Instructions for use

Barter Transactions and Reproduction of the Economy:
A Theoretical Approach on the Basis of Russian Cases

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1. Introduction

In May 1992 the share of barter transactions in the sales of Russian industrial companies was only 4 percent, whereas the share increased to 54 percent in August 1998 but decreased to 21 percent in August 2000 and to 6 percent in 2005.[1] The transaction costs for in-kind settlement usually exceed those of currency settlement because of the problem of "double coincidence of wants". Why was the share of barter transactions in Russia at such a high level? One of the controversies argued in former studies lies in whether widespread barter transactions were caused mainly by liquidity constraint or by tax evasion. Although these studies focused on the actual circumstances of barter transactions, the influence of such transactions on economic reproduction remains unclear. This paper seeks to analyze theoretically the relationship between barter transactions and reproduction of the economy, taking into account liquidity constraint and tax evasion as the causes of barter transactions.

This paper indicates the following conclusions in the analysis. First, when firms are suffering from liquidity constraint, and currency settlement and barter exchanges are intermingled, price ratios both for currency settlement and barter exchanges are required for production of the economy, and in some cases commodities are non-equivalently bartered. Second, when firms bilaterally barter in order to evade taxes, trading commodities at intentionally raised prices, the economy falls into a trade-off between the reproducible economy and tax evasion. Third, when firms multilaterally barter in order to evade taxes through the transformation of income into expense, firms in surplus will gain profits from tax evasion and reproduction of the economy is guaranteed. Finally, when bilateral and multilateral barter transactions are combined, in some cases firms in deficit will become firms in surplus, making the economy reproducible.

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[1] Russian Economic Barometer (2000), Russian Economic Barometer (2006). The years 1999 and 2000 witnessed a decline in the share of barter transactions in sales. The OECD (2000) explains the reason for the fall in the share of money surrogates as follows. "First, during the high inflation that followed the exchange rate crisis of mid-1998, regulated energy prices declined sharply in real terms… Lower energy prices have made settlements with Gazprom and RAO EES easier for many industrial firms, and have perhaps increased the ability of the natural monopolies to insist on cash payments from relatively liquid consumers… Again, the increased liquidity of exporters, and their supporting industries, has provided an important potential source of cash revenue… Finally…a number of measures have been taken both at the federal and regional level to limit the use of surrogates in budgetary operations." (OECD, 2000, p.108)

The outline of this paper is as follows. Section 2 surveys the actual situation of barter transactions in Russia. Section 3 discusses barter transactions caused by liquidity constraint. Section 4 analyzes barter transactions for the purpose of tax evasion. The final section summarizes the conclusions.

2. Actual Situation of Barter Transactions in Russia

Section 2 will survey discussions of the major causes of barter transactions in Russia, namely liquidity constraint and tax evasion.

First, barter transactions caused by liquidity constraint will be discussed. Commander and Mumssen (1998) indicate that as bank credit dried up and firms became increasingly liquidity-constrained, they responded by paying suppliers, workers, utilities and tax authorities late (leading to arrears) and in kind (leading to the barter economy).[2] According to the questionnaire survey of Commander and Mumssen (1998) in 1998, 72% of firms responded that liquidity at their own firms was the important reason for using barter and 74% of firms responded that liquidity of partners was the important reason for using barter.[3] Furthermore, the questionnaire survey of Auktsionek (1998) at the end of 1994 demonstrates that respondents listed as main reasons for using barter either shortage of working capital (47%) or heavy tax payment (17%).[4] Woodruff (1999) points out that power companies accept in-kind payments (a kind of barter transaction),[5] which reflects the aspect of liquidity constraint of firms.

Another explanation for engaging in barter transactions is the tax evasion hypothesis. Firms in arrears on taxes had to consolidate their bank accounts into one account, from which the tax authority collected taxes,[6] and the sum of settlement by cash for one transaction was limited.[7] Furthermore, around 1998 there were some 200 identified taxes, which facilitated abuse on the part of corrupt tax inspectors.[8] Shmelev (1994) indicates that the real tax rate on profits in the Russian economy stood at 55-77%, taking

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[2] Commander and Mumssen (1998), p. 17.

[3] Commander and Mumssen (1998), p. 24.

[4] Aukutsionek (1998), C. 53.

[5] "Local electric power companies had no flexibility in setting formal prices, which were established by provincial energy commissions." (Woodruff, 1999, p. 123) "They did this [lowering prices of electricity] by allowing their customers to pay fewer of their bills (leading to a growth in payment arrears) or by accepting payment in goods that for purposes of this exchange were assigned an 'unrealistically' high price." (Woodruff, 1999, p. 121) (Words in brackets are by this author.)

[6] President Decree, No. 1212 as of August 18, 1996.

[7] For example, Direction of the Central Bank of Russia, No. 375 as of October 7, 1998 limited the settlement by cash to 10 thousand ruble for one transaction.

[8] OECD (1998), p. 57. The first general part of a tax code was finally passed by the Duma in February 1998 and became law as of 1 January 1999. This part of the Tax Code limited the number and type of possible taxes for the federal, regional, and local levels to a specific list. But the determination of various federal, regional, and local tax rates, exemptions, and revenue sharing rules is still determined in a largely discretionary manner (OECD, 2000, p. 64).

inflation into account, and in some cases it reached about 100%, whereas the legal rate was 35%.[9]

The scheme of barter transactions for tax evasion can be divided into bilateral and multilateral schemes.

