An Optimal Configuration Design of Superconducting Magnets for DC Reactor Taking Loss into Account

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Abstract—**Large-inductance DC reactors are often required in practical factories, and superconducting magnets represent an interesting alternative for the construction of such devices. It is therefore useful to obtain optimized designs for DC reactors, particularly for the minimization of the winding volume under many constraints. The superconducting magnets of DC reactors can be operated at higher temperature than that of SMES, since the rated current in the case of DC reactors is much smaller. To operate at high temperature (77 K) makes the coolant cost low. However, the flux flow loss becomes large at 77 K, and then it has to be taken into account at the design stage. In this paper, the optimized configurations of toroidal superconducting magnets for DC reactor are presented and the configurations of HTS conductor are investigated under taking the flux flow loss into account.**

***Index Terms*—DC reactor, optimal design, superconducting magnet.**

I. INTRODUCTION

LARGE-SCALE DC reactors (DCL) with large inductances are commonly required in practical factories. Employing superconducting magnets for inductance has advantages, such as large current, low loss, coreless (lightness), and so on. From the point of view of flux leakage, the toroidal magnet is an ideal structure. During the optimization of the toroidal superconducting magnet, the goal is to minimize the winding volume under many constraints such as inductance, loss, characteristics of the superconductor (e.g. B–J–θ characteristic of HTS tapes), thermal stress, Lorentz force, stabilization and protection against the quench and so on.

An optimal configuration design methods for SMES with HTS tapes have been reported [1]–[3]. The toroidal superconducting magnets for SMES were designed employing simulated annealing as the optimization method and the finite element method for numerical field computation [3]. Due to quadratic dependence of the stored energy with respect to the current ($E=LI^2/2$), it is more effective to increase the operating current than the inductance to get to a given energy. The reason is that the stored energy is proportional to the square of the operating current. In the case of the DCL design, a different approach has to be used [4]. Since a large inductance is required, the inductance has to be increased as well.

We have investigated the configuration of the toroidal superconducting magnets for the DCL [4]. Previously, the configuration of the element coils of the toroidal superconducting magnet for the DCL was optimized to minimize the winding for a 1 H inductance and a 20 kA operating current at 20 K. The toroidal magnet consisted of 8 or 12 element coils wound with YBCO tapes. However, the current load factor (operating current I_{op} / critical current I_c) was extremely low, the winding volume might be wasted.

In this paper, the operating temperature increases from 20 K to 77 K (liquid nitrogen temperature) to reduce the cooling cost. The optimization of the element coils for the DCL is described to minimize the winding volume. The toroidal magnet consists of 8, 12, 24 or 48 element coils wound with YBCO tapes. Moreover, the HTS conductor consisting of a few sheets of YBCO tape is also investigated.

II. OPTIMAL DESIGN METHOD

A. Inductance Computation

In the DCL case, the toroidal superconducting magnet consists of 8, 12, 24 or 48 element coils and is connected in series. The inductance L of the toroidal magnet is computed from

$$L = 2\frac{E}{I_{op}^2}, \tag{1}$$

where I_{op} is the operating current and E is the energy computed from

$$E = \frac{1}{2}\int_V \boldsymbol{A}\cdot\boldsymbol{J}\,\mathrm{d}V, \tag{2}$$

where $\boldsymbol{A}$, $\boldsymbol{J}$ and V are the magnetic vector potential, the current density and the volume of superconducting magnet, respectively. Here, the magnetic vector potential $\boldsymbol{A}$ is given by the superposition of the element coils and computed from

$$\boldsymbol{A} = \frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}(q)}{l(q)}\mathrm{d}V, \tag{3}$$

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where $l(q)$ is the distance between the point at which $\boldsymbol{A}$ is evaluated and the point q of the current density source.

The stored energy E and the vector potential $\boldsymbol{A}$ are obtained by numerical integration of (2) and (3), respectively.

