A Generalized Approach to Block Diagonalization for Multiuser MIMO Downlink

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***Abstract*—In multiple input multiple output (MIMO) systems, multiuser enhancement is in the implementation phase. Block diagonalization (BD) is known as the safest technique for the multiuser MIMO downlink since it suppresses the inter-user interference (IUI) perfectly. However, BD consumes most of the transmit antenna resource to form many nulls. Thus, it is very difficult to obtain the transmit diversity gain. In the paper, the adaptive selection of the number of layers is considered. Such approaches have been limited to iterative beamforming. Here, we formulate deterministic transmit beamforming as a generalized approach to the BD (GBD), based on a concept of projecting the IUI to a limited vector space. Our proposal can efficiently utilize the extra degrees of freedom, which are obtained in exchange for reducing the number of layers, to provide the diversity gain in the transmit beamforming. The high capability of the GBD is confirmed by computer simulations.**

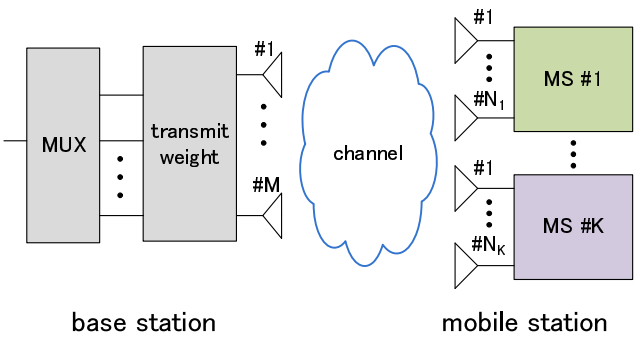


Fig. 1. A system model of the multiuser MIMO downlink

I. Introduction

Multiple input multiple output (MIMO) systems, which started from a single-user MIMO system perspective, are migrating to more complex applications. Among them, multiuser MIMO has been promised for use in 3.9G and newer mobile radio as a technology to improve the system capacity as well as the user capacity.

In multiuser MIMO systems, multiple layers (or substreams) of multiple users are accommodated in a single physical channel using spatial multiplexing and spatial division multiple access (SDMA) [1]. The downlink (or broadcast channel) of SDMA is constructed by a sort of spatial segmentation with transmit beamforming in general. Therefore, if nulls of a certain user's beam pattern are not directed to the receive antennas of any other users, inter-user interference (IUI) inevitably occurs. Obviously, the number of nulls increases not only with users but also with received antennas per user. Thus, efficient beamforming is needed for multiuser MIMO systems.

Block diagonalization (BD) is known as a technique to achieve perfect nulling [2]–[4]. This method completely suppresses IUI by directing a null to each antenna element of other users. Although the number of transmit antennas must be not less than the sum of receive antennas for all users, the simple transformation to a pure single-user MIMO channel from a multiuser MIMO channel with IUI is very attractive. Then any MIMO scheme such as spatial multiplexing or eigenmode MIMO [5]–[7] are individually applicable to each single-user MIMO channel.

However, BD consumes most of the transmit antenna resources for perfect nulling so that it is difficult to obtain transmit diversity gain from the beamforming for user segmentation. Therefore, to achieve higher diversity gain, extra antenna resources are needed as in [8].

In this paper, we utilize the partial nulling concept to ease the perfect nulling condition. The partial nulling has been previously proposed by the authors to exploit the receiver's capability of spatial filtering for IUI cancellation and thus to enable the increase in degrees of freedom at the transmitter [9]. The idea minimizing signal-to-leakage-plus-noise ratio (SLNR) also eases the perfect nulling condition [10], [11]. However, the SLNR-based beamforming considers the IUI power only. Thus, the degrees of freedom required to cancel the IUI at the receiver side is not controllable in spite of low IUI power. Our base concept is rather similar to coordinated beamforming [12]. However, our proposal can determine the transmit weight matrix uniquely without any iterative procedures.

Formerly, partial nulling was called imperfect BD and limited to single stream transmission. Here, we redefine the partial nulling concept in a generalized approach to BD and show the BD as its subset. The rest of the paper is organized as follows. In Section II, the generalized BD is described. The performance evaluation is shown in Section III. Finally, conclusions are drawn in Section IV.

II. Transmit beamforming for Multiuser MIMO

A. System Model

Here, let us consider a broadcast channel of the multiuser MIMO system where K users are connecting to a base station as shown in Fig. 1. The base station has M transmit antennas and each user has N_k ($k = 1,..., K$) receive antennas. The

following discussion is valid for any arbitrary number of receive (and transmit) antennas. For the sake of simplicity, however, we assume that the number of receive antennas is the same for all users. Thus, we redefine $N_1 = \cdots = N_K = N$.

