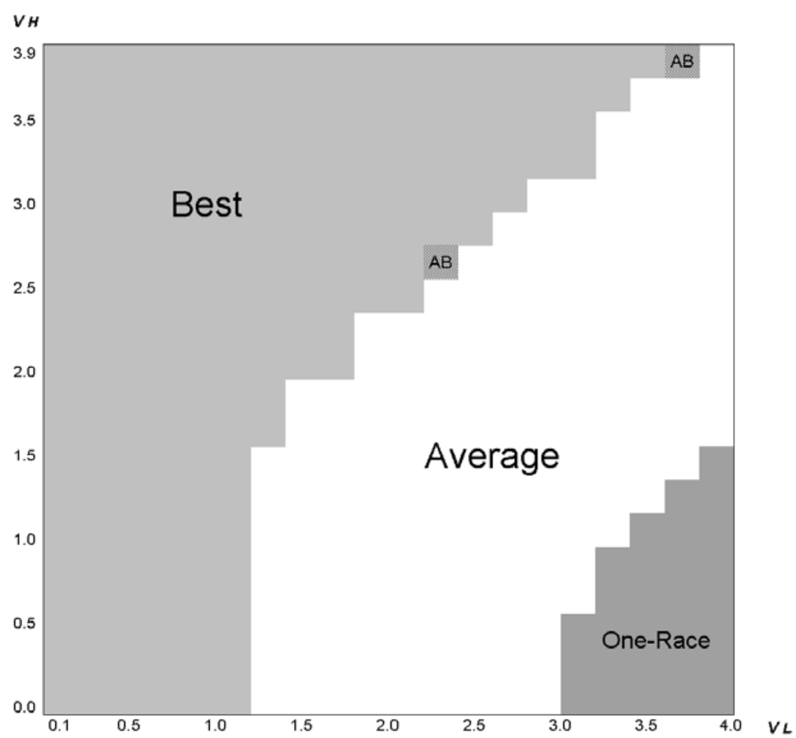
B.1.4 Findings

We set $q = 0.05$, and obtain the numerical result in Figure B1. The relationship between best-result selection and average-result selection is similar to the case of no injury (i.e., Figure 2). On the other hand, one-race selection is replaced with average-result selection in a large area when $\mu = 2.0$ (Figure B1(b)), and it disappears from Figure B1(c) for $\mu = 3.8$. This result confirms our intuition mentioned above.



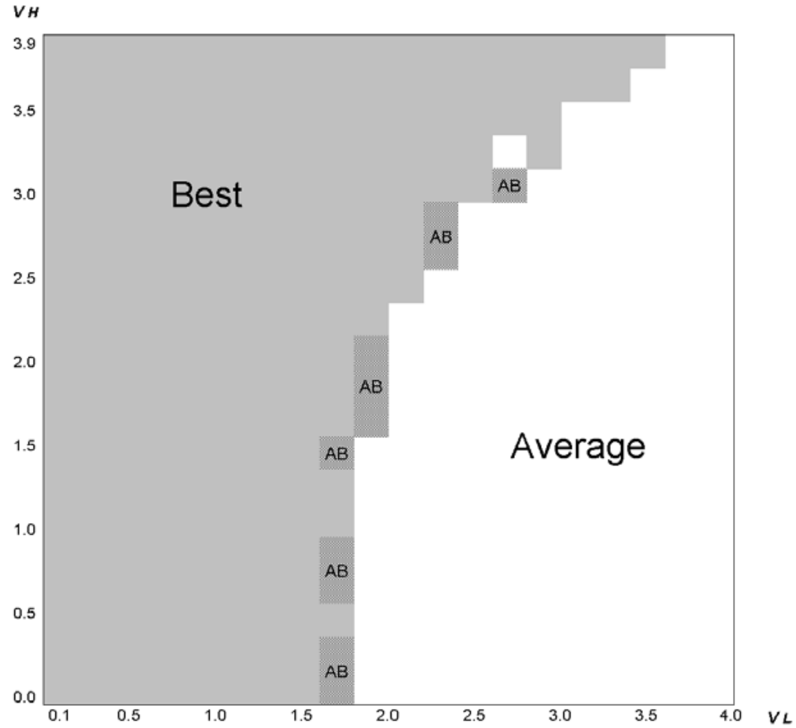
VH
3.9
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.1
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
VL
AB
AO
Best
AO
AB
One-Race
Average

(a) $\mu = 0.2$



VH
3.9
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.1
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
VL
AB
Best
AB
Average
One-Race

(b) $\mu = 2.0$



(c) $\mu = 3.8$

Figure B1: Comparison of the Three Procedures with Injury

B.2 A Simpler Model

To confirm our results analytically, here we provide a simpler version of our model in which athlete H's winning probability is explicitly derived under the three procedures although the range of parameter values is restricted. We assume $\mu = \nu_H = 0$, while α_L^j takes two values $-A < 0$, and A with probability $p \in (0.5, 1)$ and $1-p$ respectively. Then we have $E(\alpha_L^j) = (1-2p)A < 0$, and $\nu_L = 2A\sqrt{p(1-p)}$. The race-specific random variable β^j takes $-b < 0$, and b equiprobably.