The flux flow loss W_f is computed from

$$W_f = \int_v \boldsymbol{E}_p \cdot \boldsymbol{J} \mathrm{d}v \tag{4}$$

where v is the total volume of the YBCO, and $\boldsymbol{E}_p$ is the electric field given based on the percolation model [5], where the B–J–θ characteristics is taken into account. The magnetic flux density B and the angle between the tape surface and the magnetic flux density vector, θ, are computed based on the Biot-Savart law.

B. Optimization Algorithm

The simulated annealing [6] is employed as the optimization method to minimize the winding volume of the superconducting magnet. Moreover, the Augmented Lagrange multiplier method is combined with the simulated annealing in order to enable the solution of the constrained problem [7], [8]. Fig. 1 shows the flowchart of the optimization algorithm combining the simulated algorithm and the Augmented Lagrange multiplier method.

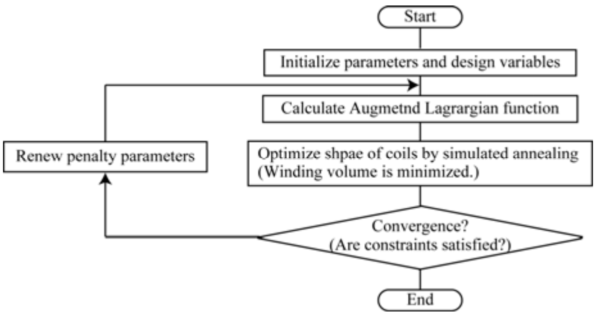

Fig. 1. Flowchart of the optimization algorithm combining the simulated annealing and the Augmented Lagrange multiplier method.

III. Optimal Configuration Design

The configuration of the toroidal superconducting magnet for the DCL is optimized to minimize the winding volume. As the mentioned earlier, the toroidal superconducting magnets to be optimized 8, 12, 24 and 48 element coils. The specifications of magnet are set as follows:

- The inductance is 1 H (within 1.0 %).
- The conductor consists of 4 sheets of YBCO tape with a thickness of 0.2 mm and a width of 10 mm, and reinforcement with thickness of 0.8 mm and a width of 10 mm.
- The operating current I_{op} in one element coil is under 2 kA.
- The inner radius, radius of toroid, number of layers and turns of each layer of element coils are design variables, as shown in Fig. 2. Fig. 3 shows the schematic view of the number of layers and turns of each layer of element coils.
- The operating temperature is 77 K (liquid nitrogen temperature).
- The flux flow loss is under 333.3 W.
- The goal of optimal design is to minimize the winding volume.

The flux flow loss is computed from (4), where the B–J–θ characteristic of YBCO tape is used based on the result of measurements presented in [2] and [5]. It was assumed that the loss of 333.3 W at 77 K corresponds to 5 kW at room temperature due to energy for cooling. In the case of the same class DCL using a copper conductor, a joule loss of 20 kW would be permissibly generated. In this paper, one forth amount of loss is targeted as compared with the copper conductor.

In the optimal design, the design variables are the distance between the center of toroid and the center of an element coil, the inner radius of each element coil and the thickness and length of each element. The thickness and the length of each coil are computed based on the number of layers and turns of each layer of coils, which are discrete variables. The objective function to be minimized in the optimal design is the winding volume in total.

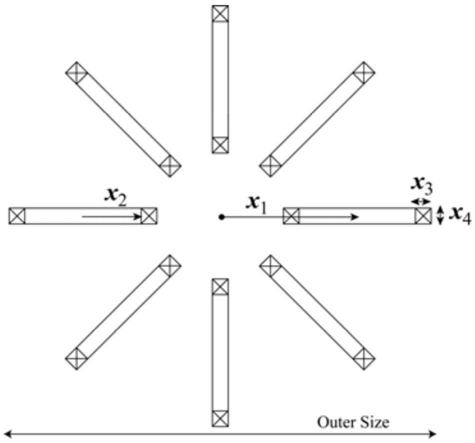

Fig. 2. Design model of toroidal magnets and 4 design variables for optimal design, x_1, x_2, x_3, and x_4 are the distance between the center of toroid and the center of an element coil, the inner radius of each element coil and the thickness and length of each element coil, respectively.