Our aim is to obtain appropriate transmit beamforming (or precoding) for the base station to accommodate K users using SDMA. Let the number of layers (or data substreams) for the kth user be L_k ($\leq N$). Then, the transmit signal vector $\boldsymbol{x}_k \in \mathbb{C}^{L_k \times 1}$ and the transmit weight matrix $\boldsymbol{T}_k = [\boldsymbol{t}_{k,1},..., \boldsymbol{t}_{k,L_k}] \in \mathbb{C}^{M \times L_k}$ can be defined for the kth user. Here, any pair of column vectors of $\boldsymbol{T}_k$ are orthogonal. Using $\boldsymbol{x}_k$ and $\boldsymbol{T}_k$, we obtain the received signal vector ($\in \mathbb{C}^{N \times 1}$) for the kth user as

$$\boldsymbol{y}_k = \sum_{j=1}^{K} \boldsymbol{H}_k \boldsymbol{T}_j \boldsymbol{x}_j + \boldsymbol{z}_k, \tag{1}$$

where $\boldsymbol{H}_k \in \mathbb{C}^{N \times M}$ is the channel matrix and $\boldsymbol{z}_k \in \mathbb{C}^{N \times 1}$ is the noise vector composed of complex Gaussian noise.

The stacked vector of $\boldsymbol{y}_1,..., \boldsymbol{y}_K$ can be expressed by

$$\begin{bmatrix} \boldsymbol{y}_1 \\ \vdots \\ \boldsymbol{y}_K \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_1 \\ \vdots \\ \boldsymbol{H}_K \end{bmatrix} \begin{bmatrix} \boldsymbol{T}_1 & \cdots & \boldsymbol{T}_K \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_K \end{bmatrix} + \begin{bmatrix} \boldsymbol{z}_1 \\ \vdots \\ \boldsymbol{z}_K \end{bmatrix} \tag{2}$$

$$= \tilde{\boldsymbol{H}} \begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_K \end{bmatrix} + \begin{bmatrix} \boldsymbol{z}_1 \\ \vdots \\ \boldsymbol{z}_K \end{bmatrix}, \tag{3}$$

where $\tilde{\boldsymbol{H}}$ is the $NK \times \sum_{k=1}^{K} L_k$ matrix for the effective channel and is represented by

$$\tilde{\boldsymbol{H}} = \begin{bmatrix} \boldsymbol{H}_1 \boldsymbol{T}_1 & \cdots & \boldsymbol{H}_1 \boldsymbol{T}_K \\ \vdots & \ddots & \vdots \\ \boldsymbol{H}_K \boldsymbol{T}_1 & \cdots & \boldsymbol{H}_K \boldsymbol{T}_K \end{bmatrix}. \tag{4}$$

Clearly, each diagonal block in the effective channel matrix expresses an individual MIMO channel per user, and non-diagonal blocks correspond to IUI components.

B. Block Diagonalization

BD is a strategy to suppress the IUI perfectly. This condition is written as

$$\boldsymbol{H}_k \boldsymbol{T}_j = \begin{cases} \boldsymbol{H}_k \boldsymbol{T}_k & k = j \\ \boldsymbol{O} & k \neq j, \end{cases} \tag{5}$$

where $\boldsymbol{O}$ is the zero matrix and $\boldsymbol{H}_k \boldsymbol{T}_k$ is a non-zero matrix. This relationship leads to a fact that every column vector of $\boldsymbol{T}_1,..., \boldsymbol{T}_{k-1}, \boldsymbol{T}_{k+1},..., \boldsymbol{T}_K$ belongs to null space of $\boldsymbol{H}_k$ or $\ker(\boldsymbol{H}_k)$, where $\ker(\ )$ denotes the kernel of the element matrix.

The singular value decomposition (SVD) of $\boldsymbol{H}_k$ is written as

$$\boldsymbol{H}_k = \boldsymbol{V}_k \boldsymbol{\Sigma}_k \boldsymbol{U}_k^H, \tag{6}$$

where we assume that the singular values are ranked in the descending order. Then, the bottom $M-N$ right-singular vectors, $\boldsymbol{u}_{k,N+1},..., \boldsymbol{u}_{k,M}$, become the basis spanning the null space of $\boldsymbol{H}_k$. Thus, we can express any column vector in $\boldsymbol{T}_1,..., \boldsymbol{T}_{k-1}, \boldsymbol{T}_{k+1},..., \boldsymbol{T}_K$ by linear combinations of $\boldsymbol{u}_{k,N+1},..., \boldsymbol{u}_{k,M}$, i.e.,

$$\boldsymbol{t}_{j,i} \in \ker(\boldsymbol{H}_k) = \mathrm{span}(\boldsymbol{u}_{k,N+1},..., \boldsymbol{u}_{k,M}), \tag{7}$$

where $j \neq k$, $i = 1,..., L_j$, and $\mathrm{span}(\ )$ denotes a vector space spanned by the element vectors.

