An Optimal Configuration Design of Superconducting Magnets with HTS Tapes for DC reactor

So Noguchi, *Member*, *IEEE* and Makoto Tsuda

***Abstract*—Large-inductance DC reactors are often required in practical factories, and superconducting magnets represent an interesting alternative for the construction of such devices. It is therefore useful to obtain optimized designs for DC reactors, particularly for the minimization of the winding volume under many constraints. In this paper the optimized configurations of toroidal superconducting magnets for DC reactor are presented and the configurations for SMES are compared.**

***Index Terms*—DC reactor, superconducting magnet, optimal design.**

I. INTRODUCTION

LARGE-SCALE DC reactors (DCL) with large inductances are commonly required in practical factories. Employing superconducting magnets for inductance has advantages, such as large current, low loss, coreless (lightness), and so on. From the point of view of flux leakage, the toroidal magnet is an ideal structure. During the optimization of the toroidal superconducting magnet, the goal is to minimize the winding volume under many constraints such as inductance, characteristics of the superconductor (e.g. B–J–θ characteristic of HTS tapes), thermal stress, Lorentz force, stabilization and protection against the quench and so on.

An optimal configuration design method for SMES with HTS tapes has been proposed [1]–[3]. The toroidal superconducting magnets for SMES were designed employing simulated annealing as the optimization method and the finite element method for numerical field computation [3]. Due to quadratic dependence of the stored energy with respect to the current, it is more effective to increase the operating current than the inductance to get to a given energy. In the case of the DCL design a different approach has to be used. Since a large inductance is needed, the inductance has to be increased as well.

We have investigated the configuration of the toroidal superconducting magnets for the DCL. Using the method proposed in [3], the configuration of the element coils of the toroidal superconducting magnet for the DCL has been optimized to minimize the winding for a 1 H inductance and a 20 kA operating current corresponding to the critical current. The toroidal magnet consists of 8 or 12 element coils and the coils are wound with YBCO tapes.

The configurations of the element coil of the toroidal HTS-SMES, as presented in [1]–[3], are flat rectangular shapes, like a disc coil. The optimization of the element coils for the DCL is described here. For comparison, the optimization of the toroidal superconducting magnet for a 20 MJ SMES is also presented.

II. Optimal Design Method

A. Inductance Computation

In the DCL case, the toroidal superconducting magnet consists of 8 or 12 element coils and connected in series. The inductance L of the toroidal magnet is calculated from

$$L = \frac{2E}{I_{op}^2}, \tag{1}$$

where I_{op} is the operating current and E is the energy computed from

$$E = \frac{1}{2}\int_V \boldsymbol{A}\cdot\boldsymbol{J}\mathrm{d}V, \tag{2}$$

where $\boldsymbol{A}$ is the magnetic vector potential, $\boldsymbol{J}$ is the current density, and V is the volume of superconducting magnet. Here, the magnetic vector potential $\boldsymbol{A}$ is given by the superposition of the element coils and calculated from

$$\boldsymbol{A} = \frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}(q)}{l(q)}\mathrm{d}V, \tag{3}$$

where $l(q)$ is the distance between the point at which $\boldsymbol{A}$ is evaluated and the point q of the current density source.

The energy E and vector potential $\boldsymbol{A}$ are obtained by numerical integration of (2) and (3), respectively.

B. Optimization Algorithm

The simulated annealing [4] is employed as the optimization method to minimize the winding volume of the superconducting magnet. Moreover, the Augmented Lagrange multiplier method is combined with the simulated annealing in order to enable the solution of the constrained problem [5].

Manuscript received 20 October 2009.

S. Noguchi is with Hokkaido University, Kita 14 Nishi 9, Kita-ku, Sapporo, Hokkaido 060-0814, Japan (phone: +81-11-706-7671; fax: +81-11-706-7670; e-mail: noguchi@ssi.ist.hokudai.ac.jp).

M. Tsuda is with Tohoku University, 6-6-05 Aoba Aramaki, Aoba-ku, Sendai, Miyagi 980-8579, Japan (e-mail: tsuda@ecei.tohoku.ac.jp).

