Luminescence of a Cooper Pair

Yasuhiro Asano,¹ ² Ikuo Suemune,³ ⁴ Hideaki Takayanagi,⁴ ⁵ ⁶ and Eiichi Hanamura⁷

¹Department of Applied Physics, Hokkaido University, Sapporo 060-8628, Japan
²Center of Education & Research for Topological Science & Technology, Hokkaido University, Sapporo 060-8628, Japan
³Research Institute for Electronic Science, Hokkaido University, Sapporo 001-0021, Japan
⁴CREST, Japan Science and Technology Agency, Kawaguchi 332-0012, Japan
⁵Department of Applied Physics, Tokyo University of Science, Tokyo 162-8601, Japan
⁶International Center for Nanoarchitectonics, NIMS, Tsukuba 305-0044, Japan
⁷Japan Science and Technology Agency, Kawaguchi 332-0012, Japan

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This Letter theoretically discusses the photon emission spectra of a superconducting p-n junction. On the basis of the second order perturbation theory for electron-photon interaction, we show that the recombination of a Cooper pair with two p-type carriers causes enhancement of the luminescence intensity. The calculated results of photon emission spectra explain characteristic features of observed signal in an recent experiment. Our results indicate high functionalities of superconducting light-emitting devices.

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Light-emitting diodes (LEDs) usually fabricated on semiconductors have been an important element of modern technologies. Recent researches seem to focus on producing a well controlled photon and an entangled photon pair [1,2] for realizing quantum information. Superconducting devices have a great advantage in producing entangled quantum states because of its coherent nature [3–5]. Superconducting LEDs [6] have been originally proposed [1,2] for realizing quantum information. Superconducting devices have a great advantage in producing entangled quantum states because of its coherent nature [3–5]. Superconducting LEDs [6] have been originally proposed [1,2] for realizing quantum information. Superconducting devices have a great advantage in producing entangled quantum states because of its coherent nature [3–5].

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the speed of light. The \( p \)-type semiconductor is described by

\[
H_p = \sum_{k, \sigma} \varepsilon_p(k) b_{k, \sigma}^\dagger b_{k, \sigma},
\]

where \( \varepsilon_p(k) = k^2/(2m_p) + E_v + eV_{sd}/2 \), \( m_p \) is the effective mass, and \( b_{k, \sigma}^\dagger (b_{k, \sigma}) \) is the creation (annihilation) operator of a \( p \)-type carrier with a wave number \( k \) and spin \( \sigma = \uparrow \) or \( \downarrow \). The photon states are described by

\[
H_{ph} = \sum_{\mathbf{q}, \omega} (a_{\mathbf{q}}^\dagger a_{\mathbf{q}} + 1/2),
\]

where \( a_{\mathbf{q}}^\dagger (a_{\mathbf{q}}) \) is the creation (annihilation) operator of a photon with a wave number \( \mathbf{q} \) and an energy \( \omega. \) The normal state in a metal is described by

\[
H_n = \sum_{k, \sigma} \left( \frac{k^2}{2m_n} + E_c + \frac{eV_{sd}}{2} \right) c_{k, \sigma}^\dagger c_{k, \sigma},
\]

where \( c_{k, \sigma} \) is the creation (annihilation) operator of an \( n \)-type carrier and \( m_n \) is the effective mass. The electron-photon interaction Hamiltonian in the dipole approximation is given by

\[
H_I = \sum_{k, \sigma, \mathbf{q}} B_{k, \mathbf{q}} c_{k, \sigma}^\dagger a_{\mathbf{q}} + \text{H.c.},
\]

where \( B_{k, \mathbf{q}} \) is the coupling energy. On the basis of the second order perturbation theory, the number of emitting photon \( N_{ph} = \sum q a_{\mathbf{q}}^\dagger a_{\mathbf{q}} \) is calculated as

\[
\langle N_{ph} \rangle = \langle N_{ph}(1) \rangle + \langle N_{ph}(2) \rangle,
\]

\[
\langle N_{ph}(1) \rangle = \int dt_1 \int dt_2 \chi_0 H_I(t_1) N_{ph} H_I(t_2) \chi_0,
\]

\[
\langle N_{ph}(2) \rangle = \int dt_1 \int dt_2 \int dt_3 \int dt_4 I(2),
\]

\[
I(2) = \langle \chi_0 | H_I(t_1) H_I(t_2) N_{ph} H_I(t_3) H_I(t_4) | \chi_0 \rangle.
\]

where \( \chi_0 = 0 \) is the zero photon state.

