Are Self-Organised Critical Dislocation Dynamics Relevant to Ice Sheet Flow?

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Abstract: It was recently shown that crystals (including ice) plastically deform in an intermittent manner in usual laboratory conditions. The present paper aims at discussing whether such self-organised critical dynamics still apply to polar ice sheet conditions. Field data suggest that grains should contain between zero and one dislocation moving at a time. However, this is nothing but an average estimate. Field data also show that strong back-stresses are present, corresponding to a significant density of potentially mobile dislocations. These findings, together with the very low loading level, are consistent with critical dislocation dynamics, in which collective motion events occur for a short time, followed by long periods of inactivity during which grain growth, rotation recrystallization and other recovery processes contribute to the reduction of the long range internal stress field. The stress and grain size dependencies of the strain rate are derived on this basis. The applicability of the Hall-Petch law is also discussed.

Key words: creep, dislocations, critical dynamics, ice sheets, Hall-Petch law, normal grain growth, rotation recrystallization

1. Introduction

Polar ice sheets contain unique data on atmosphere and climate evolution during the last hundred thousand years. However, a precise dating of ice core samples requires a trustable understanding of ice creep laws.

It is generally agreed that ice in polar sheets mostly deforms through intracrystalline slip associated with normal grain growth (NGG), rotation recrystallization (RR) and migration recrystallization (MR) processes [1]. In the case of the GRIP ice core, characterized by a constant temperature from the surface down to approximately 1600-1700 m, the average grain size gradually increases from the surface down to about 700 m and stabilizes (essentially through a combination of NGG and RR) to a constant value of about 4 mm.

The stress dependence of the strain rate is still unclear. In usual "high stress" laboratory creep tests, it is given by a classical power law $\dot{\varepsilon} \propto \sigma^n$, with a stress exponent n=3. In the slowest laboratory creep tests the stress exponent n is close to 2 or even less [2]. There is some evidence from in situ measurements [3; 4] that this is still true in the extreme low stress and strain rate conditions of ice sheet flow (see below). A convincing explanation is still lacking for these particular values of the stress exponent. However, a reduction of the internal stress by grain boundary migration and recrystallization was invoked to explain this small value of the stress exponent [1].

A grain size dependence of the flow law at low stresses, if not shown by laboratory tests carried out in the conditions of the flow of glaciers and ice sheets, has been established from in situ measurements [5]. As a consequence, the flow law can be written:

$$\dot{\varepsilon} \propto \sigma^n \mathrm{f(D)} \qquad (1)$$

where f(D) is an ill-defined decreasing function of D, that can be approximated by $\mathrm{f(D)} \cong \mathrm{D}^{-\alpha}$ with rough α estimates between 1 and 2 [5].

On the other hand, recent laboratory experiments, conducted at "high" stresses, reveal that ice single and polycrystals respond to a constant applied load by intermittent flow involving large numbers of interacting dislocations (avalanches), and that strain bursts amplitudes obey a scale-free distribution in the case of single crystals. These findings mean that usual micro-macro procedures used to derive macroscopic properties from microscopic ones, based on a "mean field" assumption, i.e. on the possible existence of a characteristic size above which the material can be treated as an average medium, are questionable in this case.

However, compressive stresses used in laboratory tests are generally above 0.1 MPa, corresponding to strain rates around 10^{-6} s^{-1} in single crystals favourably oriented for basal slip, and higher than $10^{-9}s^{-1}$ in polycrystals, at temperatures between –3 °C and –20°C. Such strain rates and stresses are significantly larger than those prevailing in polar ice sheets, respectively below 10^{-10} s^{-1}, and lower than 0.05 MPa, for temperatures ranging between –30 to –50 °C. One may therefore wonder whether or not grains in polar ice conditions deform in a continuous or in an intermittent manner.

After a brief recall of our recent findings on intermittent flow of ice crystals, we shall use ice sheet flow numerical data to answer three questions: i) do ice sheets deform by movements of individual dislocations or by avalanches involving many dislocations? ii) how can one derive the stress exponent characteristic of ice

sheet flow, and iii) is the Hall-Petch law still valid in such conditions?