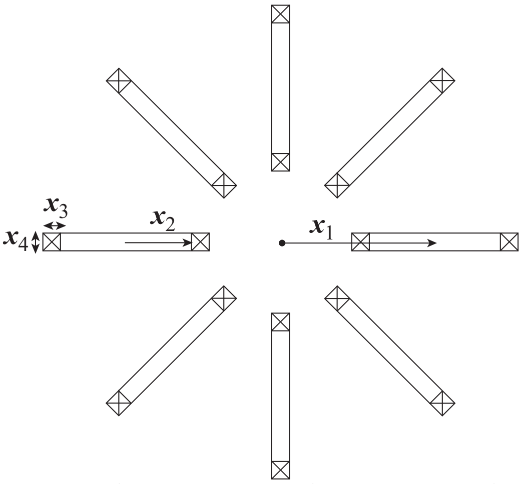


Fig. 1. Design model of toroidal magnets and four design variables for optimal design, x_1, x_2, x_3 and x_4 are the distance between the center of toroid and the center of an element coil, the inner radius of each element coil and the thickness and length of each element coil, respectively.

TABLE I. SPECIFICATIONS OF TOROIDAL SUPERCONDUCITNG MAGNET for DCL

Case	8-DCL	12-DCL	Case	8-DCL	12-DCL
No. of coils	8	12	Operating Current I_{op} (kA)	2.0	2.0
Inductance L (H)	1.0	1.0	No. of layers	39	35
Radius of toroid x_1 (mm)	487	476	No. of turns of each layer	9	9
Inner radius of coil x_2 (mm)	217	163	No. of turns of YBCO tape	1404 (39x9x4)	1260 (35x9x4)
Thickness of coil x_3 (mm)	156	140	Total length of YBCO tape (km)	20.8	22.1
Length of coil x_4 (mm)	90	90	Winding volume (m^3)	0.208	0.221

III. OPTIMAL CONFIGURATION DESIGN

The configuration of the toroidal superconducting magnet for the DCL is optimized to minimize the winding volume. As the mentioned earlier, the toroidal superconducting magnets to be optimized consist of 8 and 12 element coils. The specifications of the magnet are set as follows:

- The inductance is 1 H (within 1.0%).
- The conductor consists of 4 sheets of YBCO tape with a thickness of 0.2 mm and a width of 10 mm, and reinforcement with thickness of 0.8 mm and width of 10 mm. The B–J–θ characteristic of YBCO tape is computed based on the result of measurements presented in [2] and [6].
- The operating current, I_{op}, is under 2 kA.
- The inner radius, radius of toroid, number of layers and turns of each layer of element coils are design variables, as shown in Fig. 1.
- The operating temperature is 20 K.
- The goal of optimal design is to minimize the winding volume.

The critical current is decided from the B–J–θ characteristic based on the percolation model [6], where the magnetic flux density B and the angle between the tape surface and the magnetic flux density vector, θ, are computed based on the Biot-Savart law. In the optimal design, the design variables are the distance between the center of toroid and the center of an element coil, the inner radius of each element coil and the thickness and length of each element coil. The thickness and the length of each coil are computed based on the number of layers and turns of each layer of coils, which are discrete variables. The objective function to be minimized in the optimal design is the winding volume in total.

The specifications of the optimized toroidal superconducting magnets of the 1 H DCL are shown in Table I. The optimized configurations of the toroidal superconducting magnet with an inductance of 1H are shown in Fig. 2. Of course, all constraints in the result are satisfied.

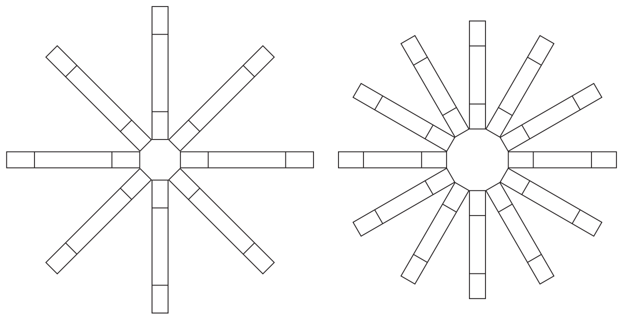


Fig. 2. Optimized configuration of the toroidal superconducting magnet for DCL. The inductance is 1 H, the operating current is 2 kA

IV. COMPARISON BETWEEN DCL AND SMES

A. Optimal Configuration Design for SMES

The configurations of SMES coils presented in papers [1]-[3] are flat rectangular shape, like a disc. However, the configurations obtained for the DCL are no so flat, compared with SMES coils. For confirmation and comparison, the configurations of the toroidal SMES coils are also optimized under a few different conditions.

