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Luminescence of a Cooper Pair

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This Letter theoretically discusses the photon emission spectra of a superconducting p-n junction. On the basis of the second order perturbation theory for electron-photon interaction, we show that the recombination of a Cooper pair with two p-type carriers causes enhancement of the luminescence intensity. The calculated results of photon emission spectra explain characteristic features of observed signal in an recent experiment. Our results indicate high functionalities of superconducting light-emitting devices.

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Light-emitting diodes (LEDs) usually fabricated on semiconductors have been an important element of modern technologies. Recent researches seem to focus on producing a well controlled photon and an entangled photon pair [1,2] for realizing quantum information. Superconducting devices have a great advantage in producing entangled quantum states because of its coherent nature [3–5]. Superconducting LEDs [6] have been originally proposed [1,2] for realizing quantum information. Superconducting devices have a great advantage in producing entangled photon pairs [7]. A recent theoretical study predicts the Josephson radiation in a superconducting p-n junction [8]. Thus hybrids of superconductor and semiconductor LED undoubtedly have a possibility to provide a key technology in the next generation.

The radiative recombination of Cooper pairs has been observed recently in an InGaAs/InP p-n junction attaching onto a superconductor Nb [9]. The electroluminescence becomes drastically large at low temperatures below the superconducting transition temperature $T_c$ of Nb electrode. Surprisingly the degree of enhancement in the luminescence intensity is 1 order of magnitude. Although the experiment has shown clearly effects of superconductivity on the radiative recombination, the mechanism has been an open question. This Letter theoretically addresses this issue. We study the emission spectra of photon in a superconducting p-n junction by using the second order perturbation expansion for electron-photon interaction. In the second order, we find that a peculiar recombination process to superconductivity enhances the luminescence intensity. In that recombination process, two electrons recombine with two p-type carriers as a Cooper pair. The theoretical results explain characteristic features of the experimental findings [9]. This Letter not only figures out a mechanism of the large luminescence intensity but also gives a guide for designing highly functional superconducting light-emitting devices.

Let us consider a p-type semiconductor-superconductor junction under the applied bias voltage $eV_J$ as shown in Fig. 1(a). The energy is measured from the horizontal line indicated by “0”. The sign of energy in a p-type semiconductor is chosen to be opposite to that in a superconductor. We assume that a semiconductor and a superconductor are in their local equilibrium which are characterized by the local chemical potential $\mu_p$ and $\mu_n$, respectively. The edges of the conduction and valence bands are $E_c$ and $E_v$, respectively. In what follows, we use a unit of $h = k_B = c = 1$, where $k_B$ is the Boltzmann constant and $c$ is

FIG. 1 (color online). Schematic energy diagram of forward biased p-n junctions. A theoretical model used for calculation is shown in (a). In (c), a realistic junction in experiments is illustrated. Predicted results of the photon spectra are shown in (b) and (d).
the speed of light. The \( p \)-type semiconductor is described by
\[
H_p = \sum_{k,\sigma} \varepsilon_p(k) b_{k,\sigma}^\dagger b_{k,\sigma},
\]
where \( \varepsilon_p(k) = k^2/(2m_p) + E_v + eV_{sd}/2, m_p \) is the effective mass, and \( b_{k,\sigma}^\dagger (b_{k,\sigma}) \) is the creation (annihilation) operator of a \( p \)-type carrier with a wave number \( k \) and spin \( \sigma = \uparrow \) or \( \downarrow \). The photon states are described by
\[
H_{ph} = \sum_q \omega_q \left( a_q^\dagger a_q + \frac{1}{2} \right),
\]
where \( a_q^\dagger (a_q) \) is the creation (annihilation) operator of a photon with a wave number \( q \) and an energy \( \omega_q \). The normal state in a metal is described by
\[
H_{nn} = \sum_{k,\sigma} \left( \frac{k^2}{2m_n} + \frac{eV_{sd}}{2} \right) c_{k,\sigma}^\dagger c_{k,\sigma},
\]
where \( c_{k,\sigma}^\dagger (c_{k,\sigma}) \) is the creation (annihilation) operator of an \( n \)-type carrier and \( m_n \) is the effective mass. The electron-photon interaction Hamiltonian in the dipole approximation is given by
\[
H_T = \sum_{k,q,\sigma} \mathcal{B}_{k,q} b_{k,q,\sigma} c_{k,\sigma}^\dagger a_q + \text{H.c.},
\]
where \( \mathcal{B}_{k,q} \) is the coupling energy. On the basis of the second order perturbation theory, the number of emitting photon \( N_{ph} = \sum_q a_q^\dagger a_q \) is calculated as
\[
\langle N_{ph} \rangle = \langle N_{ph}(1) \rangle + \langle N_{ph}(2) \rangle,
\]
\[
\langle N_{ph}(1) \rangle = \int_{-\infty}^{t_1} dt_1 \int_{-\infty}^{t_1} dt_2 \langle \chi_0 | H_T(t_1) N_{ph} H_I(t_2) | \chi_0 \rangle,
\]
\[
\langle N_{ph}(2) \rangle = \int_{-\infty}^{t_2} dt_1 \int_{-\infty}^{t_1} dt_2 \int_{-\infty}^{t_1} dt_3 \int_{-\infty}^{t_1} dt_4 I(2),
\]
\[
I(2) = \langle \chi_0 | H_T(t_1) H_I(t_2) N_{ph} H_I(t_3) H_I(t_4) | \chi_0 \rangle,
\]
where \( |\chi_0 \rangle \rightarrow |0\rangle \otimes |N\rangle \otimes |P\rangle \).