2. Self-organised critical dynamics in ice single crystals

We showed in a previous work by acoustic emission experiments [6] that plastic deformation in ice occurs in an intermittent manner. More precisely, in single crystals under constant load, strain burst amplitudes or energies exhibit power-law (i.e. scale-free) distributions, with a power law exponent of -2.0 for amplitudes (fig. 1), corresponding to -1.5 in energy distributions. Such a behaviour was shown to be quite general, as hexagonal and fcc metals were also shown to obey a similar law with the same exponent [7]. This is the signature of a close-to-criticality collective dynamics, which is encountered in several systems involving a large number of interacting entities. This behaviour was shown to be robust against stress or temperature changes, at least within the explored domain of temperatures and resolved shear stresses (-20°C < T < -3°C, 60 kPa < σ < 350 kPa).

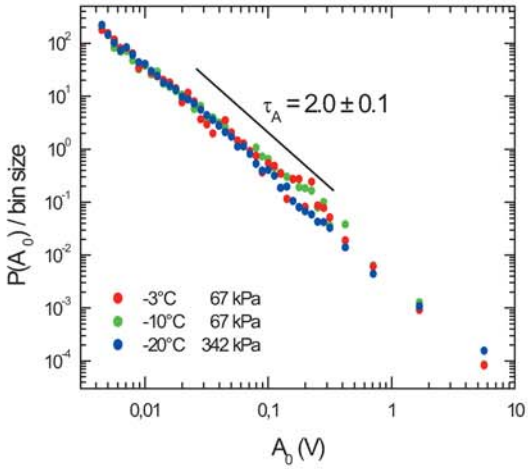

Fig. 1: Distribution of acoustic emission amplitudes in ice single crystals at different stresses and temperatures

Such scale-free distributions of strain amplitudes mean that no average value of the strain rate can be calculated. More particularly, the classical Orowan's equation $\dot{\varepsilon} = \rho b V$, where ρ is the average mobile dislocation density and V the average dislocation velocity, may not apply in this case.

3. The case of polycrystals and the Hall-Petch law

Laboratory tests on ice polycrystals [8] showed that deformation remains intermittent, but with significant modifications compared to the case of single crystals. Grain boundaries (GBs), acting as strong barriers to dislocation motion, introduce a characteristic length in the system. As a consequence, the scale-free distribution of strain amplitudes is modified in two ways: i) a cutoff appears at large amplitudes, corresponding to the larger possible avalanche that can be contained in a grain, and ii) smaller avalanches are still power law distributed, but with a smaller exponent

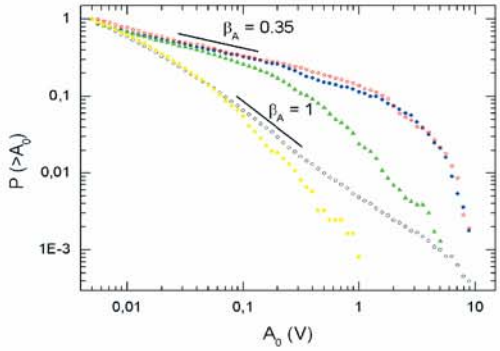

Fig 2: Cumulative distribution of acoustic emission amplitudes in ice polycrystals with decreasing grain sizes (red , blue, green and yellow curves) compared to that of single crystals (grey curve). The exponent β_A=1 in the cumulative distribution for single crystals is equivalent to an exponent τ_A=2 in the corresponding probability distribution shown in fig. 1.

These findings were explained in terms of aftershocks triggered in neighbour grains by avalanches stopped at grain boundaries [8]. The influence of grain size on mechanical properties of polycrystalline materials (Hall-Petch law) was directly derived from these findings [9]. The well known Hall-Petch (HP) law, discovered in 1951 on iron-based alloys [10, 11], states indeed that the grain size dependence of the macroscopic plastic yield stress of a polycrystal is given by:

$$\sigma_y = \sigma_\infty + k_o D^{-1/2} = \sigma_\infty + K_o \mu \sqrt{b/D} \qquad (2)$$

where D is the grain size, k_o, K_o and σ_∞ are constants, μ is the shear modulus and b is the Burgers vector modulus. The σ_∞ term is negligible compared to the grain size dependent term as long as the main obstacles to dislocation motion are GBs. In this case, the energy relaxed by a big avalanche in a given grain, proportional to the stored elastic energy (that scales as σ^2) and to the grain volume (that scales as D^3), concentrates on the GB surface (that scales as D^2). It can trigger an aftershock in a neighbour grain if the concentrated stored elastic energy (that scales as $\sigma^2 D^3 / D^2 = \sigma^2 D$) equals a given threshold k_o^2, which gives the "pure" Hall-Petch law $\sigma \propto k_o / \sqrt{D}$.