The toroidal SMES magnet to be designed consists of 8 and 12 element coils, and the stored energy is 20 MJ (10 times energy of the DCL). Three cases have been considered: I) 2 kA operating current, 10 H inductance, II) $2\sqrt{10}$ kA, 1 H, and III) the operating current is design variable but the stored energy is 20 MJ. The other specifications of the SMES magnets are the same as the DCL magnets. The specifications are as follows;

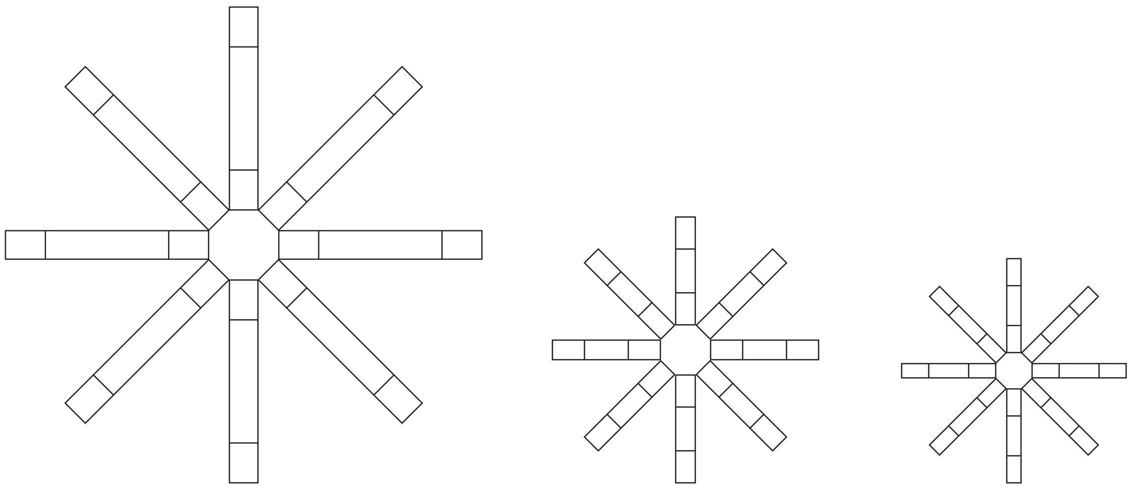
- The stored energy is under 20 MJ.
- The inductance L and operating current I_{op} are different in each case.
- The conductor consists of 4 sheets of YBCO tape and reinforcement.
- The inner radius, radius toroid, number of layers and turns of each layer of element coils are design variables, as

TABLE II. SPECIFICATIONS OF 8-TOROIDAL SMES

Case	8-SMES I	8-SMES II	8-SMES III
No. of coils	8	8	8
Operating Current I_{op} (kA)	2.0	$2\sqrt{10}$	10.5
Inductance L (H)	10.0	1.0	0.36
Radius of toroid x_1 (mm)	767	444	366
Inner radius of coil x_2 (mm)	346	123	112
Thickness of coil x_3 (mm)	224	180	152
Length of coil x_4 (mm)	160	110	80
No. of layers	56	45	38
No. of turns of each layer	16	11	8
No. of turns of YBCO tape	3584 (56x16x4)	1980 (45x11x4)	1216 (38x8x4)
Total length of YBCO tape (km)	82.5	21.2	11.5
Winding volume (m^3)	0.825	0.212	0.115