The scheme of bilateral barter transactions is as follows.[10] Assume there are two firms: Firm A in deficit and Firm B in surplus. Profit tax will be levied on the profit of Firm B. Firm B therefore tries to conceal its profit. Firm A intentionally raises the price of its commodity, which leads to the increase of costs and decrease of profit for Firm B. If Firm A does not turn surplus, profit tax will not be levied on Firm A. As a result, Firm B can reduce its taxable income (See Figure 2-1).
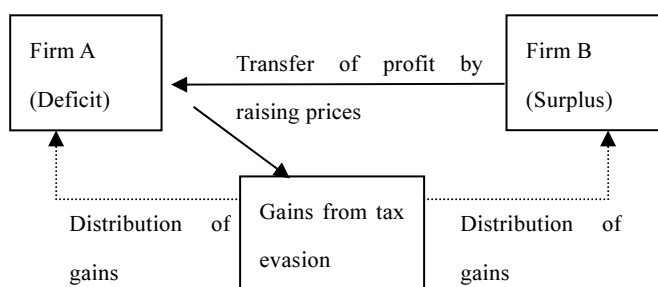

Figure 2-1   Bilateral barter transactions

The scheme of multilateral barter transactions is as follows.[11] Assume there are 3 firms: C, D, and E. Firm C sells its products to Firm D and receives commodities from Firm E as a payment for them. If Firm D is a creditor of Firm E, commodities from Firm E to Firm C are income for Firm C. However, Firm C can claim that the commodities are not income, but an expense for production. In this scheme Firm C can transform income into expense and decrease its profit (See Figure 2-2).[12]

Transaction costs should be taken into account in the analysis of barter transactions because the double coincidence of wants must be satisfied. According to Hendley et al. (1998), the cost of arranging most barter transactions is approximately 20 to 25 percent of the value of the transaction. Taking this indication into account, this paper assumes barter transaction costs are included in the price of commodities and prices are uniformly raised, for example, by 20 percent for simplification.

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[9] Shmelev (1994), p. 28.

[10] Hendley et al. (1998) was referred to for the scheme of bilateral barter transactions.

[11] Hendley et al. (1997), p. 36.

[12] For more discussion regarding barter transactions in Russia, see Maurel and Brana (1999) and Guriev and Ickes (2000).
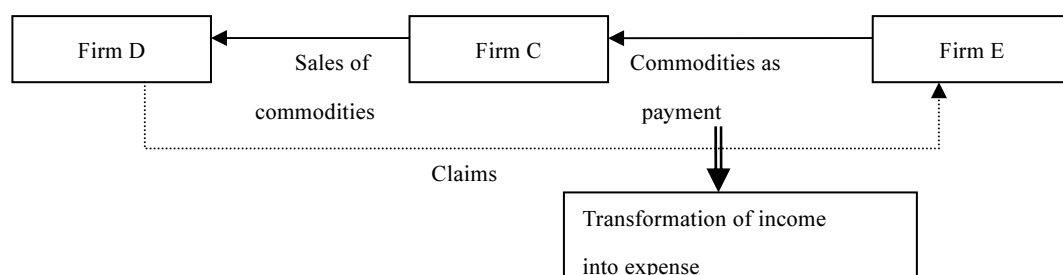
Figure 2-2　Multilateral barter transactions

This paper would contribute to the understandings of the Russian barter transactions, including power companies' in kind settlement from the viewpoints of liquidity constraint and avoidance of interbank settlement from the viewpoints of tax evasion.[13]

3. Barter Transactions Caused by Liquidity Constraint and Reproduction of the Economy

Section 3 discusses the influence of barter transactions caused by liquidity constraint on reproduction of the economy. After arguing the terms of the reproducible economy without barter transactions in Section 3.1, this paper analyzes the economy with barter transactions caused by liquidity constraint in Section 3.2.

3.1. The Terms for the Reproducible Economy without Barter Transactions

This section first examines the terms for the reproducible economy without barter transactions on the basis of the Pasinetti's theoretical analysis (see Pasinetti, 1977, 1993).

To explain the reason why Pasinetti's model is employed let us quote Pasinetti. "In the historical evolution of economic ideas since the classics, all economic theories belong to either of two alternative broad paradigms – one focusing on exchange (and more fundamentally on utility) and subjective value, the other focusing on production (and more fundamentally on labour) and objective value."[14] What is called neo-classical economics belongs to the 'exchange' paradigm whereas the 'pure' labour (production) model represents precisely that minimal and basic model that contains the very essential features of the 'production'

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[13] Donation networks can be destroyed by the existence of arbitrage transactions and the emergence of the consciousness of bilateral equivalent exchange. One of the reasons barter transactions exist in Russia is as follows. In Russia barter transactions caused by liquidity constraint are deeply associated with electric power companies, and it is difficult to arbitrage transactions with electricity. Furthermore, as mentioned above, Local electric power companies had no flexibility in setting formal prices and lowered prices of electricity by accepting payment in goods that for purposes of this exchange were assigned an 'unrealistically' high price.

[14] Pasinetti (1993), Preface p. 14.

paradigm, to which classical and Keynesian economics belongs. "There are evident symmetries between the two basic schemes. Each of them refers to a certain type of goods ('scarce' goods, and 'reproducible' goods, respectively)." [15] Because this paper will analyze terms for the reproducible economy with barter transactions, it is desirable to use Pasinetti's model.

The assumptions underlying this paper's analysis can be stated as follows. A certain economic system is observed which is in a perfectly stationary state. The methods of production are such that each sector produces a single commodity by using a certain physical quantity of labor and certain physical quantities of commodities. These commodities are completely used up in each period. The total output of the system must be used as consumption and as the replacement of those commodities which have been used up in the production process. The value added in the economic system is distributed to the members of the community at the end of each period in the forms of wages (w) and profits (π). Wages are distributed in proportion to the physical quantity of labor, and profits are distributed in proportion to the value of the means of production.

The technical methods of production are represented by a matrix of interindustry coefficients, denoted by $\mathbf{A}$, and by a row vector of direct labor coefficients, denoted by $\mathbf{a}_n$:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1,n-1} \\ a_{21} & a_{22} & \cdots & a_{2,n-1} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n-1,1} & a_{n-1,2} & \cdots & a_{n-1,n-1} \end{bmatrix} \tag{3.1.1}$$

$$\mathbf{a}_n = [a_{n1} \ a_{n2} \ \cdots \ a_{n,n-1}] \tag{3.1.2}$$

where

$$a_{ji} \geq 0, \quad i = 1, 2, \cdots, n-1 ; \quad j = 1, 2, \cdots, n \tag{3.1.3}$$

The commodity which the i th sector produce is referred to as the i th commodity, and the physical quantities of the j th commodity necessary for producing a unit of the i th commodity are denoted by a_{ji}. The prices will be defined by the system of equations

$$\begin{aligned} \left(a_{11}p_1 + a_{21}p_2 + \cdots + a_{n-1,1}p_{n-1}\right)(1+\pi) + a_{n1}w &= p_1 \\ \left(a_{12}p_1 + a_{22}p_2 + \cdots + a_{n-1,2}p_{n-1}\right)(1+\pi) + a_{n2}w &= p_2 \\ \vdots \\ \left(a_{1,n-1}p_1 + a_{2,n-1}p_2 + \cdots + a_{n-1,n-1}p_{n-1}\right)(1+\pi) + a_{n,n-1}w &= p_{n-1} \end{aligned} \tag{3.1.4}$$

where $p_1, p_2, \cdots, p_{n-1}$ are the prices of the commodities $1, 2, \cdots, n-1$.