The BCS theory describes superconducting states,
\[
H_{ns} = \sum_{k,\sigma} E_k Y_{k,\sigma}^\dagger Y_{k,\sigma},
\]
where \( E_k = \sqrt{\varepsilon_n^2(k) + \Delta^2}, \varepsilon_n(k) = k^2/2m_n - \mu_n \), \( \Delta \) is the pair potential, and \( Y_{k,\sigma}^\dagger (Y_{k,\sigma}) \) is the creation (annihilation) operator of a Bogoliubov quasiparticle. We try to consider effects of superconductivity through the Bogoliubov transformation [10]. The description in Eq. (10), however, is valid within a small energy scale near the Fermi level which is at \( \bar{\mu}_n = E_c + eV_{sd}/2 + \mu_n \) measured from 0.
\[ Q_N = \langle N | u_k e^{iE_k \gamma_k \sigma_1} e^{-iE_k \gamma_k \sigma_1} | 1 - f_k \rangle \]
\[ \times \langle \sigma_1 | u_k e^{iE_k \gamma_{k,k_2}} e^{-iE_k \gamma_{k,k_2}} | 1 - f_k \rangle \]
\[ + e^{-iE_k (\tau - t_z)} e^{iE_k (\tau - t_z)} f_k (1 - f_k) \]
\[ - e^{-iE_k (\tau - t_z)} e^{iE_k (\tau - t_z)} (1 - f_k^k) (1 - f_k^k) \]
\[ - e^{iE_k (\tau - t_z)} e^{iE_k (\tau - t_z)} f_k^k f_k^k. \]

Equation (18) describes effects of superconductivity on the emission spectra because \( \delta \sigma_1, \delta_2, \sigma_{k,k_2} \) means the destruction of two electrons as a Cooper pair. A recombination process in \( Q_N(S) \) is schematically illustrated in Fig. 2(a). The remaining eight terms in \( Q_N \) describe the emitting processes shown in Fig. 2(b) and give the luminescence intensity proportional to \( \langle N_{\text{ph}}(t) \rangle^2 \). We will show that \( Q_N(S) \) gives large contribution to the emission spectra at \( \delta q = q_1 + q_2 = 0 \) (\( \delta q = q_3 + q_4 = 0 \) in other words).

![Diagram of recombination processes](image)