The influence of an intrinsic friction stress Σ_o opposing dislocation motion (in other words the yield stress of the corresponding single crystal) was also considered in [9]. A deviation from the "pure" HP law appears for large

grain sizes, i.e. when the grain size contribution becomes comparable to the intrinsic resistance of the material. The general law giving the yield stress of polycrystals writes in this case:

$$\sigma_y = \sqrt{\Sigma_0{}^2 + \tfrac{6b}{D}\Phi^2} \qquad (3)$$

where $\Phi = k_o / \sqrt{6b}$ is the stress threshold above which new dislocations can be nucleated from a grain boundary. Two obvious limiting cases are found for $D \to \infty$ (giving the single crystal yield stress $\sigma_y = \Sigma_o$) and for $\Sigma_0 \to 0$ ("pure" HP law $\sigma_y \propto 1/\sqrt{D}$).

The generalised Hall-Petch law is a direct consequence of intermittent plastic flow in polycrystals, describing the combined effects of grain size and intrinsic strength, in a fixed grain size environment. However, in the case of ice sheet flow, GBs are mobile at the time scale of creep, and the grain size may evolve with strain. The applicability to ice sheet flow of the generalised HP law may depend on how the grain size evolves and possibly adjusts to deformation conditions. It also depends on whether or not the ice sheet deforms in an intermittent way. The question of a possible "reduction" of the generalised HP law to the "pure" HP law should depend in turn on whether or not the grain size effect is dominant compared to the yield stress of the corresponding single crystal.
It is therefore of interest to check whether or not ice sheet flow involves motion of individual uncorrelated dislocation or collective bursts of dislocation groups, and to examine how the average grain size may depend on loading conditions.

4. The specific conditions of ice sheet flow

4.1. Field data

Hexagonal ice has a shear modulus $\mu = 3 \times 10^9$ Pa, and a Burgers vector $b = 4.5 \times 10^{-10}$m. For obvious reasons, creep flow is generally slower in ice sheets than in typical laboratory experiments. In the following, we use creep data taken from the "simple" case of the GRIP ice core with a constant temperatures around – 30°C, and a vertical strain rate $\dot{\varepsilon} = 2 \times 10^{-12}\,s^{-1}$ down to a depth of about 1600-1700m.
Grain sizes are constant, around 4mm in the depth interval ranging from 700 to 1600-1700 m. As GB migration is involved in models accounting for such a constant grain size value, we need a value of the GB migration rate K: we shall take $K=3.8 \times 10^{-9}\,m^2/yr = 1.2 \times 10^{-16} m^2/s$ [1].
Shear stresses are not known precisely. The horizontal shear stress, which increases with depth, is lower than the vertical axial stress for depths smaller than 1600-1700m. A minimum value of 0.05 MPa for the equivalent stress is taken here to discuss physical mechanisms for this ice.
Dislocation velocities, extrapolated from laboratory experiments [12] down to such low stresses, are $V = 2.5 \times 10^{-8}$ m/s.

4.2. The active role of grain boundaries in ice sheet creep

The different processes involved in ice sheet creep, and more particularly those responsible for the observed constant value of the grain size, have been debated in the past.
For instance, the steady grain size value was ascribed to a balance between normal grain growth (NGG) and plastic strain of grains [13]. However, plastic deformation operates at constant volume. As a consequence, it may be responsible for a change in grain shapes, but not in grain volumes. In addition, changes in grain shapes are not observed in the present case, as they are continuously smoothed out by NGG, that tends to minimize the total GB area, i.e. favour isotropic grain shapes.
This is why, starting from the initial idea proposed by Alley [14], it is now recognized that such a constant grain size observed in ice sheet creep essentially results from a balance between NGG and rotation recrystallization (RR) mechanisms. [1].
NGG is driven by the energy decrease associated with GB surface reduction, and controlled by diffusion. It therefore leads to a grain size growth given by.