TABLE III. SPECIFICATIONS OF 12-TOROIDAL SMES

Case	12-SMES I	12-SMES II	12-SMES III
No. of coils	12	12	12
Operating Current I_{op} (kA)	2.0	$2\sqrt{10}$	6.7
Inductance L (H)	10.0	1.0	0.88
Radius of toroid x_1 (mm)	772	474	457
Inner radius of coil x_2 (mm)	326	173	149
Thickness of coil x_3 (mm)	236	184	188
Length of coil x_4 (mm)	110	60	60
No. of layers	59	46	47
No. of turns of each layer	11	6	6
No. of turns of YBCO tape	2596 (59x11x4)	1104 (46x6x4)	1128 (47x6x4)
Total length of YBCO tape (km)	86.9	22.1	20.7
Winding volume (m^3)	0.869	0.221	0.207



(a) 8-SMES I (2 kA, 10 H) (b) 8-SMES II ($2\sqrt{10}$ kA, 1 H) (c) 8-SMES III (10.5 kA, 0.36 H)

Fig. 3. The optimized configurations of the 8 toroidal SMES. The storage energy is 20 MJ.

shown in Fig. 1. In the last case, the operating current is added as a design variable.

- The operating temperature is 20 K.
- The goal of optimal design is to minimize the winding volume.

The specifications of the optimized 8- and 12-toroidal SMES magnets are shown in Table II and III, respectively. The optimized configurations are shown in Figs. 3 and 4.

These results show that increasing the operating current instead of the inductance is more efficient in terms of winding minimization. The outer size also becomes smaller as the operating current increases. There is a tendency for the cross section of the element coil to gradually become thicker and shorter.

The winding volumes of the optimized 8-toroidal SMES magnets are comparatively smaller than those of the 12-toroidal ones. The cross section of element coils of the 8-toroidal SMES is thicker and shorter than that of 12-toroidal ones, because the radius of toroid, x_1, has to be short. If the element coil is long, the radius of toroid also becomes longer, leading to an increase in magnetic flux leakage.

B. *Comparison between DCL and SMES*

Comparing the results of the DCL and the SMES magnets, we can see that roughly the same winding volume is necessary when the inductance is the same. It is approximately independent of the operating current, with both winding volume and size dependent on the inductance. In the cases of SMES design, it is possible to store a high energy in smaller spaces due to small inductances and large operating currents. In comparison with 8- and 12-SMES IIs, the 8- and 12-DCLs require a smaller number of layers of the element coils to achieve the same inductance.

The maximum magnetic field parallel to and normal to the

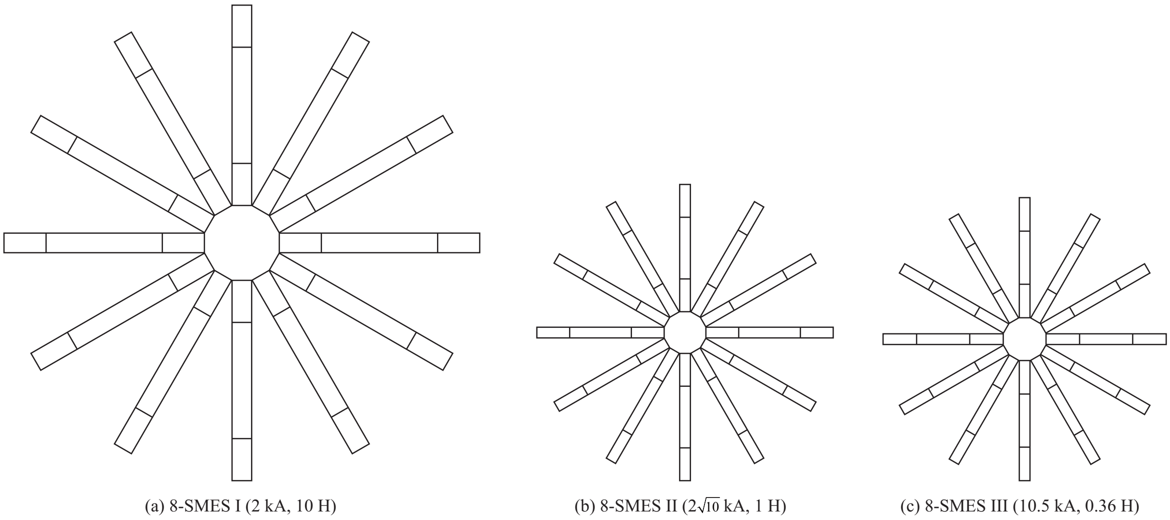


Fig. 4. The optimized configurations of the 12 toroidal SMES. The storage energy is 20 MJ.