B.2.1 One-Race Selection

Athlete H's winning probability is

$$Pr(\alpha_L^1 + \beta^1 < \alpha_H^1 + \beta^1) = Pr(\alpha_L^1 < 0) = Pr(\alpha_L^1 = -A) = p.$$

B.2.2 Best-Result Selection

Athlete H's winning probability is expressed as

$$Pr(max\{\alpha_L^1 + \beta^1, \alpha_L^2 + \beta^2\} < max\{\beta^1, \beta^2\}).$$

The following three cases can arise with respect to β^1 and β^2. First suppose $\beta^1 = \beta^2 = -b$ or $\beta^1 = \beta^2 = b$, which happens with probability 1/2. In this case, the above expression can be rewritten as

$$Pr(max\{\alpha_L^1, \alpha_L^2\} < 0) = Pr(\alpha_L^1 = -A)Pr(\alpha_L^2 = -A) = p^2.$$

Next suppose $\beta^1 = b$ and $\beta^2 = -b$, which happens with probability 1/4. The possible outcomes given $\beta^1 = b$ and $\beta^2 = -b$ are summarized in Table B3.

probability	α_L^1	α_L^2	H	L	winner
$(1-p)^2$	A	A	b	$A+b$	L
$(1-p)p$	A	$-A$	b	$A+b$	L
$(1-p)p$	$-A$	A	b	$max\{-A+b, A-b\}$	H if $A < 2b$, L if $A > 2b$
p^2	$-A$	$-A$	b	$-A+b$	H

Table B3: Possible Outcomes Given $\beta^1 = b$ and $\beta^2 = -b$

From Table B3, the winning probability of athlete H conditional on $\beta^1 = b$ and $\beta^2 = -b$ is calculated as $p(1-p) + p^2 = p$ if $A < 2b$, and p^2 if $A > 2b$.

The final case, $\beta^1 = -b$ and $\beta^2 = b$, which happens with probability 1/4, is exactly symmetric to the above case. Hence, under best-result selection, athlete H's winning probability is

$$\begin{cases} \frac{1}{2}p^2 + 2\frac{1}{4}p = \frac{1}{2}p + \frac{1}{2}p^2 & \text{if } A < 2b \\ \frac{1}{2}p^2 + 2\frac{1}{4}p^2 = p^2 & \text{if } A > 2b. \end{cases}$$

In both cases, athlete H's winning probability is lower than that for one-race selection, p.

B.2.3 Average-Result Selection

We follow the backwards-induction procedure to derive the subgame perfect equilibria.

Subgame after (N, N)**:** In this subgame, both athletes participate in the second race only, which is equivalent to one-race selection. Hence, athlete H wins with probability p.

Subgame after (P, N)**:** In this subgame, athlete L must participate in the second race.

First suppose $\beta^1 = b$. If athlete H does not participate in the second race, the possible outcomes are summarized in Table B4.

probability	α_L^2	β^2	H	L	winner
$\frac{1}{2}(1-p)$	A	b	b	$A+b$	L
$\frac{1}{2}(1-p)$	A	$-b$	b	$A-b$	H if $A < 2b$, L if $A > 2b$
$\frac{1}{2}p$	$-A$	b	b	$-A+b$	H
$\frac{1}{2}p$	$-A$	$-b$	b	$-A-b$	H

Table B4: Possible Outcomes When H Does Not Participate Given $\beta^1 = b$

If athlete H participates in the second race, Table B5 is the result.

probability	α_L^1	β^2	H	L	winner
$\frac{1}{2}(1-p)$	A	b	A	$A+b$	L
$\frac{1}{2}(1-p)$	A	$-b$	0	$A-b$	H if $A < b$, L if $A > b$
$\frac{1}{2}p$	$-A$	b	b	$-A+b$	H
$\frac{1}{2}p$	$-A$	$-b$	0	$-A-b$	H

Table B5: Possible Outcomes When H Participates Given $\beta^1 = b$

It is easy to see that athlete H does not participate in the second race if $b < A < 2b$, and is indifferent between participation and nonparticipation otherwise. His winning probability conditional on $\beta^1 = b$ is

$$\begin{cases} \frac{1}{2}(1-p) + 2\frac{1}{2}p = \frac{1}{2} + \frac{1}{2}p & \text{if } A < 2b \\ 2\frac{1}{2}p = p & \text{if } A > 2b. \end{cases}$$

