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A Chamberlinian Agglomeration Model with External Economies of Scale*

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Abstract

We investigate the effects of a reduction in trade costs on industrial location and welfare in an economy with external economies of scale. We propose a Chamberlinian agglomeration model with footloose capital, which is analytically-solvable. With respect to industrial location, we demonstrate that a reduction in trade cost is likely to lead to agglomeration. With respect to welfare, we show that agglomeration makes a country with agglomeration better off, and the country without agglomeration better or worse off, depending on the degree of external economies of scale. We also prove that agglomeration makes the overall economy better off.

Keywords: New economic geography; Agglomeration; Footloose capital, External economies of scale

JEL classifications: F12; F15; F21; R12

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1 Introduction

For the last two decades, industrial location has been studied in the field of new economic geography. Krugman (1991) is well-known as a seminal work. His model, a so called "core-periphery (CP) model," focuses on agglomeration in the industrial sector with trade cost, increasing returns to scale, and monopolistically competitive markets. One of the features of the CP model is forward and backward linkages; industrial workers are mobile between regions or countries, and shifting of workers entails a shift in demand and subsequently a shift in firms. He shows that a reduction in trade cost leads to drastic agglomeration, which is called the "catastrophic result," at the threshold trade cost level even in two countries that are initially symmetric. Another feature of the CP model is that it is not analytically-solvable. Thus, analysis in the literature based on the CP model has been conducted mainly by using numerical simulation.

Some studies have provided an analytically-solvable framework (*e.g.*, Martin and Rogers, 1995; Forslid and Ottaviano, 2002). Among them, Pfluger (2004) proposes an analytically-solvable model by using the quasi-linear utility function, which removes all income effects in the industrial sector, and assuming technology where human capital is used only for fixed cost. According to Pfluger's model, a "subcritical pitchfork bifurcation," *i.e.*, non catastrophic and gradual agglomeration, is obtained even under forward and backward linkages.

Pfluger (2004) explains industrial location in the real world using an analytically-solvable framework with CP features. However, the following two points should be noted. First, although Pfluger (2004) presumes a situation where movement of labor between countries or regions is smooth, that is not necessarily true. For example, rates of labor movement in Asian countries have been quite low (OECD, 2010). Second, some empirical studies show that external economies of scale are crucial for industrial agglomeration (*e.g.*, Andretsch and Feldman, 1996; Rosenthal and Stranger, 2001; Lu and Tao, 2009). We thus need to consider an agglomeration model specifically to analyze situations where movement of labor is not smooth and external economies of scale are explicitly considered.

Further, in the field of new economic geography, there is only a small amount of literature that deals with economic welfare brought by industrial location (*e.g.*, Amiti, 2005; Chalot, et al, 2006; Pfluger and Sudekum, 2008). This is because the main focus is on industrial location. The existing studies on welfare do not take into account external economies to scale.

This paper proposes another analytically-solvable framework to investigate industrial location and welfare. The model is based on the "footloose capital" model by Martin and Rogers (1995), and external economies of scale are incorporated. We consider the agricultural and manufacturing sectors in two countries, which are initially symmetric. In the former sector, a homogeneous good is produced by using only labor under a constant returns to scale technology, the market is perfectly competitive, and no trade costs are necessary. In the latter sector, differentiated goods are produced by using labor and capital under an increasing returns to scale technology with external economies of scale. The market is monopolistically competitive, and iceberg trade costs must be incurred when the good is traded. Capital is mobile between the countries, and international distribution of capital, and thus industrial location, is endogenously determined.

Under the setup, we obtain the following results: With respect to industrial location, a reduction in trade cost is likely to lead to agglomeration in the manufacturing sector (Proposition 1). In particular, if the trade cost becomes lower than a threshold level, agglomeration occurs drastically. That is, the "catastrophic" agglomeration as in the CP model is obtained in our analytically-solvable framework.

With respect to welfare, the country with agglomeration becomes unambiguously better off, while the country without agglomeration becomes better off or worse off (Proposition 2). This result implies that agglomeration may or may not be Pareto improving. The above results are obtained depending on the degree of initial trade cost and the degree of external economies of scale. Finally, agglomeration makes the overall economy better off (Proposition 3). This result shows that the agglomeration is potentially Pareto-improving; that is, even if agglomeration reduces welfare in the country without agglomeration, some adequate income transfer can make both countries better off.