This system contains $(n-1)$ equations in $(n+1)$ unknowns. Therefore two of the unknowns

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[15] Pasinetti (1993), Preface p. 15.

can be fixed exogenously. For example, setting $\pi = \bar{\pi}$ and the price of one of the commodities equal to unity, the system can be rendered determinate.

On the other hand, the physical quantities to be produced will be defined as follows:

$$a_{11}Q_1 + a_{12}Q_2 + \cdots + a_{1,n-1}Q_{n-1} + Y_1 = Q_1$$
$$a_{21}Q_1 + a_{22}Q_2 + \cdots + a_{2,n-1}Q_{n-1} + Y_2 = Q_2$$
$$\vdots$$
$$a_{n-1,1}Q_1 + a_{n-1,2}Q_2 + \cdots + a_{n-1,n-1}Q_{n-1} + Y_{n-1} = Q_{n-1}$$

$$(3.1.5)$$

where $Q_1, Q_2, \cdots, Q_{n-1}$ denote the physical quantities of commodities $1, 2, \cdots, n-1$ and $Y_1, Y_2, \cdots, Y_{n-1}$ denote those portions of the total physical quantities of commodities $1, 2, \cdots, n-1$ which are in surplus. If it is assumed that $Q_1, Q_2, \cdots, Q_{n-1}$ are determined exogenously, $Y_1, Y_2, \cdots, Y_{n-1}$ will be also determined exogenously[16].

If the number of workers employed per period is denoted by L, the following equation can be obtained.

$$a_{n1}Q_1 + a_{n2}Q_2 + \cdots + a_{n,n-1}Q_{n-1} = L \qquad (3.1.6)$$

This economic system is in a stationary state and commodities must be used up in each period. Therefore the sum of demand should be equal to the sum of supply in each sector. Then the terms for reproducible economy will be as follows:

$$\left(a_{11}p_1 + a_{21}p_2 + \cdots + a_{n-1,1}p_{n-1}\right)Q_1(1+\pi) + a_{n1}Q_1 w =$$
$$\left(a_{11}Q_1 + a_{12}Q_2 + \cdots + a_{1,n-1}Q_{n-1} + Y_1\right)p_1$$
$$\left(a_{12}p_1 + a_{22}p_2 + \cdots + a_{n-1,2}p_{n-1}\right)Q_2(1+\pi) + a_{n2}Q_2 w =$$
$$\left(a_{21}Q_1 + a_{22}Q_2 + \cdots + a_{2,n-1}Q_{n-1} + Y_2\right)p_2$$
$$\vdots$$
$$\left(a_{1,n-1}p_1 + a_{2,n-1}p_2 + \cdots + a_{n-1,n-1}p_{n-1}\right)Q_{n-1}(1+\pi) + a_{n,n-1}Q_{n-1} w =$$
$$\left(a_{n-1,1}Q_1 + a_{n-1,2}Q_2 + \cdots + a_{n-1,n-1}Q_{n-1} + Y_{n-1}\right)p_{n-1}$$

$$(3.1.7)$$

$$p_i, w, \pi \geq 0 \qquad (3.1.8)$$

When it is assumed that in each sector the sum of commodities in surplus is equal to wages, and profits, the following equation will be yielded.

$$\Pi_i + a_{ni}Q_i w = Y_i p_i \qquad i = 1, 2, \cdots, n-1 \qquad (3.1.9)$$

where

$$\Pi_i = \left(a_{1i}p_1 + a_{2i}p_2 + \cdots + a_{n-1,i}p_{n-1}\right)Q_i \pi \qquad i = 1, 2, \cdots, n-1 \qquad (3.1.10)$$

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[16] In this paper we assume that commodities to be produced are totally sold up and the economic system is in a perfectly stationary state. Therefore, our analysis does not cover barter transactions of commodities without demand.

This paper analyzes the economic system which consists of 3 sectors for simplification. Then following price ratio will be yielded from equations (3.1.7), (3.1.9) and (3.1.10).

$$\frac{(b_{12} + b_{13})b_{32} + b_{12}b_{31}}{(b_{21} + b_{23})b_{31} + b_{21}b_{32}} p_1 = p_2 = \frac{(b_{12} + b_{13})b_{32} + b_{12}b_{31}}{(b_{21} + b_{23})b_{13} + b_{12}b_{23}} p_3 \tag{3.1.11}$$

where

$$b_{ji} = a_{ji}Q_i \tag{3.1.12}$$

This price ratio is referred to as Price Ratio 1, and is not prices which rise under the existence of excess demand and fall under the existence of excess supply as Leon Walras assumed. This price ratio is prices which equalize the sum of the demand and the supply under the given quantities of demand and supply.

3.2. Barter Transactions Caused by Liquidity Constraint and the Terms of the Reproducible Economy

This section analyzes the situation when each sector is liquidity- constrained for some reason and transactions are partly settled by currency, with the remaining transactions settled in kind. As stated above, the economic system is analyzed which produces the commodities necessary for reproduction of the economy. In this case, each sector settles in kind because of (1) liquidity constraint, (2) reduction of the prices of the commodities, the prices of which are not independently decided (public utilities such as electricity) (Items (1) and (2) will be discussed in this section), and (3) acquisition of undue profit through tax evasion (Item (3) will be discussed in the next section).