Since (7) must be satisfied for any j, k, we can draw another result for column vectors of the kth transmit weight matrix $\boldsymbol{T}_k$ as:

$$\begin{aligned} \boldsymbol{t}_{k,i} &\in \ker(\boldsymbol{H}_1) \cap \cdots \cap \ker(\boldsymbol{H}_{k-1}) \\ &\quad \cap \ker(\boldsymbol{H}_{k+1}) \cap \cdots \cap \ker(\boldsymbol{H}_K) && (8) \\ &= \ker([\boldsymbol{H}_1^T,..., \boldsymbol{H}_{k-1}^T, \boldsymbol{H}_{k+1}^T,..., \boldsymbol{H}_K^T]^T) && (9) \\ &= \ker(\overline{\boldsymbol{H}_k}), && (10) \end{aligned}$$

where $i = 1,..., L_k$ and $\cap$ denotes intersection. Since $\mathrm{rank}(\overline{\boldsymbol{H}_k}) \leq N(K-1)$, the minimum dimension of the null space of $\overline{\boldsymbol{H}_k}$ becomes $M - N(K-1)$. To satisfy the BD condition (5), thus, we have $M \geq N(K-1) + L_k$. Moreover, when $L_k = N$, M should be not less than NK to ensure $\mathrm{rank}(\boldsymbol{H}_k \boldsymbol{T}_k) = N$.

C. Partial Nulling

As discussed above, the dimension of the null space of $\overline{\boldsymbol{H}_k}$ is much less than M. This means that transmit beamforming consumes many degrees of freedom.

Example 1. When $M = NK$ and $L_k = N$, the dimension of $\ker(\overline{\boldsymbol{H}_k})$ is N. Since the column space of $\boldsymbol{T}_k$ should be a subset of $\ker(\overline{\boldsymbol{H}_k})$ in the BD, $\boldsymbol{T}_k$ is uniquely determined by the orthonormal basis of $\ker(\overline{\boldsymbol{H}_k})$. Thus, no extra degrees of freedom are left.

Full consumption of the degrees of freedom implies that no diversity gain is obtained by the transmit beamforming for multiuser multiplexing[1]. Furthermore, the individual MIMO channel of the specific user may not have enough channel gain.

Example 2. Transmit weight vectors, $\boldsymbol{t}_{k,1},..., \boldsymbol{t}_{k,L_k}$ are determined not by $\boldsymbol{H}_k$ but by $\ker(\overline{\boldsymbol{H}_k})$. If those belong to the null space of $\boldsymbol{H}_k$, $\boldsymbol{H}_k \boldsymbol{T}_k$ falls to a zero matrix. In that case, no diversity gain is expected at the receiver side in spite of having N receive antennas.

Although the BD condition realizes the IUI free channel, perfect nulling is too strict. To ease the required condition, we have proposed partial nulling in a single stream transmission [9]. Here, the partial nulling is extended to multi-stream transmission as a generalized approach to BD.

First, let us consider a case of $L_k = N - 1$. Then, the receiver needs only to spend one degree of freedom to suppress the IUI in spatial filtering. The condition of partial nulling is expressed by

$$\boldsymbol{H}_k \boldsymbol{T}_j = \begin{cases} \boldsymbol{H}_k \boldsymbol{T}_k & k = j \\ \boldsymbol{\Delta}_{k,j} & k \neq j, \end{cases} \tag{11}$$

[1]The diversity gain through single-user MIMO channel is still available.

where $\boldsymbol{\Delta}_{k,j}$ is not a zero-matrix but a small-valued matrix. Then, the received signal vector of the kth user is given by

$$\boldsymbol{y}_k = \boldsymbol{H}_k \boldsymbol{T}_k \boldsymbol{x}_k + \sum_{j=1, j\neq k}^{K} \boldsymbol{\Delta}_{k,j} \boldsymbol{x}_j + \boldsymbol{z}_k. \tag{12}$$