Next suppose $\beta^1 = -b$. If athlete H does not participate in the second race, the possible outcomes are summarized in Table B6. If athlete H participates in the second race, Table B7 is the result. It is easy to see that athlete H

probability	α_L^1	β^2	H	L	winner
$\frac{1}{2}(1-p)$	A	b	$-b$	$A+b$	L
$\frac{1}{2}(1-p)$	A	$-b$	$-b$	$A-b$	L
$\frac{1}{2}p$	$-A$	b	$-b$	$-A+b$	L if $A<2b$, H if $A>2b$
$\frac{1}{2}p$	$-A$	$-b$	$-b$	$-A-b$	H

Table B6: Possible Outcomes When H Does Not Participate Given $\beta^1 = -b$

probability	α_L^1	β^2	H	L	winner
$\frac{1}{2}(1-p)$	A	b	0	$A+b$	L
$\frac{1}{2}(1-p)$	A	$-b$	$-b$	$A-b$	L
$\frac{1}{2}p$	$-A$	b	0	$-A+b$	L if $A<b$, H if $A>b$
$\frac{1}{2}p$	$-A$	$-b$	$-b$	$-A-b$	H

Table B7: Possible Outcomes When H Participates Given $\beta^1 = -b$

participates in the second race if $b < A < 2b$, and is indifferent between participation and nonparticipation otherwise. His winning probability conditional on $\beta^1 = -b$ is

$$\begin{cases} \frac{1}{2}p & \text{if } A < b \\ 2\frac{1}{2}p = p & \text{if } A > b. \end{cases}$$

To sum up, athlete H's winning probability after (P, N) is

$$\begin{cases} \frac{1}{2}\left(\frac{1}{2}+\frac{1}{2}p\right)+\frac{1}{2}\frac{1}{2}p = \frac{1}{4}+\frac{1}{2}p & \text{if } A < b \\ \frac{1}{2}\left(\frac{1}{2}+\frac{1}{2}p\right)+\frac{1}{2}p = \frac{1}{4}+\frac{3}{4}p & \text{if } b < A < 2b \\ \frac{1}{2}p+\frac{1}{2}p = p & \text{if } A > 2b. \end{cases}$$

Subgame after (N, P)**:** In this subgame, athlete H must participate in the second race.

First, suppose that $\alpha_L^1 = A$ and $\beta^1 = b$. Then athlete L wins with certainty by not participating in the second race. Therefore, in this case, athlete H never wins.

Second, suppose that $\alpha_L^1 = -A$ and $\beta^1 = -b$. In this case, athlete L can win only when he participates in the second race and $\alpha_L^2 = A$ and $\beta^2 = -b$

are realized, which results in a tie. Therefore, athlete H's winning probability is

$$1 - \frac{1}{2}\frac{1}{2}(1-p) = \frac{3}{4} + \frac{1}{4}p.$$

Third, suppose that $\alpha_L^1 = A$ and $\beta^1 = -b$. If athlete L participates in the second race, the possible outcomes given $\alpha_L^1 = A$ and $\beta^1 = -b$ are summarized in Table B8.

probability	α_L^2	β^2	H	L	winner
$\frac{1}{2}(1-p)$	A	b	b	A	H if $A < b$, L if $A > b$
$\frac{1}{2}(1-p)$	A	$-b$	$-b$	$A-b$	L
$\frac{1}{2}p$	$-A$	b	b	0	H
$\frac{1}{2}p$	$-A$	$-b$	$-b$	$-b$	tie

Table B8: Possible Outcomes When L Participates Given $\alpha_L^1 = A$ and $\beta^1 = -b$

If athlete L does not participate in the second race, Table B9 is the result.

probability	β^2	H	L	winner
$\frac{1}{2}$	b	b	$A-b$	H if $A < 2b$, L if $A > 2b$
$\frac{1}{2}$	$-b$	$-b$	$A-b$	L

Table B9: Possible Outcomes When L Does Not Participate Given $\alpha_L^1 = A$ and $\beta^1 = -b$

From Tables B8 and B9, athlete H's winning probabilities according to athlete L's decision and parameter values are summarized in Table B10. Since athlete L tries to reduce the probability of athlete H winning, this probability can be

L	$A < b$	$b < A < 2b$	$A > 2b$
P	$\frac{1}{2} + \frac{1}{4}p$	$\frac{3}{4}p$	$\frac{3}{4}p$
N	$\frac{1}{2}$	$\frac{1}{2}$	0