**FIG. 2** (color online). Recombination processes in the second order perturbation expansion, where solid, broken, and wavy lines represent the propagation of an electron, a \( p \)-type carrier, and a photon, respectively. In (a), a recombination of a Cooper pair in \( Q_N(S) \) is shown. In (b), a recombination process other than \( Q_N(S) \) is illustrated.
due to elastic impurity scatterings $\tau_0$. At $T = 0$, we obtain
$I_0 = I_{00}(0)2^{\alpha^2/(\sqrt{1 + \alpha^2}(\alpha + \sqrt{1 + \alpha^2}))}$, where $\alpha = \tau_0 \Delta_0$. $\Delta_0$ is the pair breaking at the zero temperature and $I_{00}(0) = \pi N_0/2 \Delta_0$ is Eq. (20) at $T = 0$ and $1/\tau = 0$. At $T \approx T_c$, we find
\[ \frac{I_0}{I_{00}(0)} = \frac{c_\alpha \alpha^2(\Delta/\Delta_0)^2 \Delta_0/T}{\alpha \Delta^2(\Delta_0)^2} \alpha \gg 1, \]
where $c_\alpha$ is a constant of the order of unity. In Fig. 3(a), we show $I_0$ as a function of temperature for several choices of $\alpha$, where we describe the dependence of $T_c$ on temperature by the BCS theory. The amplitude of $I_0$ at $T = 0$ is suppressed in the dirty limit as shown in a result with $\alpha = 0.2$. The amplitude at $T = 0$ increases with increasing $\alpha$. At $\alpha = 1$, $I_0(0)$ has almost the same amplitude as $I_{00}(0)$. When we increase $\alpha$ up to 2.0, the results show a bump just below $T_c$. Next we consider inelastic scatterings described by $1/\tau_{ie} = C_{ie}(T/T_c)^p$, where $C_{ie}$ is a coupling constant and $p$ depends on scattering sources such as $p = 1$ for electron-phonon scatterings and $p = 2$ for repulsive electron-electron interaction. In Fig. 3(b), we calculate $I_0$ for several choices of $C_{ie}$ and $p$. Since $1/\tau_{ie} = 0$ at $T = 0$, the amplitude is close to $I_{00}(0)$ at $T = 0$. When we decreases $C_{ie}$, the bump appears below $T_c$. For $1/\tau \leq \Delta_0$, the luminescence intensity at $T \approx T_c$ is then given by
\[ \langle N_{ph}(2) \rangle = \frac{4\pi c_\alpha |B|^4 N_0 (\tau \Delta)^2}{T} \sum_q \delta(\Omega_{k_F,q}). \]

Finally we modify Eq. (22) to describe the photon spectra in realistic junctions as shown in Fig. 1(c). A superconductor is attached to an $n$-type semiconductor whose thickness $L_n$ is about 30–50 nm [9]. The proximity effect enables the penetration of Cooper pairs into the $n$-type semiconductor. In experiments, photons are emitted mainly from a quantum well which is sandwiched by the $p$- and $n$-type semiconductor. The pair amplitude in the quantum well can be proportional to $\Delta e^{-L_n/\xi_T}$ with $\xi_T = \sqrt{D/2\pi T}$ and $D$ being the diffusion constant in the $n$-type semiconductor. The quantum well would be replaced by a quantum dot near future. The level in the quantum well (dot) $E_w$ should coincide with the Fermi level in the $n$-type semiconductor $\mu_n$. Namely, $|E_w - \mu_n|$ must be less than both the Thouless energy $E_{Th} = D/L_n^2$ and $\Delta$. This resonant condition is particularly important for a Cooper pair to penetrate into the quantum well (dot). The emission spectra have a peak at $\omega_q$ and the peak width is given by $\Gamma = \Gamma_0 N_0$, where $\Gamma_0$ is the transfer integral between the quantum well (dot) and the semiconductor. The argument above is summarized by an equation for $T \approx T_c$
\[ \langle N_{ph}(2) \rangle = \left| B \right|^4 N_0 \Gamma \sum_{q,\sigma} \frac{\Delta \tau^2 \tau e^{-2L_n/\xi_T}}{(\omega_q - \omega_0)^2 + (\Gamma)^2}. \]
where we introduce the Lorentz resonant function by hand.

In conclusion, we have studied the photon emission spectra in a superconducting $p$-$n$ junction based on the second order perturbation theory for electron-photon interaction. We have found in the second order expansion that a peculiar recombination process to superconductivity enhances the luminescence intensity. The theoretical results explain temperature dependence of the luminescence intensity observed in an recent experiment.

[11] The speed of light is much larger than the Fermi velocity. In such case, this condition becomes $|q_1| \sim |q_2|$. 

FIG. 3 (color online). Temperature dependence of luminescence intensity in Eqs. (19) and (20). In (a), we consider relaxation time due to the elastic impurity scatterings by $\alpha = \tau_0 \Delta_0$. In (b), we consider the relaxation due to inelastic scatterings.

\[ N_{ph}(2) \]