$$D^2 - D_o{}^2 = Kt \qquad (4)$$

where K is the GB migration rate. This theoretical result slightly differs from what is observed at GRIP and Dome C ice cores, for which the grain size exponent is m = 2.4 or 2.5 instead of 2, probably due to bubble pinning [15,16], as exponents larger than 2 are qualitatively equivalent to lower migration rates. For the sake of simplicity, we shall assume an exponent of 2, and mention in section 7 the possible consequences of an exponent m of 2.4 or 2.5 on the grain size dependence of the strain rate.
The GB migration velocity can be deduced from the derivation of eq. (4):

$$dD/dt = K/2D \qquad (5)$$

On the other hand, rotation recrystallisation (RR) results from accommodation of strain gradients through formation of (essentially tilt) subboundaries, that may evolve into GBs for a sufficiently large misorientation θ_c, usually taken of about 5°.
Starting from small grains close to the surface and going downwards into the ice core, the average grain size gradually increases, but the NGG rate, that scales as 1/D, slows down. A steady state is reached when NGG is exactly balanced by RR, that is responsible for generation of new GBs.
The resulting steady state grain size D can be estimated considering that such a steady state is achieved when the time t_{ngg} necessary to increase the grain volume by a

factor 2 (i.e. the grain diameter by a factor 2/3) through NGG equals the time t_{rr} required to build a new boundary with a characteristic misorientation θ_c through RR, which brings the grain volume back to its initial value.
t_{ngg} is given by:

$$t_{ngg} \propto 2D\Delta D / K = (2D^2 / K)(\Delta D / D) \propto D^2 / K \quad (6)$$

t_{rr} can be estimated considering that subboundaries do not directly result from deformation, but from deformation gradients stemming from strain incompatibilities between grains. It can be reasonably assumed that the incompatible part of the deformation is proportional to the deformation:

$$\varepsilon_{inc} = \gamma\varepsilon \quad (7)$$

with $\gamma < 1$ [1]. A grain boundary is considered to be formed from a subboundary when the bending resulting from the incompatible part of the strain reaches a characteristic value θ_c, which gives:

$$\varepsilon_{inc} = \gamma\varepsilon = \gamma\dot{\varepsilon}t_{rr} = \theta_c \quad (8)$$

giving:

$$t_{rr} = \theta_c / \gamma\dot{\varepsilon} \quad (9)$$

Equating t_{rr} and t_{ngg} gives:

$$D^2 = 3K\theta_c / 4\gamma\dot{\varepsilon} \quad (10)$$

A similar equation was already derived in [17]. If most dislocations are "geometrically necessary dislocations" (GNDs), γ=1. Owing to the very large plastic anisotropy of ice, this is likely to be true in ice polycrystals. This equation and eq. (13) in ref [1] are equivalent in this case.
Eq. (10) shows that $f(D) = D^{-2}$ in eq. (1), in reasonable agreement with observations that give $1<\alpha<2$.

4.3. What can be directly deduced from field data

Let us consider a group of dislocations piled-up on a GB. The time required for the dislocation at the head of the pile-up to be absorbed by the GB is the time taken by the GB to move on a distance of the order of $b = 4.5 \times 10^{-10}$m. From field data recalled in section 4.1, it is of the order of:

$$2bD/K = 3 \times 10^4 \text{ s.} \quad (11)$$

On the other hand, assuming that all dislocations travel across the whole grain, each of them providing a strain b/D, the average arrival time of dislocations on a GB, which is equal to the nucleation time on the opposite GB, is given by:

$$\delta t_{arr} = \delta t_{nucl} = 1/\nu = b / D\dot{\varepsilon} = 5\times10^4 s \quad (12)$$

where ν is the nucleation frequency. It is remarkable that the absorption and nucleation times are quite comparable. This is probably not a pure coincidence. A steady state is likely to be obtained through a self adjustment of nucleation and annihilation rates. Yet, no direct quantitative indication can be drawn from this finding about the number of potentially mobile and simultaneously moving dislocations, i.e. on whether we are facing a single (individual) or a many (avalanche type) dislocation problem.
However, the fact that these times have similar values means that some back stresses had developed up to a level that enables a reduction of the nucleation rate, in order to adjust it to the NGG rate.
Experiments show indeed a reduction of the strain rate by several orders of magnitude during transient creep [18] (though ice sheet conditions are not necessarily identical). Such back stresses (long range coupling) are likely to exist in ice sheet conditions, and suggest that a significant number of dislocations interact, and may possibly propagate in the form of avalanches.