Table B10: H's Winning Probabilities Given $\alpha_L^1 = A$ and $\beta^1 = -b$

written for $\alpha_L^1 = A$ and $\beta^1 = -b$ as

$$\begin{cases} \frac{1}{2} & \text{if } A < b \\ \frac{1}{2} & \text{if } b < A < 2b,\ p > \frac{2}{3} \\ \frac{3}{4}p & \text{if } b < A < 2b,\ p < \frac{2}{3} \\ 0 & \text{if } A > 2b. \end{cases}$$

Finally, suppose that $\alpha_L^1 = -A$ and $\beta^1 = b$. If athlete L participates in the second race, the possible outcomes given $\alpha_L^1 = -A$ and $\beta^1 = b$ are summarized in Table B11.

probability	α_L^2	β^2	H	L	winner
$\frac{1}{2}(1-p)$	A	b	b	b	tie
$\frac{1}{2}(1-p)$	A	$-b$	$-b$	0	L
$\frac{1}{2}p$	$-A$	b	b	$-A+b$	H
$\frac{1}{2}p$	$-A$	$-b$	$-b$	$-A$	L if $A < b$, H if $A > b$

Table B11: Possible Outcomes When L Participates Given $\alpha_L^1 = -A$ and $\beta^1 = b$

If athlete L does not participate in the second race, Table B12 is the result.

probability	β^2	H	L	winner
$\frac{1}{2}$	b	b	$-A+b$	H
$\frac{1}{2}$	$-b$	$-b$	$-A+b$	L if $A < 2b$, H if $A > 2b$

Table B12: Possible Outcomes When L Does Not Participate Given $\alpha_L^1 = -A$ and $\beta^1 = b$

From Tables B11 and B12, athlete H's winning probabilities according to athlete L's decision and parameter values can be summarized in Table B13. Therefore, athlete H's winning probability conditional on $\alpha_L^1 = -A$ and $\beta^1 = b$ is

$$\begin{cases} \frac{1}{4} + \frac{1}{4}p & \text{if } A < b \\ \frac{1}{2} & \text{if } A < b < 2b \\ \frac{1}{4} + \frac{3}{4}p & \text{if } A > 2b. \end{cases}$$

L	$A < b$	$b < A < 2b$	$A > 2b$
P	$\frac{1}{4} + \frac{1}{4}p$	$\frac{1}{4} + \frac{3}{4}p$	$\frac{1}{4} + \frac{3}{4}p$
N	$\frac{1}{2}$	$\frac{1}{2}$	1

Table B13: H's Winning Probabilities Given $\alpha_L^1 = -A$ and $\beta^1 = b$

To sum up, athlete H's winning probabilities for each parameter values are written in Table B14.

probability	$A < b$	$b < A < 2b$, $p > \frac{2}{3}$	$b < A < 2b$, $p < \frac{2}{3}$	$A > 2b$
$\frac{1}{2}(1-p)$	0	0	0	0
$\frac{1}{2}(1-p)$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{4}p$	0
$\frac{1}{2}p$	$\frac{1}{4} + \frac{1}{4}p$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4} + \frac{3}{4}p$
$\frac{1}{2}p$	$\frac{3}{4} + \frac{1}{4}p$	$\frac{3}{4} + \frac{1}{4}p$	$\frac{3}{4} + \frac{1}{4}p$	$\frac{3}{4} + \frac{1}{4}p$

Table B14: H's Winning Probabilities After (N, P)

From Table B14, we can calculate athlete H's winning probability after (N, P) as

$$\begin{cases} \frac{1}{4} + \frac{1}{4}p + \frac{1}{4}p^2 & \text{if } A < b \\ \frac{1}{4} + \frac{3}{8}p + \frac{1}{8}p^2 & \text{if } A < b < 2b, p > \frac{2}{3} \\ p - \frac{1}{4}p^2 & \text{if } A < b < 2b, p < \frac{2}{3} \\ \frac{1}{2}p + \frac{1}{2}p^2 & \text{if } A > 2b. \end{cases}$$

Subgame after (P, P)**:** Suppose that neither athlete participates in the second race. Then the athlete with the better result in the first race wins with certainty. That is, athlete H wins with certainty if $\alpha_L^1 = -A$, while he loses with certainty if $\alpha_L^1 = A$.

Suppose that athlete H participates in the second race while athlete L does not. Then athlete H wins if

$$\alpha_L^1 + \beta^1 < \frac{1}{2}\beta^1 + \frac{1}{2}\beta^2,$$

which can be rewritten as

$$2\alpha_L^1 < \beta^2 - \beta^1.$$

Therefore, the relationship between the realization of variables, α_L^1 and β^1, and athlete H's winning probability, is described in Table B15.