The BCS theory describes superconducting states,

\[
H_{ns} = \sum_{k, \sigma} E_k \gamma_{k, \sigma}^\dagger \gamma_{k, \sigma},
\]

where \( E_k = \sqrt{\frac{\Delta^2}{2} + \xi_n(k)} = k^2/2m_n - \mu_n, \) \( \Delta \) is the pair potential, and \( \gamma_{k, \sigma}^\dagger \) is the creation (annihilation) operator of a Bogoliubov quasiparticle. We try to consider effects of superconductivity through the Bogoliubov transformation [10]. The description in Eq. (10), however, is valid within a small energy scale near the Fermi level which is at \( \mu_n = E_c + eV_{sd}/2 + \mu_n \) measured from 0.
\[ Q_P = \langle P|b_{p, \sigma}^\dagger b_{p, \sigma}^\dagger b_{p', \sigma'} b_{p, \sigma}|P \rangle, \]
\[ = f_P^{p'} f_{p'}^{p''} (\delta_{i_1}^{q_1} \delta_{i_2}^{q_2} \delta_{i_3}^{q_3} \delta_{i_4}^{q_4} \delta_{i_5}^{q_5} \delta_{i_6}^{q_6} \delta_{i_7}^{q_7} \delta_{i_8}^{q_8}), \]
where \( \delta_{ij} = \delta_{q_i, q_j}, \delta_{ij} = \delta_{\sigma_i, \sigma_j}, \delta_{ij} = \delta_{p_i, p_j}, \) and \( p_j = k_j - q_j. \) By applying the Bogoliubov transformation, we find,
\[ Q_N = \langle N| u_{k_1} e^{i E_{k_1}} \gamma_{k_1, \sigma_1} \gamma_{k_2, \sigma_2} \sigma_3 \sigma_4 \gamma_{k_1, -\sigma_1} \gamma_{k_2, -\sigma_2} \rangle \]
\[ \times (u_{k_2} e^{i E_{k_2}} \gamma_{k_2, \sigma_2} \sigma_3 \sigma_4 \gamma_{k_2, \sigma_2} \gamma_{k_3, \sigma_3}) \]
\[ \times (u_{k_3} e^{-i E_{k_3}} \gamma_{k_3, \sigma_3} \sigma_3 \sigma_4 \gamma_{k_3, \sigma_3} \gamma_{k_4, \sigma_4}) \]
\[ \times (u_{k_4} e^{-i E_{k_4}} \gamma_{k_4, \sigma_4} \sigma_3 \sigma_4 \gamma_{k_4, \sigma_4} \gamma_{k_5, -\sigma_4}). \]
\[ (17) \]
which gives 12 terms. In what follows, we extract the most dominant contribution in Eq. (17). The average of \( Q_N \)
includes the following four terms
\[ Q_N(S) = u_{k_1} u_{k_2} u_{k_3} u_{k_4} \delta_{\sigma_1, \sigma_2} \delta_{\sigma_3, \sigma_4} \delta_{k_1, -k_2} \]
\[ \times \delta_{k_3, -k_4} \sigma_3 \sigma_4 \left[ e^{i E_{k_3}} (1 - f_{k_3}) - e^{i E_{k_4}} (1 - f_{k_4}) \right] \]
\[ + e^{-i E_{k_3}} (1 - f_{k_3}) - e^{-i E_{k_4}} (1 - f_{k_4}) \]
\[ - e^{-i E_{k_1}} e^{-i E_{k_4}} (1 - f_{k_1}) (1 - f_{k_4}) \]
\[ - e^{-i E_{k_1}} e^{-i E_{k_4}} (1 - f_{k_1}) (1 - f_{k_4}) \].
\[ (18) \]
Equation (18) describes effects of superconductivity on the emission spectra because \( \delta_{\sigma_1, \sigma_2} \delta_{k_1, -k_2} \) means the destruction of two electrons as a Cooper pair. A recombination process in \( Q_N(S) \) is schematically illustrated in Fig. 2(a).
The remaining eight terms in \( Q_N(S) \) describe the emitting processes shown in Fig. 2(b) and give the luminescence intensity proportional to \( \langle N_{ph}(\Omega) \rangle^2 \). We will show that \( Q_N(S) \) gives large contribution to the emission spectra at \( \delta q = q_1 + q_2 = 0, (\delta q = q_3 + q_4 = 0 \) in other words).