To be able to suppress the IUI in (12) with one degree of freedom

$$\text{rank}\left(\sum_{j=1, j\neq k}^{K} \boldsymbol{\Delta}_{k,j} \boldsymbol{\Delta}_{k,j}^H \right) = 1. \tag{13}$$

must be satisfied. This means that every column vector in $\boldsymbol{\Delta}_{k,j}$ is parallel to a certain vector $\boldsymbol{q}$. Such a condition can be easily realized as an extension of (7) as follows:

$$\boldsymbol{t}_{j,i} \in \ker(\boldsymbol{H}_k) + \text{span}(\boldsymbol{p}) \tag{14}$$

$$= \text{span}(\boldsymbol{u}_{k,N+1},..., \boldsymbol{u}_{k,M}, \boldsymbol{p}), \tag{15}$$

where $\boldsymbol{p}$ is a vector which satisfies that $\boldsymbol{H}_k \boldsymbol{p}$ is a constant multiple of $\boldsymbol{q}$. It should be noted that the space spanned by $\boldsymbol{p}$ is no longer available for signal transmission due to IUI.

Example 3. A reasonable choice for $\boldsymbol{q}$ is the Nth left-singular vector $\boldsymbol{v}_{k,N}$ that corresponds to the minimum singular value since the loss of channel gain becomes minimum. Then, we have $\boldsymbol{p} = \boldsymbol{u}_{k,N}$.

Let us consider this example case here. We may rewrite (15) as

$$\boldsymbol{t}_{j,i} \in \text{span}(\boldsymbol{u}_{k,N}, \boldsymbol{u}_{k,N+1},..., \boldsymbol{u}_{k,M}), \tag{16}$$

where $j \neq k$ and $i = 1,..., L_k$.

If $L_k = N - 1$ and $\boldsymbol{p} = \boldsymbol{u}_{k,N}$ hold for any k, we have a final condition for the transmit weight vector of the kth user as

$$\begin{aligned} \boldsymbol{t}_{k,i} \in\ & \text{span}(\boldsymbol{u}_{1,N}, \boldsymbol{u}_{1,N+1},..., \boldsymbol{u}_{1,M}) \cap \cdots \\ & \cap \text{span}(\boldsymbol{u}_{k-1,N}, \boldsymbol{u}_{k-1,N+1},..., \boldsymbol{u}_{k-1,M}) \\ & \cap \text{span}(\boldsymbol{u}_{k+1,N}, \boldsymbol{u}_{k+1,N+1},..., \boldsymbol{u}_{k+1,M}) \cap \cdots \\ & \cap \text{span}(\boldsymbol{u}_{K,N}, \boldsymbol{u}_{K,N+1},..., \boldsymbol{u}_{K,M}) \end{aligned} \tag{17}$$

$$= \overline{\mathcal{S}_k}^{\perp}, \tag{18}$$

where

$$\begin{aligned} \overline{\mathcal{S}_k} = \text{span}(&\boldsymbol{u}_{1,1},..., \boldsymbol{u}_{1,N-1}, \boldsymbol{u}_{2,1},..., \boldsymbol{u}_{k-1,N-1}, \\ &\boldsymbol{u}_{k+1,1},..., \boldsymbol{u}_{K,N-1}). \end{aligned} \tag{19}$$

The dimension of the right hand side of (18) is $M - (N - 1)(K - 1)$. Since $L_k = N - 1$, the degrees of freedom for transmit beamforming becomes $M - (N - 1)K$. Thus, the partial nulling exploits the extra degrees of freedom to improve the channel gain in exchange for decreasing L_k.

Example 4. When $M = NK$ and $L_k = N - 1$ for all k, the dimension of $\mathcal{S}^{\perp}$ becomes $K + N - 1$. Since the dimension of the column space of $\boldsymbol{T}_k$ is $N - 1$, K degrees of freedom are remaining.

When $L_k = 1$ for all users, the maximum diversity gain for transmit beamforming is obtained. This special case was our previous proposal, called imperfect BD [9].

D. Generalized BD

Considering the above discussion, we can say that the BD is a subset of partial nulling where $L_k = N$ for all k. Here, we propose generalized BD (GBD) as a unified strategy based on the new definition of the required condition:

$$\boldsymbol{H}_k \boldsymbol{T}_j = \begin{cases} \boldsymbol{H}_k \boldsymbol{T}_k & k = j \\ \boldsymbol{\Delta}_{k,j} & k \neq j, \end{cases} \tag{20}$$

where the column space of $\boldsymbol{\Delta}_{k,j}$ is a subset of $\text{span}(\boldsymbol{v}_{k,1},..., \boldsymbol{v}_{k,N})$. The subset is defined by the base vectors extracted from $\boldsymbol{v}_{k,1},..., \boldsymbol{v}_{k,N}$ for each k. Here, the numbers of selected vectors are equal to $N - L_k$. When the subset is null (i.e., $L_k = N$) for all k, $\boldsymbol{\Delta}_{k,j}$ becomes a zero matrix. Then, GBD is equivalent to BD.