4.4. What is not known without any further assumption

As discussed above, the applied shear stress in polar ice sheets is probably less than 0.1 MPa. Owing to the probable existence of back stresses as discussed above, the effective stress experienced by dislocations is even smaller, and may have values down to 0.05 MPa.
The dislocation velocity V of an individual dislocation can be estimated assuming that the stress is known, and that the linear relation $V = \beta\sigma$ (with $\beta=5\times10^{-7}$ m/s.MPa) obtained by [19] from laboratory data at -32°C can be linearly extrapolated to the ice sheet stress range.
The dislocation velocity range corresponding to a typical stress $\sigma \geq 0.05$ MPa as mentioned above is:

$$V \geq 2.5 \times 10^{-8} \text{ m/s} \quad (13)$$

The number N_{fly} of simultaneously flying dislocations is given by:

$$N_{flight} = \delta t_{flight}/\delta t_{nucl} = (D/V)\,\nu \quad (14)$$

where δt_{flight} =D/V is the dislocation flight time. which corresponds to a moving[2] dislocation density:

[1] Most dislocations travel from a GB to the opposite one, ensuring crystal deformation. Owing to strain gradients, a fraction of them (γ) get stuck in the grain interior and participate to subboundary formation. The large plastic anisotropy of ice suggests that strain is mostly incompatible, i.e. strain gradients are large, and γ is very close to 1.

[2] instead of the classical "mobile" term, we prefer using "moving", which means "simultaneously moving". By contrast, the term "mobile" will refer here to "potentially mobile" dislocations, i.e. dislocations that are able to move from time to time.

$\rho_{flight} = N_{flight} / D^2$ (15)

5. Is ice sheet flow a single or a many-dislocation problem?

5.1. Arguments in favour of a single dislocation behaviour

The dislocation travel time through a 4mm grain is obtained from eq. (13):

$\delta t_{flight} = D/V < 1.6 \times 10^5$ s (16)

The average number of dislocations travelling simultaneously is estimated from eqs. (12) and (16):

$N=(\delta t_{flight} / \delta t_{nucl}) < 3$ (17)

As a consequence, there is probably no more than one dislocation traveling at a time in a grain in average.

5.2. Arguments in favour of a many-dislocation behaviour

i) The values of nucleation and relaxation times (eqs (11) and (12)) are surprisingly similar. This is probably not fortuitous, and is a clue for a control of dislocation nucleation by GB migration. This has to be achieved through long range back stresses, as mentioned in section 4.3.

ii) In transient regimes [18] the strain rate decreases over at least 2 orders of magnitude, due to the development of strong back stresses. Though this is a transient that probably differs from what occurs in ice sheet conditions, it may be used to obtain a rough estimate of dislocation density and dislocation number in a grain. Taking the observed stress dependence of the strain rate given by eq. (1) with n=2 ($\dot{\varepsilon} \propto \sigma^2$) one can write $(\sigma - \sigma_i)^2 / \sigma^2 < 10^{-2}$, or equivalently:

$(\sigma - \sigma_i)/\sigma < 10^{-1}$ (18)

where σ_i is the internal stress. The internal stress is thus of the order of the applied stress (only 10% less).

The corresponding dislocation density can be estimated through Taylor's relation, that directly results from the 1/r dependence of the dislocation stress field:

$\sigma \approx \sigma_i \approx 0.5 \mu b \sqrt{\rho}$ (19)

Taking $\sigma \cong 0.05$ MPa, one gets:
$\rho \cong 6 \times 10^9 m^{-2}$.