When sectors in the economy are liquidity-constrained and they can select the priority of currency settlement, the share of sales under currency settlement will differ by sector. In this case, in order to guarantee reproduction of the economy each sector has to exchange by barter those commodities that are not settled by currency.[17] One question then arises: are these bartered commodities equivalently exchanged? To answer this question equivalent exchanges must be defined. This paper uses the definition of Krause (1982) that "if no commodity-owners can augment their stocks through mere exchange in a given exchange-structure, then exchange within this structure may be called *equivalent-exchange*."[18] Furthermore, Krause (1982) states as follows. "Equivalence as defined here relates not to a single individual act of exchange, but to the exchange-structure as a whole. In particular, the equivalence of two quantities of commodities does *not* mean that they embody an equal amount of labor-time or something similar. The term

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[17] If the currency which each sector has should not be run out, the assumption has to be added that the sum which each sector receives should be equal to the sum which each sector pays.

[18] Krause (1982), p. 32.

'equivalent exchange' can be applied to an individual act of exchange only if it is one act of a structure or equivalent-exchange."[19]

Therefore, it cannot be identified whether commodities are equivalently exchanged in each transaction or not. Furthermore, we do not have the scale with which we measure the sum of transactions in order to confirm the equivalency of transactions. Accordingly, selecting the commodities exchanged in kind, let us calculate the price ratio equalizing the sum of bartered demand with the sum of bartered supply in each sector, and use this price ratio as the scale for the measurement of the equivalency. The price ratio will be yielded as follows.

$$\frac{(b'_{12}+b'_{13})b'_{32}+b'_{12}b'_{31}}{(b'_{21}+b'_{23})b'_{31}+b'_{21}b'_{32}}\,p_1 = p_2 = \frac{(b'_{12}+b'_{13})b'_{32}+b'_{12}b'_{31}}{(b'_{21}+b'_{23})b'_{13}+b'_{12}b'_{23}}\,p_3 \tag{3.2.1}$$

where b'_{21} and b'_{31} denote the quantities of commodity 1 settled in kind in some period with sector 2 and 3, respectively. Similarly b'_{12} and b'_{32} denote the quantities of commodity 2 settled in kind in some period with sector 1 and 3, respectively. b'_{13} and b'_{23} denote the quantities of commodity 3 settled in kind in some period with sector 1 and 2, respectively. This price ratio is referred to as Price Ratio 2.

When the sum of transactions between sectors in terms of commodity 1 is calculated, the result indicates that one sector donates a given sum to another sector, and this donation passes through sectors, subsequently returning to the original sector. That is to say, Commodity 1 is overestimated (underestimated) in relation to Commodity 3, Commodity 3 is overestimated (underestimated) in relation to Commodity 2, and Commodity 2 is overestimated (underestimated) in relation to Commodity 1. In spite of the strange exchanges, the economy will be reproducible (Figure 1).[20] In this scheme there exists a possibility that commodity-owners can augment their stocks through mere exchange, and therefore commodities are not equivalently exchanged. This system is referred to as a real non-equivalent exchange.

To return to the subject of the Price Ratio 2, quantities of traded commodities are equal both in the case when all transactions are settled using currency and in the case when currency settlement and barter transactions are used. However, when the commodities traded in kind are picked out and the price ratio equalizing the sum of demand and supply in barter transactions is calculated, in some cases this price ratio is different from the price ratio employed in the currency settlement; that is, Price Ratio 2 is different from

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[19] Krause (1982), pp. 32-33. Assume that one unit of Commodity A is exchanged for 8 units of Commodity C, one unit of Commodity B is exchanged for 3 units of Commodity C, and one unit of Commodity A is exchanged for 2 units of Commodity B. In this case the owner of Commodity A receives 8 units of Commodity C for one unit of Commodity A, and receives 8/3 units of Commodity B for 8 units of Commodity C, and finally receives 4/3 units of Commodity A for 8/3 units of Commodity B. Because the owner of Commodity A can augment his stocks through mere exchange, this scheme is not regarded as equivalent exchange.

[20] This scheme was discussed on the basis of Nishibe (2001).

$$\frac{b'_{13}b'_{32}b'_{21} - b'_{12}b'_{23}b'_{31}}{(b'_{21} + b'_{23})b'_{31} + b'_{21}b'_{32}} p_1$$

donation

$$\frac{b'_{13}b'_{32}b'_{21} - b'_{12}b'_{23}b'_{31}}{(b'_{21} + b'_{23})b'_{31} + b'_{21}b'_{32}} p_1$$

donation

$$\frac{b'_{13}b'_{32}b'_{21} - b'_{12}b'_{23}b'_{31}}{(b'_{21} + b'_{23})b'_{31} + b'_{21}b'_{32}} p_1$$
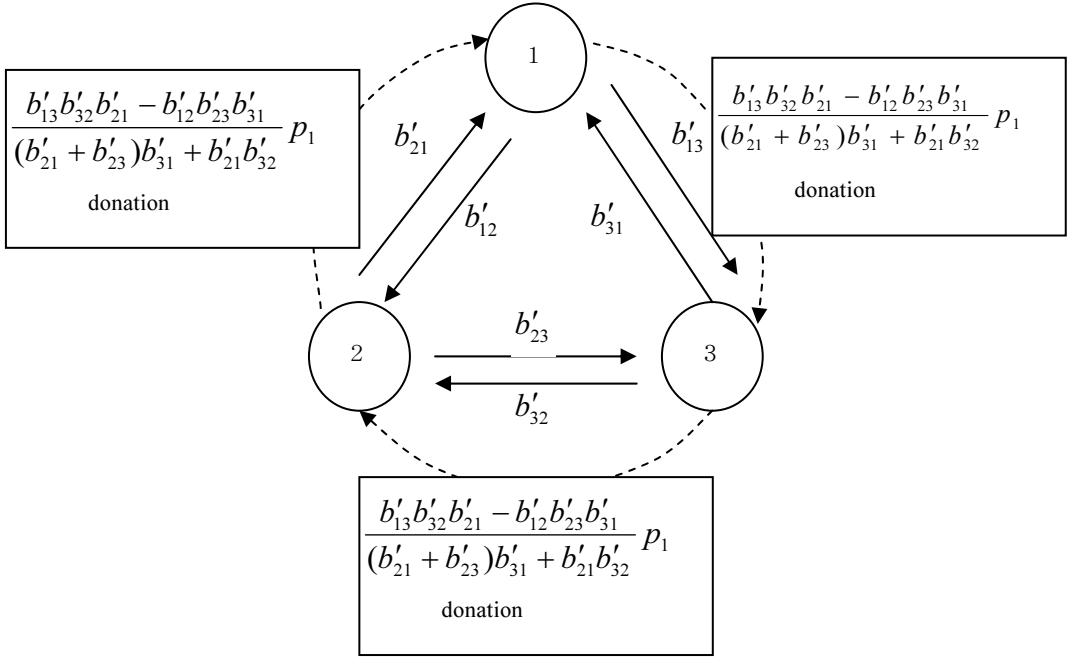
donation

Figure 1.   Real non-equivalent exchange

Price Ratio 1. Therefore, it appears commodities are non-equivalently bartered if only Price Ratio 1 is paid attention to. This scheme is referred to as a nominal non-equivalent exchange.