Subset selecting rules are discussed later. Once a certain subset is selected, the transmit weight matrices are determined as follows. First, we define matrices expressing the dominant component of each channel as

$$\boldsymbol{D}_k = [\underbrace{\boldsymbol{u}_{k,1}, \boldsymbol{u}_{k,2},..., \boldsymbol{u}_{k,N}}_{L_k}], \tag{21}$$

for all k where the element vectors are chosen from N-top right-singular vectors in (6). The indices of vectors excluded in $\boldsymbol{D}_k$ correspond to those of $(N - L_k)$ vectors spanning the column space of $\boldsymbol{\Delta}_{k,j}$, selected from $\boldsymbol{v}_{k,1},..., \boldsymbol{v}_{k,N}$.

Before obtaining the kth transmit weight matrix, we construct a matrix needed for projection as

$$\overline{\boldsymbol{D}_k} = [\boldsymbol{D}_1,..., \boldsymbol{D}_{k-1}, \boldsymbol{D}_{k+1},..., \boldsymbol{D}_K]. \tag{22}$$

This expresses the dominant space other than the kth one. Then, a primitive matrix for the kth transmit weight is given by the complementary projection onto $\overline{\boldsymbol{D}_k}(\overline{\boldsymbol{D}_k}^H \overline{\boldsymbol{D}_k})^{-1} \overline{\boldsymbol{D}_k}^H$ as

$$\boldsymbol{T}_k^{(p)} = \left\{ \boldsymbol{I} - \overline{\boldsymbol{D}_k}(\overline{\boldsymbol{D}_k}^H \overline{\boldsymbol{D}_k})^{-1} \overline{\boldsymbol{D}_k}^H \right\} \boldsymbol{D}_k, \tag{23}$$

where the same concept of this operation is shown in (18) and (19). The projection source must be $\boldsymbol{D}_k$ here since the subspace of $\boldsymbol{D}_k$ is the null space of $\boldsymbol{H}_k$ or the space which will be corrupted by the IUI. Therefore, no usable signals are transmitted.

Finally, the transmit weight matrix is determined as

$$\boldsymbol{T}_k \leftarrow \text{orthonormalized column vectors of } \boldsymbol{T}_k^{(p)}. \tag{24}$$

This operation and complementary projection (23) may be performed jointly by using Gram-Schmidt orthonormalization.

E. Subset Selection for GBD

The subset selection is done under a certain criterion such as system capacity and user throughput. If the second term in (12) is regarded as colored noise, the user specific MIMO channel capacity can be evaluated [13]. Or, we may simply apply the zero-forcing spatial filter as post-coding for IUI suppression. The weight matrix of the kth user for post-coding is given by

$$\boldsymbol{R}_k = [\underbrace{\boldsymbol{v}_{k,1}, \boldsymbol{v}_{k,2},..., \boldsymbol{v}_{k,N}}_{L_k}], \tag{25}$$

where the indices of element vectors in (25) are equal to those in (21). Since the column space of $\boldsymbol{\Delta}_{k,j}$ in (12) is in the null space of $\boldsymbol{R}_k$, the IUI is perfectly removed by $\boldsymbol{R}_k$.

The kth received signal after post-coding is written as

$$\begin{aligned}\boldsymbol{y}'_k &= \boldsymbol{R}_k^H \boldsymbol{y}_k \\ &= \boldsymbol{R}_k^H \boldsymbol{H}_k \boldsymbol{T}_k \boldsymbol{x}_k + \boldsymbol{R}_k^H \boldsymbol{z}_k \\ &= \boldsymbol{H}'_k \boldsymbol{x}_k + \boldsymbol{z}'_k, \end{aligned} \tag{26}$$

where $\boldsymbol{H}'_k \in \mathbb{C}^{L_k \times L_k}$ denotes the equivalent matrix for a pure single-user MIMO channel. Thus, any quality assessments are applicable to $\boldsymbol{H}'_k$.

In this paper, we select the best subset from all the candidates based on a certain quality measurement. Clearly, the total number of subset candidates for all users reaches to

$$\left(\sum_{l=0}^{N-1} \binom{N}{l} \right)^K . \tag{27}$$

This number becomes enormous when K and N are large. For example, it is 50,626 when $K = N = 4$. However, this search method is effective when the singular vectors corresponding to large singular values are close to those of other users. We call this as full search hereinafter.