Estimates from distortion measurements using hard X-ray diffraction on Vostok ice cores [20] gave GND dislocation densities of the same order of magnitude. The corresponding number of GNDs per grain (for a grain size of 4 mm) is therefore of the order of:

$N_{GND} \cong 10^5$. (20)

These arguments show that the applied stress is likely to be very close to the internal stress. It also reveals that a significant number of dislocations (moving or not) are present on average in a grain. This is strongly in favour of a many-dislocation behaviour

5.3. Discussion

The fact that only one dislocation is supposed to travel at a time is very similar to what occurs in polycrystals with nanosized grains [9], which is a single-dislocation problem.

In the case of nanograins however the only dislocation present in a grain is the moving one. As a consequence, no back stress opposes dislocation nucleation nor motion. The situation is quite different here. The moving dislocation density is indeed only part of the "potentially mobile" dislocation density. Back stresses result from some storage of potentially mobile dislocations, even if the average number of moving dislocations per grain is lower than unity.

The number of simultaneously moving dislocations N=1 is in average only. Deformation may proceed through a series of bursts involving several interacting dislocations, separated by long periods of inactivity.

Where can these dislocations come from? Owing to the strong plastic anisotropy, forest dislocations do not exist (except for a few possible growth defects). Dislocation dipoles may be formed by interaction of opposite dislocations, and subboundaries may result from strain heterogeneities. A sudden burst in a grain changes the internal stress field, and may destabilize dipoles or subboundaries or any other metastable dislocation structure in a neighbour grain, triggering dislocation avalanches through a domino effect.

In acfdgddition, critical dynamics are supposed to apply when the loading rate is small compared to the relaxation rate, which is the case here.

The above arguments support intermittent deformation through a many dislocation behaviour.

6. The stress exponent in ice sheet flow conditions

As discussed above, classical "mean field" methods used to derive macroscopic properties from microscopic mechanisms are not supposed to work in the case of self-organised critical dynamics. They cannot be directly used to determine the stress exponent n. In particular, Orowan's equation, that assumes that an average dislocation velocity can be defined, may not be applicable. The case of polycrystals is probably less stringent, as the introduction of a characteristic size (grain size) may allow averaging velocities and dislocation densities over both time and space. However, a derivation of the stress exponent will be achieved here using a different type of argument.

As mentioned in section 4.3, the internal stress σ_i is very close to the applied stress σ, and is gradually relieved by grain boundary migration, allowing further plastic flow to occur, which restores the previous internal stress

level. Such a steady GB migration averages spatial heterogeneities arising from intermittent flow, which allows a mean field treatment of stress relaxation through GB migration. A decrease $d\sigma_i$ of internal stresses allows an increment of plastic strain $d\varepsilon \propto d\sigma_i$. The corresponding strain rate is therefore:

$$\dot{\varepsilon} \propto d\sigma_i / dt \qquad (21)$$

Strain rate is thus controlled by internal stress relaxation, i.e. by dislocation annihilation, which obeys a first order kinetics since the dislocation absorption mechanism at GBs involves single dislocations:

$$d\rho / dt \propto \rho \qquad (22)$$

Using eq. (22) and eq. (19) (Taylor's relation), one gets:

$$d\sigma_i / dt \propto \left(1/\sqrt{\rho}\right) d\rho / dt \propto \sqrt{\rho} \propto \sigma_i \qquad (23)$$

and from eq. (21):

$$\dot{\varepsilon} \propto d\sigma_i / dt \propto \sigma_i \approx \sigma \qquad (24)$$

We get in this case a stress exponent n = 1. Plastic flow should therefore be newtonian at low stresses.
By contrast, at larger stresses, relaxation of internal stresses is essentially ensured by dislocation pair annihilation. In this case, the kinetics is a 2^d order one, and eq. (22) is replaced by:

$$d\rho / dt \propto \rho^2 \qquad (25)$$

which leads to:

$$\dot{\varepsilon} \propto d\sigma_i / dt \propto \sigma_i^3 \approx \sigma^3 \qquad (26)$$