TABLE IV. MAGNETIC FIELD APPLYING TO YBCO TAPE AND LOAD FACTOR

Case	8-DCL	8-SMES I	8-SMES II	8-SMES III
$B_{parallel}$ (T)	4.75	7.63	18.27	23.59
B_{normal} (T)	1.86	3.00	6.81	8.84
I_{op} / I_c	0.087	0.111	0.529	1.000
Case	12-DCL	12-SMES I	12-SMES II	12-SMES III
$B_{parallel}$ (T)	5.23	7.62	15.46	16.46
B_{normal} (T)	1.56	2.25	4.44	4.71
I_{op} / I_c	0.078	0.093	0.417	1.000

YBCO tape surface, as well as the load factor, I_{op} / I_c, are shown in Table IV. In every case, the critical current is decided from the maximum normal magnetic field. The load factors of the DCL cases are very low. Therefore, it would be possible for higher currents to flow, that is, the winding volume is being used ineffectively. When the operating current increases, we observe increases in both the maximum normal magnetic field and the load factor. It is therefore necessary to take a safety margin and Lorentz force into account when designing the SMES. In this paper, the Lorentz force is disregarded because of the simple conceptual design.

The winding volume and the size of the toroidal superconducting magnets for the DCL are determined by the desired inductance. On the other hand, the size of toroidal SMES is dependent on the operating current, i.e., the design concepts are different between the DCL and the SMES magnets.

V. CONCLUSION

We have optimized the 8 and 12 toroidal superconducting magnets for 1 H DC reactor and 20 MJ SMES in order to winding volume. In the DC reactor case, the winding volume and the size are strongly dependent on the desired inductance. On the other hand, the winding volume and the size of SMES are dependent on the operating current. The design concept is different between the DCL magnet and the SMES magnet.

As the number of element coils and the operating current increase, the cross section of the element coil becomes a flat rectangular shape, like a disc. As the result, the leakage of the magnetic flux becomes small due to the small radius of toroid.

In the future, it will be necessary to design large scale superconducting magnets for DC reactor and SMES to take the maximum Lorentz force and the AC loss into account.

REFERENCES

[1] S. Noguchi, A. Ishiyama, S. Akita, H. Kasahara, Y. Tatsuta, and S. Kouso, “An optimal configuration design method for HTS-SMES coils,” *IEEE Trans. Appl. Supercond.*, vol. 15, no. 2, pp. 1927–1930, Jun. 2005.

[2] K. Higashikawa, T. Nakamura, K. Shinkimachi, N. Hirano, S. Nagaya, T. Kiss, and M. Inoue, “Conceptual design of HTS coil for SMES using YBCO coated conductor,” *IEEE Trans. Appl. Supercond.*, vol. 17, no. 2, pp. 1990–1993, Jun. 2007.

[3] S. Noguchi, Y. Inaba, H. Igarashi, “An optimal configuration design method for HTS-SMES coils taking account of thermal and electromagnetic characteristics,” *IEEE Trans. Appl. Supercond.*, vol. 18, no. 2, pp. 762–765, Jun. 2008.

[4] E. Aarts and J. Korst, *Simulated Annealing and Boltzmann Machines: A Stochastic Approach to Combinational Optimization and Neural Computing*, John Wiley & Sons Ltd., Jan. 1989.

[5] S. Noguchi and A. Ishiyama, “An optimal design method for high field superconducting magnets,” *Physica B*, vol. 216, nos. 3&4, pp. 212–214, Jan. 1996.

[6] K. Yamafuji, T. Kiss, “A new interpretation of the glass–liquid transition of pinned fluxoids in high-T_c superconductors,” *Physica C*, vol. 258, nos. 3&4, pp. 197-212, Feb. 1996.