A METHOD OF PHASE DIFFERENCE DETERMINATION
IN ELECTROENCEPHALOGRAM

Ryosaku Kusachi, Muneo Shonai, Shigeo Mishima
and Katashi Owada

Department of Physiology
Faculty of Veterinary Medicine,
Hokkaido University, Sapporo, Japan.
(Received for publication, April 15, 1959)

In essence, information obtained from electroencephalogram (EEG) may be summarized as frequency components and these phases. Generally, EEG is divided into frequency components and the division is significant. However, due to the nature of EEG, it is meaningless to determine the phases of these components.

The problem in question in the present paper is the relation of phases in two simultaneous registrations of EEG. In order to deal with this problem as precisely as possible, the best method is to determine the coefficient of cross correlation. The coefficient can also be determined by two frequency spectrums. However, under the present circumstances in Japan, in most cases it is impossible to use a computer suitable for the case. Thus, a large amount of time and effort must be used to determine the required results.

The present method as reported here while not being exact, is a simple method for calculating the phase difference in proximity.

METHOD OF ANALYSIS

Two EEGs taken at random sites are amplified simultaneously and lead to both axis of the deflecting plate of a cathode-ray tube. The spot migrates with a two dimensional motion. When the motion of this spot is exposed for a given time on film, generally an oval shaped elliptic figure results, due to superposition of Lissajous' figure which is also due to the fact that the frequency spectrum of EEG has a certain width. This is analysed and the phase difference is determined. We will now consider a simple example. When phenomenon \( f(t) = a \sin \omega t \) is lead to horizontal axis, and phenomenon \( g(t) = b \sin (\omega t - \varphi) \) which has a phase difference \( \varphi \) against \( f(t) \), is lead to vertical axis, the phase difference \( \varphi \) can be calculated readily by the following formula.

\[
\sin \varphi = \frac{4}{\pi} \frac{S_E}{S_0}
\]

\( S_E \) represents the dimension of the oval. \( S_0 \) is the dimension of the rectangular enclosing the oval with one side parallel to the combined axis. Since \( \varphi \) is the quantity from 0

through to \( \pi \), 2 values satisfying the above formula are obtained. The differentiation depends on the direction on which the long axis of the oval is pointed and as shown in fig. 1, it is determined whether it is under \( \frac{\pi}{2} \) or over \( \frac{\pi}{2} \).

**FIG. 1. Decision of Quadrant which Phase Difference Belongs**

Now, while the actual EEG, as described later, is not a simple harmonic nature, the outer fringes of the combined elliptic figure may be considered as an approximation of Lissajous' figure combined from two simple harmonics, and the phase difference is determined from the above formula.

**EXPERIMENTAL RESULTS AND DISCUSSION**

The following was conducted on a healthy adult man. Indifferent electrode was attached to the left ear lobe and different electrode was attached to the proc. occipitalis. The resulting EEG was lead to vertical axis. In addition, the median line between proc. occipitalis and glabella was divided into 8 sections and different electrodes were attached at each point. The resulting EEG lead from the 9 points were connected with horizontal axis. The elliptic pattern as seen in fig. 2, is the resulting motion of the spot on a cathode-ray tube as exposed to film for 10 seconds. The figures in fig. 2, indicate the pattern which appeared when the different electrode was attached at each of the 9 points on the aforementioned median line beginning from proc. occipitalis, and the resulting EEG was lead to the horizontal axis.

Inspection of these patterns shows the phase difference relationship to a certain extent. When calculations were made using the above method and by adjusting the size of the spot, the results were as in fig. 3.

It is noted that when the distribution of phase difference as determined from the report of MOTOKAWA and TSUZIGUCHI, was compared with the above results, a considerable similarity was present. As to the difference between the two, in the authors' method even
when the different electrode was attached to the vertex, the value of the phase difference was smaller than $\pi$. Next, considerations are made to determine how the elliptic pattern, as obtained by the authors' method, is made. Now, if the EEG taken at 2 points simultaneously are expressed respectively as $f(t)$, $g(t)$, the frequency components may be calculated as follows.

$$A_f(\nu, t_0) = \frac{1}{4T} \int_{t_0 - \Delta T}^{t_0 + \Delta T} f(t) e^{-i\nu t} dt$$

$$A_g(\nu, t_0) = \frac{1}{4T} \int_{t_0 - \Delta T}^{t_0 + \Delta T} g(t) e^{-i\nu t} dt$$

$\nu$ in the present equation stands for frequency, $4T$ and $t_0$ stand for time of analysis and for time of the origin respectively, and $i$ stands for a imaginary unit. $A_f(\nu, t)$ and $A_g(\nu, t)$
respectively indicate the EEG frequency components and since they depend on the location of analysis, they are the function of time of the origin and of analysis. According to the authors' experiments, MT is set at 10 seconds.

Thus, the components of a given $\nu_0, t_0$ are determined and $A_r(\nu_0, t_0)$ and $A_\theta(\nu_0, t_0)$ are obtained respectively. The above appears as a Lissajous' figure on the cathode-ray tube with a sine wave with respective amplitudes of $|A_r(\nu_0, t_0)|$ and $|A_\theta(\nu_0, t_0)|$.

Now, since, generally speaking, EEG has a large number of frequency components, various Lissajous' figures are formed according to varying values of $\nu$. Hence, elliptic patterns as seen in the present experiment are obtained by superposition of the Lissajous' figures.

When $\nu_a$, a component which is equivalent to the $a$ wave, is predominant in both EEGs, the outer fringe of the elliptic pattern, which is actually produced by the Lissajous' figure, is formed by the phase difference in the sine waves which have an amplitude of $|A_r(\nu_a, t_0)|$ and $|A_\theta(\nu_a, t_0)|$. Thus, other components enter into the picture as modifiers and become the factors for the error of the present method. It is suspected that this is the reason why the EEG phase difference between the vertex and occiput has a smaller value than $\pi$. Also discussing the problem from the results of Motokawa and Tsuziguchi, it may be said that the wider the width of the peak which is formed by the phase difference distribution, the larger the discrepancies. Therefore, according to the authors' method, when both EEGs have the same remarkable frequency component in common, the rate of proximity is extremely high.
Phase Difference Determination in Electroencephalogram

Summary

Two simultaneously registered EEGs are lead to both axis of the deflecting plate of a cathode-ray tube and the combined oval pattern was analysed. The present paper deals with an approximate method of determining the resulting phase difference. In addition, the degree of proximity in such a method was discussed.

References