α_L^1	β^1	$Pr(Hwins)$
A	b	0
A	$-b$	$\frac{1}{2}$ if $A < b$, 0 if $A > b$
$-A$	b	$\frac{1}{2}$ if $A < b$, 1 if $A > b$
$-A$	$-b$	1

Table B15: H's Winning Probabilities When H Participates While L Does Not

Suppose that athlete H does not participate in the second race while athlete L does. When $\alpha_L^1 = A$ and $\beta^1 = b$, the possible outcomes are described in Table B16.

probability	α_L^2	β^2	H	L	winner
$\frac{1}{2}(1-p)$	A	b	b	$A+b$	L
$\frac{1}{2}(1-p)$	A	$-b$	b	A	H if $A < b$, L if $A > b$
$\frac{1}{2}p$	$-A$	b	b	b	tie
$\frac{1}{2}p$	$-A$	$-b$	b	0	H

Table B16: Possible Outcomes Given $\alpha_L^1 = A$ and $\beta^1 = b$

According to Table B16, athlete H's winning probability is $\frac{1}{2} + \frac{1}{4}p$ if $A < b$, and $\frac{1}{4} + \frac{3}{4}p$ if $A > b$. Similarly, when $\alpha_L^1 = A$ and $\beta^1 = -b$, we can draw Table B17. According to Table B17, athlete H's winning probability is $\frac{1}{4}p$. When $\alpha_L^1 = -A$ and $\beta^1 = b$, we can draw Table B18. According to Table

probability	α_L^2	β^2	H	L	winner
$\frac{1}{2}(1-p)$	A	b	$-b$	A	L
$\frac{1}{2}(1-p)$	A	$-b$	$-b$	$A-b$	L
$\frac{1}{2}p$	$-A$	b	$-b$	0	L
$\frac{1}{2}p$	$-A$	$-b$	$-b$	$-b$	tie

Table B17: Possible Outcomes Given $\alpha_L^1 = A$ and $\beta^1 = -b$

probability	α_L^2	β^2	H	L	winner
$\frac{1}{2}(1-p)$	A	b	b	b	tie
$\frac{1}{2}(1-p)$	A	$-b$	b	0	H
$\frac{1}{2}p$	$-A$	b	b	$-A+b$	H
$\frac{1}{2}p$	$-A$	$-b$	b	$-A$	H

Table B18: Possible Outcomes Given $\alpha_L^1 = -A$ and $\beta^1 = b$

B18, athlete H's winning probability is $1 - \frac{1}{4}(1-p) = \frac{3}{4} + \frac{1}{4}p$. Finally, when $\alpha_L^1 = -A$ and $\beta^1 = -b$, we can draw Table B19.

probability	α_L^2	β^2	H	L	winner
$\frac{1}{2}(1-p)$	A	b	$-b$	0	L
$\frac{1}{2}(1-p)$	A	$-b$	$-b$	$-b$	tie
$\frac{1}{2}p$	$-A$	b	$-b$	$-A$	L if $A < b$, H if $A > b$
$\frac{1}{2}p$	$-A$	$-b$	$-b$	$-A-b$	H

Table B19: Possible Outcomes Given $\alpha_L^1 = -A$ and $\beta^1 = -b$

According to Table B19, athlete H's winning probability is $\frac{1}{4} + \frac{1}{4}p$ if $A < b$, and $\frac{1}{4} + \frac{3}{4}p$ if $A > b$.

Suppose that both athletes participate in the second race. Then athlete H wins only if

$$\frac{1}{2}(\alpha_L^1 + \beta^1) + \frac{1}{2}(\alpha_L^2 + \beta^2) \leq \frac{1}{2}\beta^1 + \frac{1}{2}\beta^2,$$

which is rewritten as

$$\alpha_L^1 + \alpha_L^2 \leq 0.$$

When $\alpha_L^1 = A$, this inequality holds with equality if $\alpha_L^2 = -A$, while not if $\alpha_L^2 = A$ (i.e., athlete H wins with probability $\frac{1}{2}p$ when $\alpha_L^1 = A$). When $\alpha_L^1 = -A$, this inequality holds strictly if $\alpha_L^2 = -A$, and it holds with equality if $\alpha_L^2 = A$ (i.e., athlete H wins with probability $p + \frac{1}{2}(1-p) = \frac{1}{2} + \frac{1}{2}p$ when $\alpha_L^1 = -A$).

Now we are ready to evaluate strategic forms in the second race according to the first-race outcomes, where the row and column players are athletes H

and L respectively.