Furthermore, situations are analyzed in which Price Ratio 1 is not applied and products of Sector 1 are underestimated in relation to products of Sectors 2 and 3. In this case the profit of Sector 1 will be decreased. However, if Sector 1 produces some surplus, its obligation can be met and the reproduction of the economy will be guaranteed. On the other hand, if Sector 1 falls into deficit, the reproduction of the economy will not be guaranteed.

Moreover, the situation is analyzed that Sector 2 receives the products of Sector 1 at a higher price. For simplification we assume that Sector 2 does not fall into deficit when it donates a part of its profit to Sector 1. This paper denotes by α the price-rise-rate of Commodity 1 traded to Sector 2, by μ the profit tax rate, by s the share of in-kind payment in the sum of Commodity 1 traded to Sector 2, by t the share of in-kind payment in the sum of Commodity 2 traded to Sector 1 (on the terms of the price before intentional price rise). For simplification only profit tax levied at a fixed rate is taken into account. It is assumed that collected tax will be spent for the purchase of consumer goods, in order to satisfy the assumption of the stationary economy. The sum of produced commodities will be as follows (The profit is

indicated separately ($(1-\mu)\Pi_i$ and $\mu\Pi_i$) in order to clarify the levied tax).

$$(a_{11}p_1 + a_{21}p_2 + a_{31}p_3)Q_1 + a_{41}Q_1w + (1-\mu)\Pi_1 + \mu\Pi_1 = Q_1p_1$$
$$(a_{12}p_1 + a_{22}p_2 + a_{32}p_3)Q_2 + a_{42}Q_2w + (1-\mu)\Pi_2 + \mu\Pi_2 = Q_2p_2 \qquad (3.2.2)$$
$$(a_{13}p_1 + a_{23}p_2 + a_{33}p_3)Q_3 + a_{43}Q_3w + (1-\mu)\Pi_3 + \mu\Pi_3 = Q_3p_3$$

The nominal sum of the produced commodities under the donation from Sector 2 to Sector 1 will be as follows:

$$\{a_{11}p_1 + (1-s)a_{21}p_2 + sa_{21}p_2 + a_{31}p_3\}Q_1 + a_{41}Q_1w + (1-\mu)\Pi_1' + \mu\Pi_1' = Q_1p_1 + ta_{12}Q_2\alpha p_1$$
$$\{(1-t)a_{12}p_1 + ta_{12}(1+\alpha)p_1 + a_{22}p_2 + a_{32}p_3\}Q_2 + a_{42}Q_2w + (1-\mu)\Pi_2' + \mu\Pi_2' = Q_2p_2$$
$$(a_{13}p_1 + a_{23}p_2 + a_{33}p_3)Q_3 + a_{43}Q_3w + (1-\mu)\Pi_3 + \mu\Pi_3 = Q_3p_3$$

$$(3.2.3)$$

where the third member of the first equation and the second member of the second equation show the barter transaction and we get

$$s\,a_{21}Q_1p_2 = t\,a_{12}Q_2(1+\alpha)p_1 \qquad (3.2.4)$$

Furthermore, Π_1' and Π_2' denote the profit of Sector 1 and Sector 2 after donation from Sector 2 to Sector 1:

$$\Pi_1' = \Pi_1 + ta_{12}Q_2\alpha p_1 \qquad (3.2.5)$$
$$\Pi_2' = \Pi_2 - ta_{12}Q_2\alpha p_1 \qquad (3.2.6)$$

After all, the sum of $ta_{12}Q_2\alpha p_1$ was donated from Sector 2 to Sector 1. The following equations should be satisfied with non-minus prices, wages, and profits in order to render the economy reproducible.

$$(a_{21}p_2 + a_{31}p_3)Q_1 + a_{41}Q_1w + (1-\mu)(\Pi_1 + ta_{12}Q_2\alpha p_1) + \mu(\Pi_1 + ta_{12}Q_2\alpha p_1) =$$
$$(a_{12}Q_2 + a_{13}Q_3 + Y_1)p_1 + t\,a_{12}Q_2\alpha p_1$$
$$(a_{12}p_1 + a_{32}p_3)Q_2 + a_{42}Q_2w + (1-\mu)(\Pi_2 - ta_{12}Q_2\alpha p_1) + \mu(\Pi_2 - ta_{12}Q_2\alpha p_1) =$$
$$(a_{21}Q_1 + a_{23}Q_3 + Y_2)p_2 - t\,a_{12}Q_2\alpha p_1$$
$$(a_{13}p_1 + a_{23}p_2)Q_3 + a_{43}Q_3w + (1-\mu)\Pi_3 + \mu\Pi_3 = (a_{31}Q_1 + a_{32}Q_2 + Y_3)p_3$$

$$(3.2.7)$$

When the sum of deficit of Sector 1 is smaller than the donation, namely

$$-\Pi_1 < ta_{12}Q_2\alpha p_1 \qquad (3.2.8)$$

is satisfied, Sector 1 can maintain liquidity.

4. Barter Transactions to Evade Taxes and the Reproducible Economy

In the analysis barter transactions are divided into bilateral and multilateral transactions. Section

Barter Transactions and Reproduction of the Economy

4.1 discusses bilateral barter transactions, and Section 4.2 analyzes trilateral barter transactions as a case of multilateral barter transactions for simplification. Finally, Section 4.3 argues trilateral barter transactions with the elements of bilateral barter transactions.