The stress exponent n=3 agrees with experiments performed at high stresses.
As both types of internal stress relaxation work in parallel, the general expression of the strain rate writes:

$$\dot{\varepsilon} = A\sigma + B\sigma^3 \qquad (27)$$

This expression is schematised and compared with experimental data in fig 3. Such a model is consistent with stress exponents of 1.8 or 2 observed on both laboratory experiments and in situ inclinometry observations [21, 3].
One may wonder whether this is still valid in "dirty" ice, though no data are available in this case. In dirty ice, obstacles (impurities, precipitates) may disturb both dislocation motion and GB migration. Considering dislocation motion, the above calculation is valid as long as Taylor's relation (eq. (19)) applies. Localised obstacles to dislocation motion should not change the critical dislocation dynamics, as elastic coupling between dislocations still exists. They may add a contribution to the resistance to dislocation motion, but such a contribution is not polarised, which means that, as a first approximation, it is not involved in eq. (23). Eq. (27) should therefore still be valid. By contrast, impurities may hinder GB migration, and consequently internal stress relaxation. Dislocations should be essentially eliminated through pair annihilation, as in the high stress case, and one may expect a stress exponent close to 3.

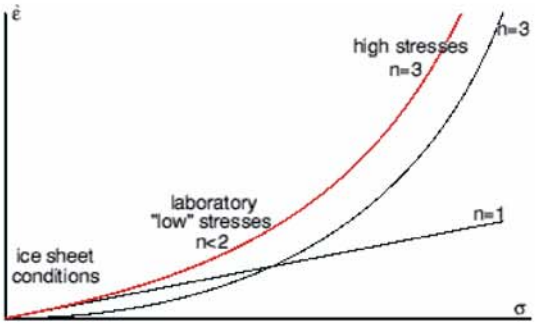

Fig. 3: Schematic representation of the stress dependence of the strain rate (red curve), combining a linear dependence at low stresses and a cubic one at large stresses (black curves), according to eq. (27).

7. The Hall-Petch law and the grain size dependence of the strain rate

As discussed in section 3, the generalised Hall Petch law (eq. (3)) is obeyed in the case of self-organised critical dislocation dynamics, and related to the relaxation of internal (incompatibility) stresses developed during polycrystal deformation. The existence of a possible lattice friction does not change critical dynamics, as shown by acoustic emission experiments at –3, -10 and –20°C [22]. Such internal stresses are reduced by NGG in steady state conditions. They nevertheless stabilise at a significant level, as mentioned in section 5.2, which suggests that the HP law may apply in ice sheet conditions.
However, in steady state conditions, eq. (10) shows that small-grained ice is softer than large-grained one, which contradicts the HP law tendency. A similar trend is found experimentally: with grain sizes in the 0.5 to 10 mm range, and strain rates between 10^{-10} and $10^{-11}s^{-1}$ [23-25], the viscosity of small-grained ice is found larger than that of large-grained one in transient creep, but the tendency is reversed as the system becomes closer to the steady state.
Actually, the Hall-Petch law is usually found to apply in a fixed grain size environment. In the present case of ice sheet flow, the steady state described by eq. (10) results from a balance between RR and NGG. As a consequence, the grain size is not fixed, but

continuously adjusts to the strain rate. Combining eqs (1) and (10) gives:

$$\dot{\varepsilon} \propto \sigma^n K D^{-2} \qquad (28)$$

The grain size dependence of the stress at constant strain rate and in steady state conditions is therefore:

$$\sigma \propto D^{2/n} / K^{1/n} \qquad (29)$$

Such a grain size dependence of stress can be considered as an inverse Hall-Petch behaviour. Fig. 4 schematically shows this unusual behaviour.

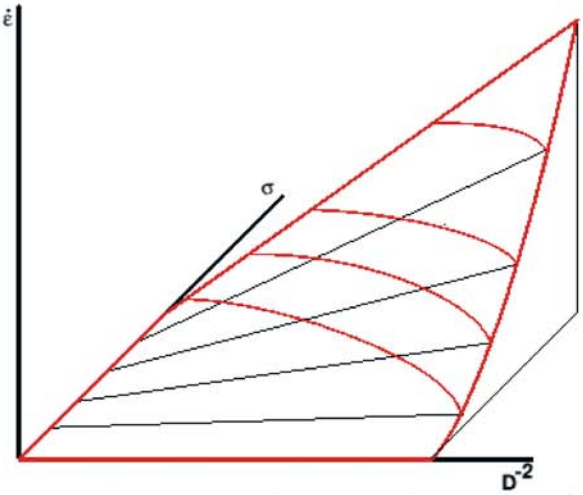

Fig 4 : Schematic stress / strain rate / grain size surface based on i) a linear dependence of strain rate vs D^{-2} (eq. (10)), and a supposed parabolic dependence of strain rate vs stress (eq. (1)). The grain size dependence of stress at constant strain rate is given by horizontal sections of the surface (thin red curves). It corresponds to an increase of stress with grain size at constant strain rate, i.e. to an inverse Hall-Petch behaviour.