Case 1 ($\alpha_L^1 = A$ *and* $\beta^1 = b$): The strategic form is drawn as follows:

if $A < b$,

	P	N
P	$\frac{1}{2}p$	0
N	$\frac{1}{2}+\frac{1}{4}p$	0

if $A > b$,

	P	N
P	$\frac{1}{2}p$	0
N	$\frac{3}{4}p$	0

Hence, athlete L chooses not to participate in the second race, and athlete H never wins.

Case 2 ($\alpha_L^1 = A$ *and* $\beta^1 = -b$): The strategic form is drawn as follows:

if $A < b$

	P	N
P	$\frac{1}{2}p$	$\frac{1}{2}$
N	$\frac{1}{4}p$	0

if $A > b$

	P	N
P	$\frac{1}{2}p$	0
N	$\frac{1}{4}p$	0

Hence, if $A < b$, both athletes participate in the second race, and athlete H wins with probability $\frac{1}{2}p$. If $A > b$, athlete L chooses not to participate in the second race, and athlete H never wins.

Case 3 ($\alpha_L^1 = -A$ *and* $\beta^1 = b$): The strategic form is drawn as follows:

if $A < b$,

	P	N
P	$\frac{1}{2}+\frac{1}{2}p$	$\frac{1}{2}$
N	$\frac{3}{4}+\frac{1}{4}p$	1

if $A > b$,

$$\begin{array}{c|cc} & P & N \\ \hline P & \frac{1}{2}+\frac{1}{2}p & 1 \\ N & \frac{3}{4}+\frac{1}{4}p & 1 \end{array}$$

Hence, athlete L chooses to participate while athlete H does not, which results in athlete H's winning probability $\frac{3}{4}+\frac{1}{4}p$.

Case 4 ($\alpha_L^1 = -A$ *and* $\beta^1 = -b$): The strategic form is drawn as follows:

if $A < b$,

$$\begin{array}{c|cc} & P & N \\ \hline P & \frac{1}{2}+\frac{1}{2}p & 1 \\ N & \frac{1}{4}+\frac{1}{4}p & 1 \end{array}$$

if $A > b$

$$\begin{array}{c|cc} & P & N \\ \hline P & \frac{1}{2}+\frac{1}{2}p & 1 \\ N & \frac{1}{4}+\frac{3}{4}p & 1 \end{array}$$

Hence, both athletes choose to participate, which results in athlete H's winning probability $\frac{1}{2}+\frac{1}{2}p$.

From these four cases, we can write athlete H's winning probability in this subgame as

$$\begin{cases} \frac{1}{2}p\left(\frac{3}{4}+\frac{1}{4}p\right)+\frac{1}{2}p\left(\frac{1}{2}+\frac{1}{2}p\right)+\frac{1}{2}(1-p)\frac{1}{2}p = \frac{7}{8}p+\frac{1}{8}p^2 & \text{if } A < b \\ \frac{1}{2}p\left(\frac{3}{4}+\frac{1}{4}p\right)+\frac{1}{2}p\left(\frac{1}{2}+\frac{1}{2}p\right) = \frac{5}{8}p+\frac{3}{8}p^2 & \text{if } A > b. \end{cases}$$

The First Race: Based on the analysis of the second race, we can draw the strategic form for the first race as follows.

if $A < b$,

$$\begin{array}{c|cc} & P & N \\ \hline P & \frac{7}{8}p+\frac{1}{8}p^2 & \frac{1}{4}+\frac{1}{2}p \\ N & \frac{1}{4}+\frac{1}{4}p+\frac{1}{4}p^2 & p \end{array}$$

if $b < A < 2b$ and $p > \frac{2}{3}$,

$$\begin{array}{c|cc} & P & N \\ \hline P & \frac{5}{8}p+\frac{3}{8}p^2 & \frac{1}{4}+\frac{3}{4}p \\ N & \frac{1}{4}+\frac{3}{8}p+\frac{1}{8}p^2 & p \end{array}$$

if $b < A < 2b$ and $p < \frac{2}{3}$,

	P	N
P	$\frac{5}{8}p + \frac{3}{8}p^2$	$\frac{1}{4} + \frac{3}{4}p$
N	$p - \frac{1}{4}p^2$	p

if $A > 2b$,

	P	N
P	$\frac{5}{8}p + \frac{3}{8}p^2$	p
N	$\frac{1}{2}p + \frac{1}{2}p^2$	p

Simple algebra yields the Nash equilibrium first-race action profiles and athlete H's winning probabilities in Table B20.