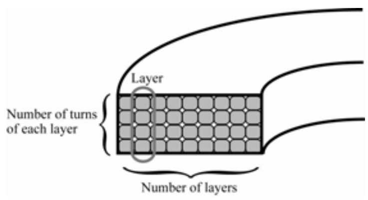

Fig. 3. Schematic view of the layer and the turn of each layer of the element coil. The numbers of layers and turns of each layer are the design variables.

The specifications of the optimized toroidal superconducting magnets of the 1 H DCL are shown in Table I. The optimized configurations of the toroidal superconducting magnet with an inductance of 1 H are shown in Fig. 4. All constrains in the results are, of course, satisfied. The computed flux flow loss is nearly 0.0 since the electric field is too low. As seen in Table I, as the number of element coils increases, the winding volume and the outer size also increase. Though it tends increasing of the number of element coils in SMES magnet [1], [2], a few element coils are enough in DCL.

TABLE I
SPECIFICATIONS OF TOROIDAL SUPERCONDUCTING MAGNETS FOR DCL AT 77 K

Case	8-DCL	12-DCL	24-DCL	48-DCL
No. of coils	8	12	24	48
Inductance L (H)	1.0	1.0	1.0	1.0
Radius of toroid x_1 (mm)	597	621	713	1,499
Inner radius of coil x_2 (mm)	357	372	270	1,326
Thickness of coil x_3 (mm)	164	168	396	80
Length of coil x_4 (mm)	60	40	10	10
Outer size (mm)	2,236	2,322	2,758	5,810
Operating current I_{op} (kA)	2.0	2.0	2.0	2.0
Critical current I_c (kA)	5.4	6.6	9.8	6.7
No. of layers	41	42	99	20
No. of turns of each layer	6	4	1	1
No. of turns of YBCO tape	984 (41x6x4)	672 (42x4x4)	396 (99x1x4)	80 (20x1x4)
Total length of YBCO tape (km)	21.7	23.1	28.0	33.0
I_{op}/I_c	0.739	0.604	0.409	0.597
Flux Flow Loss (W)	0.0	0.0	0.0	0.0
Winding volume (m^3)	0.217	0.231	0.280	0.330

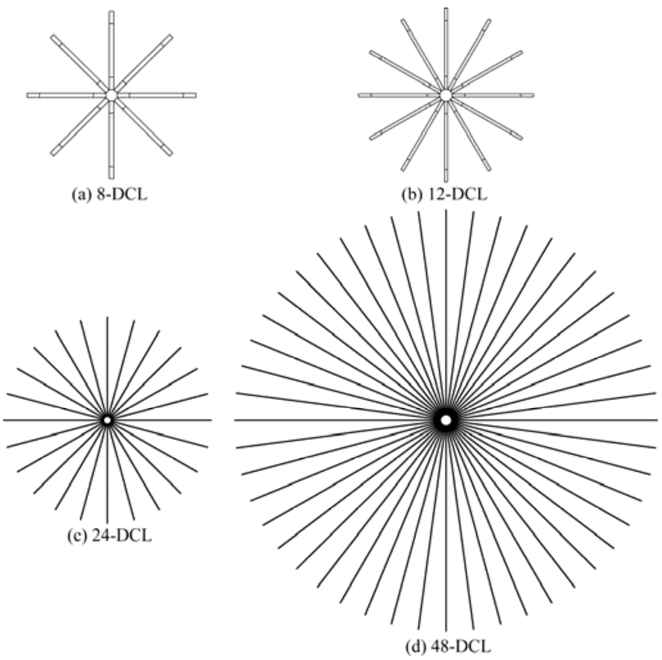

Fig. 4. Optimization configurations of the toroidal superconducting magnet for DCL. The operating temperature is 77 K. The figures show the actual proportion of coils.