Taking the example of GRIP or Dome C, where the measured grain size exponent in eq. (10) is m = 2.4 or 2.5 instead of 2, it can be easily shown that eq. (29) becomes:

$$\sigma \propto D^{m/n} \qquad (30)$$

which still corresponds to an inverse Hall-Petch behaviour.

Taking now the case of ice impure enough to totally hinder GB migration, eq (10) is no more valid, since GBs cannot adjust their size any more to the strain rate. The HP law should apply in this case, the strain rate being then controlled by strain transmission at GBs. More precisely, the generalised HP law (eq. (3)) reduces to the "pure" HP law $\sigma_y \propto 1/\sqrt{D}$ if the contribution of GBs to the yield stress is dominant compared to the intrinsic grain resistance. This situation occurs for small grains and large dislocation mobility, i.e. small lattice friction. Fairly low temperatures favour a significant lattice friction, but the very low stresses and strain rates (compared to Shearwood & Whitworth experiments [19]) are favourable to the validity of the pure HP law. On the other hand, grain sizes are large compared to "usual" cases of metals for instance that obey the HP law. It is therefore difficult to decide from such arguments whether or not the generalised HP law may reduce to the "pure" one in impure ice core conditions.

For a different reason from the high stress case discussed in section 6, dislocation pair annihilation should be the dominant dislocation elimination mechanism in impure ice, and a stress exponent of 3 is expected. Combining the HP relation $\sigma \propto D^{-1/2}$ with $\dot{\varepsilon} \propto \sigma^3$ gives:

$$\dot{\varepsilon} \propto D^{-3/2} \qquad (31)$$

instead of the D^{-2} dependence given in eq. (10).

8. Summary and conclusions

The goal of the present paper was to decide whether or not ice sheets were likely to deform through a succession of discrete dislocation avalanches, characteristic of self-organised critical dynamics, as ice crystals do during laboratory experiments.

We have shown from field data estimates that grains should contain between zero and one dislocation moving at a time. However, this is nothing but an average. There is circumstantial evidence that strong back-stresses develop during ice sheet deformation, corresponding to a significant density of potentially mobile dislocations, larger than 10^9 m^{-2}, in agreement with X-ray diffraction estimates conducted on Vostok ice cores. Together with the very low loading level, this is consistent with critical dislocation dynamics, in which collective motion events occur for a short time, followed by long periods of inactivity during which grain growth, rotation recrystallization and other recovery processes contribute to the reduction of the internal stress field.

Due to the non-linear behaviour of the system, Orowan's law, on which classical derivation of creep laws are usually based, may not be applicable in such conditions (although the case of polycrystals may be less stringent). We made use of the fact that deformation is controlled by gradual relaxation of back stresses by dislocation absorption during continuous grain boundary migration to derive the stress exponent of the creep law. It is found that the stress exponent is n=1 in the extreme case where this mechanism for dislocation annihilation is dominant. In cases where the efficiency of this process is reduced compared to dislocation pair annihilation (larger stresses, grain boundary pinning by impurities), the stress exponent is found between 1 and 3, and should be n=3 if pair annihilation is the only relaxation mechanism. This is not inconsistent with experimental findings and field observations, giving n of the order or less than 2 at low stresses.

The applicability of the Hall-Petch law was also discussed. In spite of the self-organised critical dynamics, which usually leads to the Hall-Petch law, an inverse Hall-Petch behaviour is found in the case where the grain size adjusts to the strain rate through a balance between RR and NGG. By contrast, a normal Hall-Petch law is expected to apply either at high stresses, or when GBs are strongly pinned by impurities.

Part of these findings are prospective, and should be confirmed by dedicated experiments or observations. More particularly, measurements of grain size dependence of the yield stress at constant (large and small) strain rates, and of the stress exponent at very low stresses on impure ice, should be of interest.

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