	$A < b,\ p > \frac{\sqrt{17}-3}{2}$	$A < b,\ p < \frac{\sqrt{17}-3}{2}$
first race	(mix, mix)	(P, P)
Pr(H *wins*)	$\frac{1+3p-11p^2}{8-18p+2p^2}$	$\frac{7}{8}p + \frac{1}{8}p^2$
$b < A < 2b,\ p > \frac{3}{5}$	$b < A < 2b,\ p < \frac{3}{5}$	$A > 2b$
(P, P)	(N, P)	(P, P)
$\frac{5}{8}p + \frac{3}{8}p^2$	$p - \frac{1}{4}p^2$	$\frac{5}{8}p + \frac{3}{8}p^2$

Table B20: Equilibrium First-Race Action Profiles and H's Winning Probabilities under Average-Result Selection

This summary of action profiles in Table B20 is also depicted in Figure B2.

B.2.4 Comparing the Three Procedures

Let us compare athlete H's winning probabilities under the three procedures. First of all, we have already seen that athlete H's winning probability is greater under one-race selection (i.e., p) than under best-result selection (i.e., $\frac{1}{2}p + \frac{1}{2}p^2$ if $A < 2b$ and p^2 if $A > 2b$). Next, the previous subsection tells us that athlete H's winning probability is also greater under one-race selection than under average-result selection (i.e., p is greater than any of $\frac{1+3p-11p^2}{8-18p+2p^2}$, $\frac{7}{8}p + \frac{1}{8}p^2$, $\frac{5}{8}p + \frac{3}{8}p^2$ and $p - \frac{1}{4}p^2$). These results are consistent with the area near the horizontal axis in Figure 2.

Finally, we compare best-result selection and average-result selection. Simple algebra shows that athlete H's winning probability is greater under best-

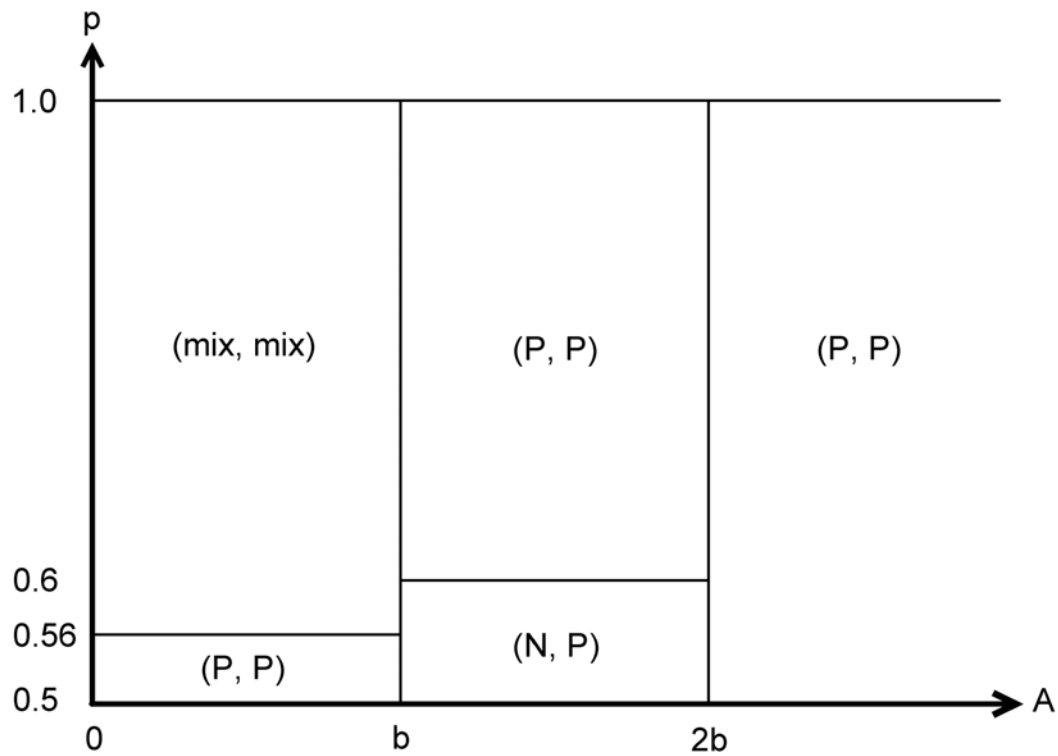


Figure B2: Equilibrium First-Race Action Profiles under Average-Result Selection in the Simpler Model

result selection than under average-result selection only if $A < b$ and $p > 0.781752$ as illustrated in Figure B3.