Table II shows the outer size, the current load factor and the winding volumes of the DCL, which is minimized at 20 K, presented in the previous paper [4]. The design specifications are the same, except for the operating temperature. Fig. 5 shows the minimized winding volumes with the different operating temperatures. As seen in Fig. 5, the winding volumes obtained in this optimal design are almost the same as those in [4], although the outer size of the DCL magnets at 20 K is a little smaller. That is, the lower temperature is not so effective for DCL magnet with 1 H inductance, since the current load factor (I_{op}/I_c) at 20 K is extremely low [4].

TABLE II
WINDING VOLUME OF TOROIDAL SUPERCONDUCTING MAGNETS FOR DCL AT 20 K

Case	8-DCL	12-DCL
Outer size (mm)	1,720	1,558
I_{op}/I_c	0.087	0.078
Winding volume (m^3)	0.208	0.221

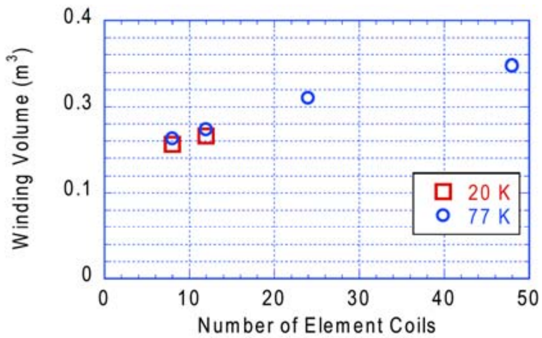

Fig. 5. The relation between the number of element coils and the winding volume at the operating temperatures 20 K and 77 K.

IV. REDUCTION OF WINDING VOLUME

On the results mentioned above, there is still a room to improve in the current load factor, as shown in Table I. Hence, the composition of the conductor is changed to reduce the winding volume of DCL magnet more. Two kinds of the conductor are investigated. One consists of 4 sheets of YBCO tape with a width of 5 mm (Conductor B), another 2 sheets of YBCO tape with a width of 10 mm (Conductor C), as shown in Fig. 6. The cross-section area of the conductors is a half of the previous conductor. The other design specifications are the same as the previous (Conductor A). The specifications of the optimized toroidal superconducting magnets of the 1 H DCL using Conductors B and C are shown in Table II and III, respectively. The winding volume of the DCL magnets using the Conductors A, B and C is plotted in Fig. 7. As seen in Fig. 7, the reduction of the cross-section area of the conductor effectively enables to reduce the winding volume. However, from the viewpoint of the winding volume, the results of the Conductors B and C are almost the same.

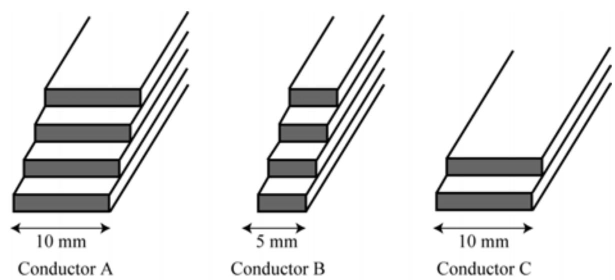

Fig. 6. The conductors used in optimal design of DCL magnet. The Conductor A consists of 4 sheets of YBCO with a width of 10 mm, the Conductor B 4 sheets of YBCO with a width of 5 mm, and Conductor C 2 sheets of YBCO with a width of 10 mm.