If we consider the loss of channel gain, i.e., singular values in (6), the ordered search as discussed in **Example 3** may be useful. In this case, the kth subset is uniquely determined for each L_k from the $N - L_k$ $(< N)$ bottoms of $\boldsymbol{v}_{k,2}, \ldots, \boldsymbol{v}_{k,N}$. Thus, the total number of subset candidates for the ordered search is only N^K. However, the ordered search would cause some performance degradation due to vector space similarity between users, for example, in the case that users are located closely. The performance comparison will be discussed later.

III. Numerical Analysis

A. Simulation Parameters

In the following, the performance of GBD is evaluated using computer simulations. Table I shows the simulation parameters used in the paper. Applying the GBD to a multiuser MIMO channel and using the post-coding given by (25) yields K single-user MIMO channels. In the paper, we applied the eigenmode transmission to each single-user MIMO channel $\boldsymbol{H}'_k$ in (26). The resource adaptation for the eigenmode transmission was carried out with a predicted bit error rate (BER) from the singular values of $\boldsymbol{H}'_k$ [7]. The prepared modulation schemes are listed in Table I. The resource adaptation is user-independent except the constraint that the total data rate per user is the same among all users.

For each subset candidate in (20) and (21), the predicted BER was calculated according to the optimum resource adaptation described above. Then, the subset giving the minimum predicted BER was selected for both ordered and full searches. In addition, the fixed L_k cases in the ordered search are evaluated as a simplified procedure. It should be noted that the case of $L_k = N$ is equivalent to BD. To distinguish the fixed and non-fixed cases, we call the non-fixed case as "adaptive" hereinafter.

TABLE I
Simulation Parameters

number of antennas	$M = 4, 6, 8, \quad N = 2, 3$
number of users	$K = 2, 3, 4$
spatial multiplexing per user	eigenmode transmission
data rate	4 bits/symbol, 8 bits/symbol
modulation	QPSK, 16QAM, 64QAM, 256QAM
signal detection	MMSE spatial filtering (IUI rejection and eigenmode reception)
data frame length	256 symbols
channel	i.i.d. Rayleigh fading
CSI	perfectly known
number of trial frames	100,000

We assumed that the channel information is perfectly known at the transmitter and receivers. As the SNR measure, we used the normalized transmit power per user which was divided by the transmit power yielding an average E_s/N_0 of 0 dB in the single antenna transmission.

B. Diversity Gain Availability of GBD with Ordered Search

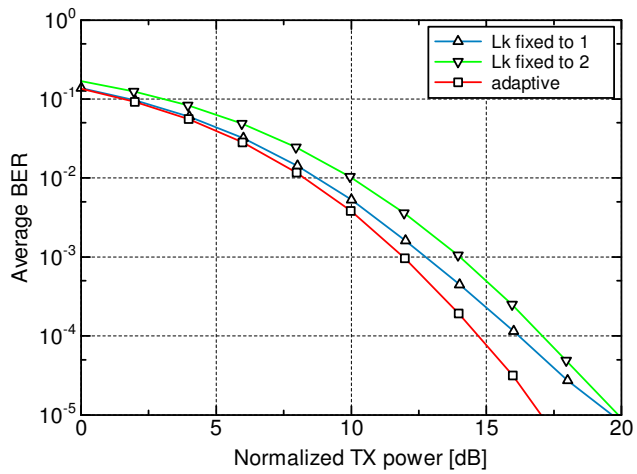
First, we would like to show the diversity gain availability of GBD with ordered search. Figure 2 shows the BER performance for $M = 4$ and 8 where $K = N = 2$.

In the $M = 4$ $(= NK)$ case, no degrees of freedom are left for BD, i.e., the case of $L_k = N$. From Fig. 2(a), we can see that the GBD with $L_k = 1$ provides some gain compared to BD ($L_k = 2$). This comes from the extra degrees of freedom for the GBD. Here, it should be noted that the smaller L_k enforces the higher modulation level. Therefore, the diversity gain and BER penalty is a trade-off. As a result, adaptive L_k selection shows the best performance since it can reduce the modulation level to QPSK by double-layer transmission when the channel condition is good.