Let us examine the consistency between this result and what was obtained from the general model in the main text. Figure B4 describes which of the two procedures results in a higher winning probability for athlete H in the general model with $\nu_H = 0$, according to the values of ν_L (horizontal axis) and μ (vertical axis). As we can see in the figure, best-result selection is superior to average-result selection for selecting athlete H when μ is relatively large compared with ν_L. For the simpler model, on the other hand, the superiority of best-result selection holds for sufficiently small A and large p. First of all, note that a smaller $E(\alpha_L^j)$ in the simpler model is interpreted as a larger μ in the general model because both imply a larger difference between the two athletes' expected results. If we decrease A, ν_L becomes small while $E(\alpha_L^j)$ becomes large (i.e., μ becomes small). In addition, if we increase p, ν_L becomes much smaller and $E(\alpha_L^j)$ also becomes small (i.e., μ becomes large). As a result, we reach the northwestern area in Figure B4, where best-result

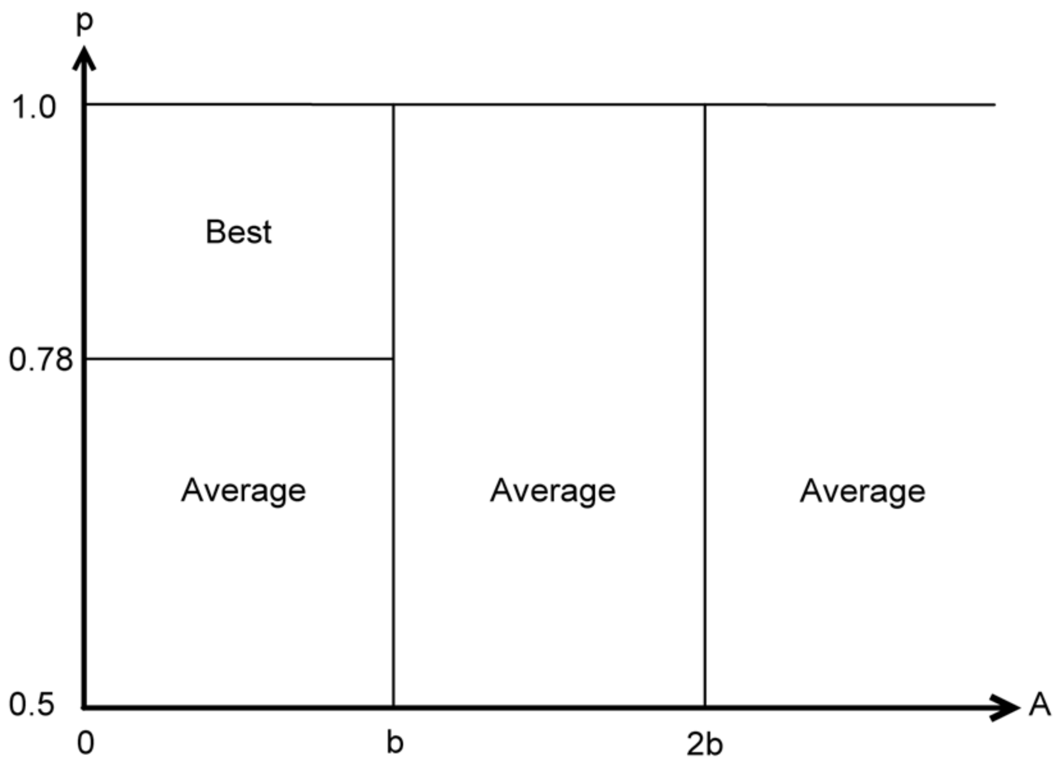


Figure B3: Comparison between Best-Result Selection and Average-Result Selection in the Simpler Model

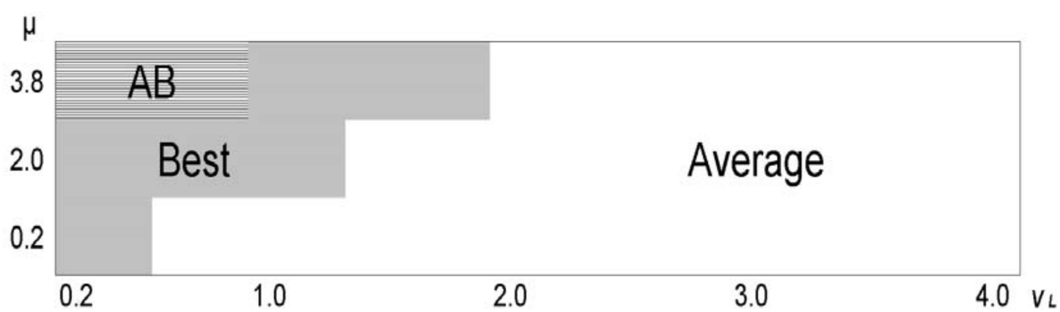


Figure B4: Comparison between Best-Result Selection and Average-Result Selection in the General Model with $\nu_H = 0$

selection is superior. We thus observe that the region in which best-result selection is preferable in Figure B3 corresponds to the region in which best-result selection is preferable in Figure B4.

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