**FIG. 2** (color online). Recombination processes in the second order perturbation expansion, where solid, broken, and wavy lines represent the propagation of an electron, a \( p \)-type carrier, and a photon, respectively. In (a), a recombination of a Cooper pair in \( Q_N(S) \) is shown. In (b), a recombination process other than \( Q_N(S) \) is illustrated.

Substituting Eqs. (15), (16), and (18) into Eq. (14) and carrying out time integrations, we obtain
\[ \langle N_{ph}(\Omega) \rangle = 4 |B|^2 \sum_{q, \sigma} \langle \delta Q_{k, \sigma} \rangle I_0. \]
\[ (19) \]
\[ I_0 = \sum_k \left[ f_k^2 (1 - f_k^2) + f_k^4 (1 - f_k^2)^2 \right] \frac{1}{E_k^2 + (1/\tau)^2} \]
\[ + \left( f_k^2 + (1 - f_k^2)^2 \right) \]
\[ \frac{1}{E_k^2} . \]
\[ (20) \]
where a relaxation time \( \tau \) is introduced to remove effects of the perturbation at \( t \rightarrow -\infty \). We neglect dependence of \( B \) on wave numbers and assume \( f_{k, p, q} = 1 \). At \( 1/\tau = 0 \), \( I_0 = \pi N_0 / 2 \Delta \) essentially diverges for small \( \Delta \) with \( N_0 \) denoting the normal density of states in a superconductor at the Fermi energy. The singular behavior at small \( \Delta \) in Eq. (20) is a sign of the large luminescence intensity due to superconductivity.

We first show mathematical reasons of the singularity. Then we will discuss the physics behind the phenomenon. A two-photon emitting process in \( Q_N(S) \) is illustrated on Fig. 2(a).

The annihilation of a Cooper pair is described by \( c_{-k_1} c_{k_1} \) which includes an operator \( \gamma_{k_1}^\dagger \gamma_{k_1} \). Let us assume that the energy of the initial state is zero. In the first order expansion, the operation of \( \gamma_{k_1}^\dagger \) to the BCS state decreases energy by \( E_E + \mu_n \). At the same time, a \( p \)-type carrier with energy \( E_p (k - q) \) is destructed and a photon with energy \( \omega_\sigma \) is created. Thus the energy of the intermediate state \( \delta E_1 \) results in \( \delta E_1 = \omega_\sigma - E_p (k - q) - \mu_n = \Omega_{k, q} - E_k \). In the perturbation expansion, \( \delta E_1 \) becomes the energy denominator. In the second order, the operation of \( \gamma_{k_1}^\dagger \gamma_{k_1} \) destructs the \( p \)-type carrier, and the creation of a photon gain energy by \( E_k - \mu_n = E_p (q + k) \), and \( \omega_\sigma \) respectively. Therefore the difference in energy between the intermediate state and the final one becomes \( \delta E_2 = \omega_\sigma - E_p (q + k) - \mu_n = \Omega_{k, q} + E_k \). The perturbation theory requires the energy conservation between the initial state and the final one (i.e., \( \delta E_1 + \delta E_2 = 0 \)), which leads to \( 2 \Omega_{k, q} = 0 \). As a result, a small value of \( E_k \) remains in the denominator as shown in Eq. (20). The physics behind the phenomena is simple. The BCS state has an ability to emit a pair of photons with remaining in its state almost unchanged because the BCS state is the eigenstate of \( \gamma_{k_1}^\dagger \gamma_{k_1} \). The equation \( \Omega_{k, q} = 0 \) describes the emitting condition of two photons. The threshold and width of spectra are \( E_g + \mu_n \) and \( \mu_p \), respectively. In Fig. 1(b), we schematically show predicted spectra in the second order process.