4.1 Bilateral Barter Transactions and Reproducibility of the Economy

First, it is assumed that in bilateral barter transactions both sectors are in surplus. Sector 2 aims to conceal its profit by receiving products of Sector 1 at a higher price and transferring profit to Sector 1. The sum of the produced commodities will be as follows.

$$\{a_{11}p_1 + (1-s)a_{21}p_2 + sa_{21}p_2 + a_{31}p_3\}Q_1 + a_{41}Q_1 w + (1-\mu)\Pi_1' + \mu\Pi_1' = Q_1 p_1 + ta_{12}Q_2\alpha p_1$$
$$\{(1-t)a_{12}p_1 + ta_{12}(1+\alpha)p_1 + a_{22}p_2 + a_{32}p_3\}Q_2 + a_{42}Q_2 w + (1-\mu)\Pi_2' + \mu\Pi_2' = Q_2 p_2$$
$$(a_{13}p_1 + a_{23}p_2 + a_{33}p_3)Q_3 + a_{43}Q_3 w + (1-\mu)\Pi_3 + \mu\Pi_3 = Q_3 p_3$$

(4.1.1)

The third member of the first equation and the second member of the second equation mean barter transactions and $s\,a_{21}Q_1 p_2$ is equal to $t\,a_{12}Q_2(1+\alpha)p_1$. Furthermore, Π_1' and Π_2' stand for $\Pi_1 + ta_{12}Q_2\alpha p_1$ and $\Pi_2 - ta_{12}Q_2\alpha p_1$, respectively. It is supposed that Sector 1 and Sector 2 divide undue profit, guaranteeing the original profit for Sector 2. In this case the profit is transferred from a sector in surplus to a sector in surplus. Therefore, the profit tax will be levied on the profit. Obviously, in this scheme undue profit from barter transactions does not arise and the profit transferred from Sector 2 to Sector 1 is only returned to Sector 2. However, each sector is in surplus and the reproduction of the economy will be guaranteed.

Subsequently, it is assumed that Sector 1 is in deficit and other sectors are in surplus. In this case, Sector 2 can transfer its profit to Sector 1 as long as Sector 1 does not turn surplus (if Sector 1 turns surplus, tax will be levied on the profit of Sector 1, causing additional undue profit to disappear). The sum of $\mu\,t\,a_{12}Q_2\alpha p_1$ can be gained as undue profit by barter transactions. When Sector 1 and Sector 2 divide undue profit, guaranteeing the original profit for Sector 2, the share of Sector 1 cannot be larger than its deficit. The reason can be explained as follows. We denote by k the share of Sector 1 and by $(1-k)$ the share of Sector 2 in undue profit gained by tax evasion, or $\mu\,t\,a_{12}Q_2\alpha p_1$. The sum of the profit of Sector 1 after division of undue profit will be

$$\Pi_1 + k\mu\,t\,a_{12}Q_2\alpha p_1 \tag{4.1.2}$$

The sum of the profit of Sector 2 will be

$$(1-\mu)\Pi_2 + (1-k)\mu\,t\,a_{12}Q_2\alpha p_1 \tag{4.1.3}$$

Moreover, since the following inequality is satisfied

$$-\Pi_1 \ge \mu\, t\, a_{12} Q_2 \alpha p_1 \tag{4.1.4}$$

$$0 \le k \le 1 \tag{4.1.5}$$

the following expression is obtained

$$\Pi_1 + k\mu\, t\, a_{12} Q_2 \alpha p_1 \le 0 \tag{4.1.6}$$

　　　Therefore, Sector 1 remains in deficit even after it receives a portion of undue profit though tax evasion. In order to guarantee reproduction of the economy, the following equations should be satisfied with non-minus prices, wages, and profits.

$$
\begin{aligned}
\left(a_{21}p_2 + a_{31}p_3\right) Q_1 + a_{41}Q_1 w + \left(\Pi_1 + k\mu\, t\, a_{12} Q_2 \alpha p_1\right) &= \\
&\left(a_{12}Q_2 + a_{13}Q_3 + Y_1\right)p_1 + k\mu\, t\, a_{12} Q_2 \alpha p_1 \\
\left(a_{12}p_1 + a_{32}p_3\right) Q_2 + a_{42}Q_2 w + (1-\mu)\Pi_2 + (1-k)\mu\, t\, a_{12} Q_2 \alpha p_1 + \mu\left(\Pi_2 - t\, a_{12} Q_2 \alpha p_1\right) &= \\
&\left(a_{21}Q_1 + a_{23}Q_3 + Y_2\right)p_2 - k\mu\, t\, a_{12} Q_2 \alpha p_1 \\
\left(a_{13}p_1 + a_{23}p_2\right) Q_3 + a_{43}Q_3 w + (1-\mu)\Pi_3 + \mu\Pi_3 &= \left(a_{31}Q_1 + a_{32}Q_2 + Y_3\right)p_3
\end{aligned}
$$

$$\tag{4.1.7}$$

　　　However, as mentioned above, Sector 1 cannot meet its obligation and the economy will not be reproducible. Consequently, in bilateral barter transactions a kind of trade-off will arise between a reproducible economy with no undue profit, and undue profit under a non-reproducible economy.

4.2 Trilateral Barter Transactions and Reproducibility of the Economy

　　　Let us analyze trilateral barter transactions in which sectors transform their income into expense. The following analysis assumes that at equilibrium prices Price Ratio 1 is equal to Price Ratio 2 and the sum of barter transactions between sectors is equal. In other words, neither nominal non-equivalent exchange nor real non-equivalent exchange will arise.