TABLE III
SPECIFICATIONS OF TOROIDAL SUPERCONDUCTING MAGNETS FOR DCL USING CONDUCTOR B AT 77 K

Case	8-DCL	12-DCL	24-DCL	48-DCL
No. of coils	8	12	24	48
Inductance L (H)	1.0	1.0	1.0	1.0
Radius of toroid x_1 (mm)	883	963	702	1,420
Inner radius of coil x_2 (mm)	585	721	266	1,275
Thickness of coil x_3 (mm)	240	216	396	80
Length of coil x_4 (mm)	15	10	5	5
Outer size (mm)	3,416	3,800	2,728	6,990
Operating current I_{op} (kA)	2.0	2.0	2.0	2.0
Critical current I_c (kA)	2.9	2.9	3.7	2.4
No. of layers	60	54	99	20
No. of turns of each layer	3	2	1	1
No. of turns of YBCO tape	720 (60x3x4)	432 (54x2x4)	396 (99x1x4)	80 (20x1x4)
Total length of YBCO tape (km)	63.8	67.5	69.3	79.3
Flux Flow Loss (W)	322.6	330.5	329.6	311.2
Winding volume (m^3)	0.128	0.135	0.139	0.159

TABLE IV
SPECIFICATIONS OF TOROIDAL SUPERCONDUCTING MAGNETS FOR DCL USING CONDUCTOR C AT 77 K

Case	8-DCL	12-DCL	24-DCL	48-DCL
No. of coils	8	12	24	48
Inductance L (H)	1.0	1.0	1.0	1.0
Radius of toroid x_1 (mm)	1,020	784	797	1,433
Inner radius of coil x_2 (mm)	661	451	371	1,300
Thickness of coil x_3 (mm)	176	292	198	40
Length of coil x_4 (mm)	20	10	10	10
Outer size (mm)	3,714	1,527	2,732	5,546
Operating current I_{op} (kA)	2.0	2.0	2.0	2.0
Critical current I_c (kA)	2.6	3.3	3.4	2.4
No. of layers	88	146	99	20
No. of turns of each layer	2	1	1	1
No. of turns of YBCO tape	352 (88x2x2)	292 (146x1x2)	198 (99x1x2)	40 (20x1x2)
Total length of YBCO tape (km)	66.3	65.7	70.2	79.6
Flux Flow Loss (W)	331.4	324.9	330.9	327.7
Winding volume (m^3)	0.133	0.131	0.140	0.159

The computed flux flow loss in all cases is less than 333.3 W. However, it increases as compared with that of the Conductor A, since the electric field and the current density increase due to the reduction of the cross-section area of the conductor. In the case of the DCL magnets, the operating current is low enough that it is suppressed to a little amount of loss. As the result, the winding volume is also reduced. On the other hand, in the case of SMES magnet [1], [4], the large current causes the increase of the winding volume in order not to generate the loss. However, in order to store a large energy, the large current is required.

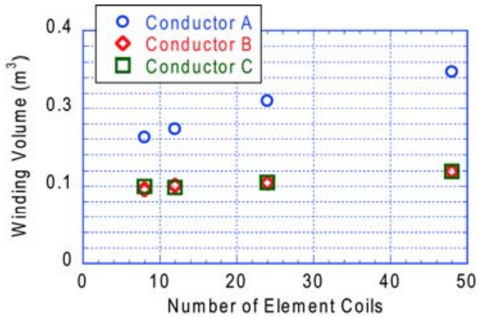

Fig. 7. The winding volume of the optimized DCL magnet using the Conductors A, B and C.

V. CONCLUSION

We have optimized the 8, 12, 24 and 48 toroidal superconducting magnets for 1 H DC reactor using the three kinds of conductors. When the cross-section area of the conductor is large, the current load factor is small enough. Therefore, it is possible to reduce the winding volume. On the other hand, when the cross-section area of the conductor is small, the winding volume is so small, but a little amount of loss is generated. However, it is small enough to be removed.

In the SMES magnet design [2], the large number of element coils was treated to store a high energy. However, in the DCL magnet case, the large number of element coils causes the increase of the winding volume. As the result, the design concept is different between the DCL magnet and the SMES magnet.

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