The singular behavior in perturbation expansion implies an importance of higher order terms for predicting the luminescence intensity quantitatively. Here we do not discuss this issue, but choose an alternative way of regularizing the obtained results. In what follows, we introduce a finite relaxation time. First we consider a mean free time.
due to elastic impurity scatterings $\tau_0$. At $T = 0$, we obtain
$I_0 = I_{00}(0)2\alpha^2/(\sqrt{1 + \alpha^2}(\alpha + \sqrt{1 + \alpha^2}))$, where $\alpha = \tau_0\Delta_0$. $\Delta_0$ is the pair breaking at the zero temperature and $I_{00}(0) = \pi N_0/2\Delta_0$ is Eq. (20) at $T = 0$ and $1/\tau = 0$. At $T \leq T_c$, we find
\[
\frac{I_0}{I_{00}(0)} = \frac{c_0\alpha^2(\Delta/\Delta_0)^2\Delta_0/T}{\alpha^2(\Delta/\Delta_0)^2} \quad \alpha \ll 1,
\]
where $c_0$ is a constant of the order of unity. In Fig. 3(a), we show $I_0$ as a function of temperature for several choices of $\alpha$, where we describe the dependence of $\Delta$ on temperature by the BCS theory. The amplitude of $I_0$ at $T = 0$ is suppressed in the dirty limit as shown in a result with $\alpha = 0.2$. The amplitude at $T = 0$ increases with increasing $\alpha$. At $\alpha = 1$, $I_0(0)$ has almost the same amplitude as $I_{00}(0)$. When we increase $\alpha$ up to 2.0, the results show a bump just below $T_c$. Next we consider inelastic scatterings described by $1/\tau_{ie} = C_{ie}(T/T_c)^p$, where $C_{ie}$ is a coupling constant and $p$ depends on scattering sources such as $p = 1$ for electron-phonon scatterings and $p = 2$ for repulsive electron-electron interaction. In Fig. 3(b), we calculate $I_0$ for several choices of $C_{ie}$ and $p$. Since $1/\tau_{ie} \rightarrow 0$ at $T = 0$, the amplitude is close to $I_{00}(0)$ at $T = 0$. When we decreases $C_{ie}$, the bump appears below $T_c$. For $1/\tau \leq \Delta_0$, the luminescence intensity at $T \leq T_c$ is then given by
\[
\overline{(N_{ph}(2))} = 4\pi c_0|B|^4N_0(\tau\Delta)^2\sum_q\delta(\Omega_{kF,q}).
\]  
Finally we modify Eq. (22) to describe the photon spectra in realistic junctions as shown in Fig. 1(c). A superconductor is attached to an $n$-type semiconductor whose thickness $L_w$ is about 30–50 nm [9]. The proximity effect enables the penetration of Cooper pairs into the $n$-type semiconductor. In experiments, photons are emitted mainly from a quantum well which is sandwiched by the $p$- and $n$-type semiconductor. The pair amplitude in the quantum well can be proportional to $\Delta e^{-L_w/\xi_T}$ with $\xi_T = \sqrt{D/2\pi T}$ and $D$ being the diffusion constant in the $n$-type semiconductor. The quantum well would be replaced by a quantum dot near future. The level in the quantum well (dot) $E_w$ should coincide with the Fermi level in the $n$-type semiconductor $\mu_n$. Namely, $|E_w - \mu_n|$ must be less than both the Thouless energy $E_{Th} = D/L_w^2$ and $\Delta$. This resonant condition is particularly important for a Cooper pair to penetrate into the quantum well (dot). The emission spectra have a peak at $\omega_0$ and the peak width is given by $\Gamma = \xi_T^2N_0$, where $\xi_T$ is the transfer integral between the quantum well (dot) and the semiconductor. The argument above is summarized by an equation for $T \leq T_c$
\[
\overline{(N_{ph}(2))} = |B|^4N_0\sum_q\frac{\Delta^2\tau^2e^{-2L_w/\xi_T}/T}{(\omega_q - \omega_0)^2 + (\Gamma)^2}.
\]  
where we introduce the Lorentz resonant function by hand.
In the experiment, $\xi_T$ estimated to be 680 nm at 4 K is much larger than $L_w$ below $T_c$. Thus the theoretical results in Fig. 3 may describe experimental results of the luminescence intensity. In fact, the experimental results of Fig. 6(b) in Ref. [9] show a very similar line shape to that in Fig. 3(a) with $\alpha = 1$.

In conclusion, we have studied the photon emission spectra in a superconducting $p$-$n$ junction based on the second order perturbation theory for electron-photon interaction. We have found in the second order expansion that a peculiar recombination process to superconductivity enhances the luminescence intensity. The theoretical results explain temperature dependence of the luminescence intensity observed in an recent experiment.

[11] The speed of light is much larger than the Fermi velocity. In such case, this condition becomes $|q_1| \sim |q_2|$. 

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