　　　It is assumed that Ω is the amount settled in kind of total sales in each sector, or $Q_i p_i \, (i = 1, 2, 3)$. Since the sum of Ω can be treated as expense, sales in the book will amount to $Q_i p_i - \Omega$. The sum settled using currency is denoted by M_i, and the profit in the book will be $Q_i p_i - 2\Omega - M_i - a_{4i} Q_i w$. While undue profit increases as the nominal profit decreases, undue profit does not increase under the negative nominal profit. Therefore, each sector gains the following undue profit, or R_i:

$$
R_i =
\begin{cases}
\mu\left(2\Omega + M_i - a_{1i}Q_i p_1 - a_{2i}Q_i p_2 - a_{3i}Q_i p_3\right) \\
\qquad\qquad \text{when } Q_i p_i - 2\Omega - M_i - a_{4i}Q_i w \ge 0 \\[1em]
\mu\left(Q_i p_i - a_{1i}Q_i p_1 - a_{2i}Q_i p_2 - a_{3i}Q_i p_3 - a_{4i}Q_i w\right) \\
\qquad\qquad \text{when } Q_i p_i - 2\Omega - M_i - a_{4i}Q_i w < 0
\end{cases}
\tag{4.2.1}
$$

In this scheme sectors in surplus can gain undue profit through tax evasion, while sectors in deficit cannot. This is due to the fact that sectors in deficit do not pay tax on profits and do not need to evade tax.

In order to guarantee the reproduction of the economy the following equations should be satisfied with non-minus prices, wages, and profits:

$$\left(a_{21}p_2 + a_{31}p_3\right)Q_1 + a_{41}Q_1 w + \left\{\left(1-\mu\right)\Pi_1 + R_1\right\} + \left(\mu\Pi_1 - R_1\right) =$$
$$\left(a_{12}Q_2 + a_{13}Q_3 + Y_1\right)p_1$$
$$\left(a_{12}p_1 + a_{32}p_3\right)Q_2 + a_{42}Q_2 w + \left\{\left(1-\mu\right)\Pi_2 + R_2\right\} + \left(\mu\Pi_2 - R_2\right) =$$
$$\left(a_{21}Q_1 + a_{23}Q_3 + Y_2\right)p_2$$
$$\left(a_{13}p_1 + a_{23}p_2\right)Q_3 + a_{43}Q_3 w + \left\{\left(1-\mu\right)\Pi_3 + R_3\right\} + \left(\mu\Pi_3 - R_3\right) =$$
$$\left(a_{31}Q_1 + a_{32}Q_2 + Y_3\right)p_3$$

$$(4.2.2)$$

where $\left(1-\mu\right)\Pi_i + R_i$ indicates real profit of Sector i, and $\mu\Pi_i - R_i$ indicates tax revenue. When all sectors are in surplus, they can meet their obligations and the economy is reproducible. When one sector is in deficit, we have to distinguish nominal deficit from real deficit. A nominal deficit sector means the sector falls into deficit through trilateral barter transactions, and therefore is actually in surplus and can meet its obligation, which guarantees the reproduction of the economy. A real deficit sector means the sector is in deficit even if we do not take into account a profit decrease effect through trilateral barter transactions. Therefore, the real deficit sector cannot meet its obligation, which makes it impossible for the economy to be reproduced.

4.3 Trilateral Barter Transactions with the Elements of Bilateral Barter Transactions

Finally, trilateral barter transactions with the elements of bilateral barter transactions (barter transactions under intentionally raised prices) is analyzed. If all sectors are in surplus, each sector can gain undue profit from tax evasion through transformation of income into expense, whereas each sector cannot gain undue profit through intentionally raised prices. Therefore, in order to gain more undue profit from bilateral and trilateral barter transactions than from trilateral barter transactions alone, there should be a sector in deficit that can receive the transfer of profit. In the same way as in the former section, this paper denotes by k the share of Sector 1 and by $\left(1-k\right)$ the share of Sector 2 in undue profit gained from bilateral barter transactions. The sum of the real profit of Sector 1 and Sector 2 will amount to $\Pi_1 + R_1 + k\mu t\, a_{12}Q_2\alpha p_1$ and $\left(1-\mu\right)\Pi_2 + R_2 + \left(1-k\right)\mu t\, a_{12}Q_2\alpha p_1$, respectively. In order to confirm reproducibility of the economy, it is required to distinguish the case when Sector 1 is nominally in deficit from the case when Sector 1 is actually in deficit. If Sector 1 is nominally in deficit, it can meet its

obligation and the reproduction of the economy will be guaranteed. Therefore, the following equations will be satisfied with non-minus prices, wages, and profits:

$$\left(a_{21}p_2 + a_{31}p_3\right)Q_1 + a_{41}Q_1 w + \left\{(1-\mu)\Pi_1 + R_1 + k\mu t\, a_{12}Q_2\alpha p_1\right\} + \left(\mu\Pi_1 - R_1\right) =$$
$$\left(a_{12}Q_2 + a_{13}Q_3 + Y_1\right)p_1 + k\mu t\, a_{12}Q_2\alpha p_1$$
$$\left(a_{12}p_1 + a_{32}p_3\right)Q_2 + a_{42}Q_2 w + \left\{(1-\mu)\Pi_2 + R_2 + (1-k)\mu t\, a_{12}Q_2\alpha p_1\right\} +$$
$$\left\{\mu\left(\Pi_2 - t\, a_{12}Q_2\alpha p_1\right) - R_2\right\} = \left(a_{21}Q_1 + a_{23}Q_3 + Y_2\right)p_2 - k\mu t\, a_{12}Q_2\alpha p_1$$
$$\left(a_{13}p_1 + a_{23}p_2\right)Q_3 + a_{43}Q_3 w + \left\{(1-\mu)\Pi_3 + R_3\right\} + \left(\mu\Pi_3 - R_3\right) = \left(a_{31}Q_1 + a_{32}Q_2 + Y_3\right)p_3$$

$$(4.3.1)$$

This scheme shows the difference from the scheme of bilateral barter transactions. In the bilateral barter scheme, the sector in deficit is required to receive transferred profit. However, this sector in deficit cannot turn a surplus even after it receives a part of undue profit, which does not guarantee the reproducible economy. On the other hand, in the scheme of trilateral barter transactions with the elements of bilateral barter transactions, the sector in deficit is also required to receive transferred profit. The sector nominally in the red and actually in surplus enables concerned sectors to gain additional undue profit and the economy to be reproduced.