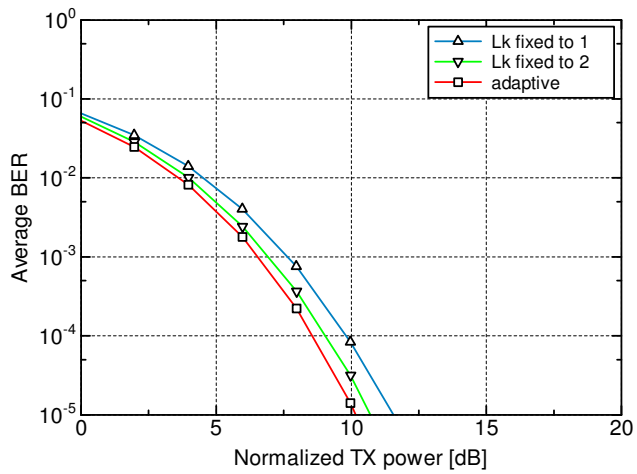
When $M = 8$, BD ($L_k = 2$) outperforms the GBD with $L_k = 1$ since we still have $(M - NK) = 4$ degrees of freedom even in the BD case. When, the transmit diversity gain is large enough for the BD, the benefit of the GBD with adaptive selection is not so highlighted although this scheme shows the best performance. From this fact, it can be stated that the GBD is effective when $M \simeq NK$.

C. Many User Case

Next, let us take a look at the dependency of the GBD to the number of users. Figure 3 shows the BER performance with ordered search for $K = 4$ where $NK = M = 8$. In comparison to Fig. 2(a), it is clearly shown that the gain from the GBD is better for $K = 4$. In both cases, no degrees of freedom are left for the BD ($L_k = 2$) due to $NK = M$. Thus, the curves for $L_k = 2$ are almost the same in these figures. In contrast, the curve for the GBD in Fig. 3 is highly improved. When the number of users are larger, the extra degrees of freedom, obtained by adaptive L_k reduction becomes larger as well. Thus, in the $K = 4$ case, about 4 dB gain is obtained at the BER of 10^{-3}, where the optimum L_k is selected by the trade-off between the diversity gain and BER penalty due to the higher modulation level.



(a) $K = 2$, $M = 4$, $N = 2$



(b) $K = 2$, $M = 8$, $N = 2$

Fig. 2. BER performance of ordered search for remaining degrees of freedom: $(M - NK) = 0$ and 4 (4 bits/symbol).

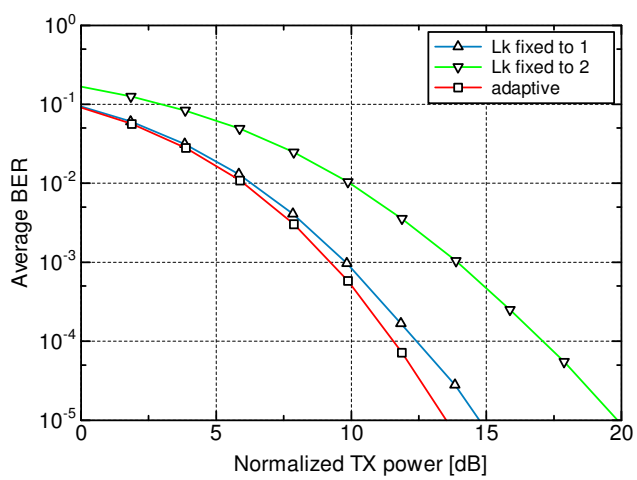


Fig. 3. BER performance for $K = 4$, $M = 8$, and $N = 2$ (4 bits/symbol).

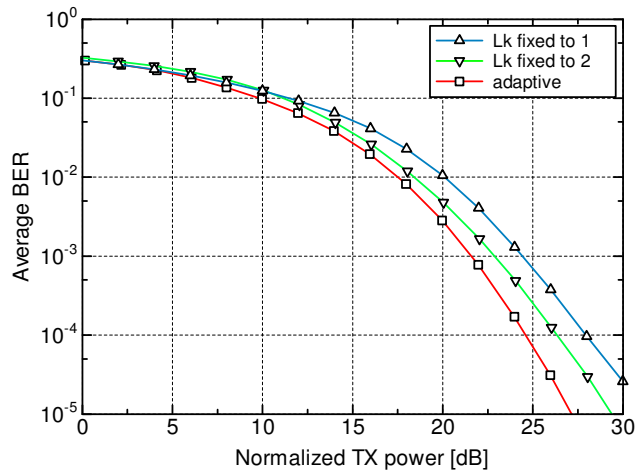


Fig. 4. BER performance for $K = 2$, $M = 4$, and $N = 2$ (8 bits/symbol).