Furthermore, the situation that Sector 1 is actually in deficit is analyzed. In the earlier sections it was indicated that the real deficit sector could not meet its obligation and the reproduction of the economy would not be guaranteed. In this case, the deficit of Sector 1 is increased by trilateral barter transactions, which means the sum increases to the extent to which Sector 2 can transfer its profit to Sector 1. If we denote by Π_1 the real loss of Sector 1, then the sum possible to transfer profit will be

$$-\Pi_1 \geq \mu t\, a_{12}Q_2\alpha p_1 \qquad (4.3.2)$$

On the other hand, in trilateral barter transactions with the elements of bilateral barter transactions, we denote by Π_1'' the intentionally increased loss of Sector 1. The sum possible to transfer profit will be

$$-\Pi_1'' \geq \mu t\, a_{12}Q_2\alpha p_1 \qquad (4.3.3)$$

If undue profit from bilateral barter transactions, namely $k\mu t\, a_{12}Q_2\alpha p_1$ satisfies the inequality

$$k\mu t\, a_{12}Q_2\alpha p_1 \geq -\Pi_1, \qquad (4.3.4)$$

then the share of undue profit for Sector 1 exceeds the actual deficit of Sector 1 and it turns surplus. As a result, even if a sector in deficit exists in the economy, the economy becomes reproducible. This conclusion clearly differs from that of the scheme of bilateral barter transactions.

5. Conclusion

This paper analyzed the relationship between barter transactions and the reproducible economy.

Barter Transactions and Reproduction of the Economy

Let us summarize reproducibility of the economy with barter transactions caused by liquidity constraint and tax evasion (See Table 1).

In an economy in which transactions are settled both using currency and in kind because of liquidity constraint, the sum of supplied commodities should be equal to the sum of demanded commodities in each sector (including bartered commodities) under a price ratio for the currency settlement. If only the bartered commodities are selected and the price ratio is calculated that equates the sum of supplied commodities to the sum of demanded commodities in each sector, then a given sum will be donated from one sector to other sectors. When the sum of donation from Sector 1 to Sector 2 is equal to the sum of donation from Sector 2 to Sector 3 and from Sector 3 to Sector 1, the economy will be reproducible. We refer to this donation among sectors as a real non-equivalent exchange. Furthermore, in some cases this price ratio is different from the price ratio employed in the currency settlement. Therefore, it appears that commodities are non-equivalently bartered if we pay attention only to the price ratio for currency settlement. This scheme was referred to as a nominal non-equivalent exchange.

Furthermore, it is supposed that the price ratio rendering the economy reproducible is not satisfied, and one sector falls into deficit. If another sector accepts the commodities of the sector in deficit as a payment at an intentionally higher price, and the sector in deficit turns surplus, then the economy will be reproducible.

Barter transactions caused by tax evasion can be summarized as follows (See Table 1). When in bilateral barter transactions both participants are in surplus, they cannot gain undue profit from tax evasion through the transfer of profit from one participant to another. This is because profit tax will be levied on sectors in surplus. The reproduction of the economy, however, will be guaranteed. When one participant in bilateral barter transactions is in deficit, participants can evade tax with an intentionally raised price, transferring profit to the deficit sector. The deficit sector remains in deficit ever after it receives undue profit through tax evasion, however, and cannot meet its obligation, which results in a non-reproducible economy. Consequently, in bilateral barter transactions a kind of trade-off will arise between a reproducible economy with no undue profit, and undue profit under a non-reproducible economy.

In trilateral barter transactions, participants can reduce their profit by treating income as expense and evade tax even when all participants in barter transactions are in surplus. If one sector is in deficit, we have to distinguish nominal deficit from real deficit. A nominal deficit sector means the sector falls into deficit through trilateral barter transactions. Therefore, it is actually in surplus and can meet its obligation, which enables the economy to be reproduced. A real deficit sector means the sector is in deficit even if we do not take into account a profit decrease effect caused by trilateral barter transactions. Therefore, the real

Table 1. Barter transactions caused by tax evasion

	Model	Tax evasion	Reproducibility
Bilateral	One firm is in surplus. Another firm is in surplus after barter transactions.	Impossible	Possible
	One firm is in surplus. Another firm is in deficit after barter transactions.	Possible	Impossible
Multilateral	All firms are in surplus.	Possible	Possible
	One firm is in surplus but in deficit after barter transactions.	Possible	Possible
	One firm is in deficit.	Possible	Impossible
Multilateral plus Bilateral	All firms are in surplus.	Possible, but no additional undue profit from intentionally raised prices	Possible
	One firm is in surplus but in deficit after barter transactions.	Possible	Possible
	One firm is in deficit.	Possible	Possible

deficit sector cannot meet its obligation, and the reproduction of the economy will not be guaranteed.

Finally, trilateral barter transactions with the elements of bilateral barter transactions (barter transactions under intentionally raised prices) are summarized. If all sectors are in surplus, each sector can evade tax by transforming income into expense. When all sectors remain in surplus after transforming income into expense, each sector cannot gain undue profit through intentionally raised prices. Therefore in order to gain more undue profit through bilateral and trilateral barter transactions than through trilateral barter transactions alone, there should be a sector in deficit that can receive the transfer of profit. In order to confirm the reproducibility of the economy, it is required to distinguish nominal deficit from real deficit. If Sector 1 is nominally in deficit, it can meet its obligation and the reproduction of the economy will be guaranteed. When Sector 1 is really in deficit, the sum of deficit in the sector is increased by trilateral barter transactions, which means the limit is expanded to the extent to which sectors in surplus can transfer their profit to the sector in deficit. If the amount of undue profit for the deficit sector exceeds the actual deficit of the sector, it turns surplus. As a result, even if there is a sector in deficit in the economy, the economy becomes reproducible.

Needless to say, the conclusion of this paper should not be directly applied to the Russian economy. A new point of view is introduced here in the analysis of the economy, however – the relationship between barter transactions and the reproducible economy, which, I believe, may offer some suggestion for understanding the economic system.

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Barter Transactions and Reproduction of the Economy:
A Theoretical Approach on the Basis of Russian Cases

Shigeki ONO

Abstract

This paper seeks to theoretically analyze the relationship between barter transactions and reproduction of the economy, taking into account causes of barter operations in Russia: liquidity constraint and tax evasion. This paper suggests that when firms are suffering liquidity constraint, price ratios both for currency settlement and barter exchanges are required and commodities are non-equivalently bartered. Furthermore, when firms barter in order to evade taxes, and bilateral and multilateral barter transactions are combined, in some cases firms in deficit become firms in surplus, making the economy reproducible.