D. Higher Level Modulation Case

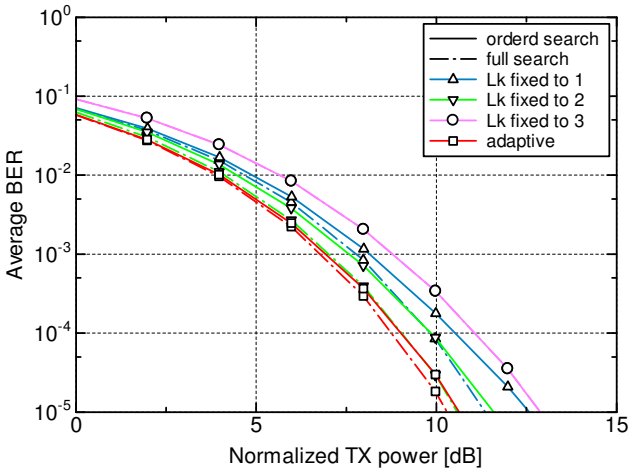
When the data rate becomes 8 bits/symbol, higher level modulation up to 256QAM is possibly used. Therefore, the number of layers should be larger to reduce the modulation level unlike the previous cases. Figure 4 shows the BER performance with ordered search for 8 bits/symbol. The performance for 4 bits/symbol with the same parameters is shown in Fig. 2(a). Clearly, all the curves in Fig. 4 degrade by about 9 dB. In addition, the GBD with $L_k = 1$ becomes worse than the BD ($L_k = 2$) case due to the use of 256QAM. Therefore, the adaptive L_k selection is necessary in actual situations using adaptive modulation.

E. Performance of Full Search

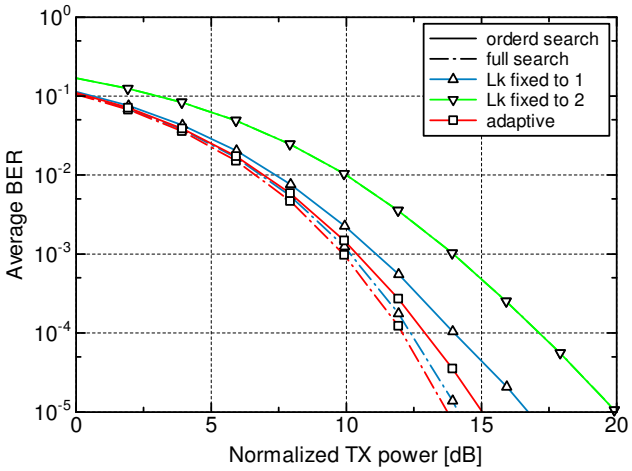
Finally, we would like to evaluate the performance of the GBD with full search. Figure 5 shows the BER performance with full and ordered searches for $M = 6 = NK$. The gain obtained by full search is very small in Fig. 5(a) but becomes larger in Fig. 5(b). When $N = 2$, ordered search always fails if $\boldsymbol{u}_{1,1}$ is similar to $\boldsymbol{u}_{2,1}$ or $\boldsymbol{u}_{3,1}$, i.e., complementary projection on to $\overline{\boldsymbol{D}_1}(\overline{\boldsymbol{D}_1}^H\overline{\boldsymbol{D}_1})^{-1}\overline{\boldsymbol{D}_1}^H$ of $\boldsymbol{D}_1$ becomes the null space. However, when $N = 3$, such a similarity of dominant space is expected to be rare enough since the dimension of the dominant space may be increased by up to two except in the BD case. Thus, we can say that the full search is useful when N is small.

IV. Conclusion

In this paper, we have extended the concept of partial nulling to the generalized expression of the BD and formulated the GBD as the subset selection for the IUI projection. Using this formula, we can uniquely determine the transmit beamforming weight where the IUI is projected to a tiny and controlled range of within the column space of each channel matrix and thus can easily be suppressed by receive beamforming. The dimension of the IUI image depends on the number of layers. If the number of layers is smaller than the number of receive antennas, we can use the extra degrees of freedom to



(a) $K = 2$, $M = 6$, and $N = 3$



(b) $K = 3$, $M = 6$, and $N = 2$

Fig. 5. BER performance for full search (4 bits/symbol).

suppress the IUI. Thus, in the GBD, the transmit beamforming weights of interfering users are controlled to be concentrated in a certain limited space. The BD is equivalent to GBD for the case when the number of layers equals to the number of receive antennas.

The GBD is capable of providing higher diversity gain in exchange for reducing the number of layers. From the numerical analysis, it has been confirmed that the GBD can utilize the extra degrees of freedom for transmit diversity. We can conclude that the GBD is an attractive technique for the multiuser MIMO broadcast. However, the subset selection for the IUI projection is not sophisticated yet. It should be looked at as part of a future work. In addition, MMSE based transmit beamforming should also be investigated as the next stage.

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