The rest of this paper is organized as follows. Section 2 proposes the basic model. Section 3 considers the short-run equilibrium and clarifies some characteristics of this economy. Section 4 derives the long-run equilibrium and examines the effects of a reduction in trade cost on industrial location and welfare. Finally, section 5 briefly concludes the paper.

2 The Model

The economy is composed of two countries, home and foreign (denoted by an asterisk); two sectors, agriculture and manufacturing sectors; and two factors of production, labor and capital.

In the agricultural sector, a homogeneous good is produced by using only labor under a constant returns to scale technology. The market is perfectly competitive. No trade costs are necessary when the good is traded between countries. In what follows, we treat the agricultural good as a numeraire.

In the manufacturing sector, differentiated goods are produced by using labor and capital under an increasing returns to scale technology. The market is monopolistically competitive. An iceberg trade cost must be incurred when the manufacturing good is traded. We assume that the iceberg trade cost from home to foreign is the same as that from foreign to home.

At the beginning, suppose that the home and foreign countries are symmetric. Both countries then have common factors of production. With respect to labor, each country is endowed with L units. Labor is mobile between the industries but immobile between the countries. With respect to capital, the overall economy is endowed with $\bar{K}$ unit. Since the two countries are symmetric, $\frac{\bar{K}}{2}$ units of capital are initially distributed to each country. Capital is mobile between the countries. In the following analysis, an international distribution of capital will be endogenously determined in the equilibrium.

2.1 Consumption

Consumers in the home country have a common Cobb-Douglas utility function:

$$
\begin{aligned}
U &= C_M^{\mu} C_A^{1-\mu}, \quad 0 < \mu < 1, \\
C_M &\equiv \left(\int_0^n c_i^{\frac{\sigma-1}{\sigma}} di + \int_0^{n^*} c_{i^*}^{\frac{\sigma-1}{\sigma}} di^* \right)^{\frac{\sigma}{\sigma-1}},
\end{aligned}
\tag{1}
$$

where C_M and C_A are consumption of the aggregate of varieties of the manufacturing good and that of the agricultural good, respectively, i and i^* are indices of the manufacturing good produced in the home and foreign countries, n and n^* are the number of home and foreign varieties, μ is the share of expenditure on the manufacturing aggregates, and $\sigma > 1$ is a constant elasticity of substitution between manufacturing varieties.

Under utility function (1), we obtain the following demand functions for varieties i and i^* of the manufacturing good, c_i and c_{i^*}:

$$
c_i = p_i^{-\sigma} G^{\sigma-1} \mu I, \tag{2}
$$

$$
c_{i^*} = \left(\tau p_{i^*}\right)^{-\sigma} G^{\sigma-1} \mu I, \tag{3}
$$

where p_i and p_{i^*} are prices on varieties i and i^* set by the home and foreign firms, respectively, $\tau > 1$ is the iceberg trade cost,

$$
G \equiv \left(\int_0^n p_i^{1-\sigma} di + \int_0^{n^*} \left(\tau p_{i^*}\right)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \tag{4}
$$

is a price index on the manufacturing aggregates, and I is consumers' income. We also obtain the demand function for agricultural goods as $C_A = (1-\mu)I$ from equation (1).

2.2 Production

In the agricultural sector, we assume that a unit of labor is necessary for a unit of production. Since we treat the agricultural good as a numeraire, the wage level becomes unity in both the countries in the equilibrium.

In the manufacturing sector, labor and capital are required for production. In our model, we consider technology such that labor is used for the marginal requirement, while capital is used for the fixed requirement. Let $B(n)$ and $F(n)$ be the marginal labor requirement and the fixed capital requirement to produce q_i. $B(n)$ and $F(n)$ are given by

$$B(n) \equiv \left(1 - \frac{1}{\sigma}\right) n^{-\beta}, \quad \beta > 0, \tag{5}$$

$$F(n) = n^{-\gamma}, \quad 0 < \gamma < 1, \tag{6}$$

where β and γ are paremeters expressing external economies of scale on the marginal requirement and the fixed requirement, respectively. Note that an increase in the number of varieties, n, reduces $B(n)$ and $F(n)$.

3 Short-run Equilibrium

Before analyzing industrial location and economic welfare, let us consider the short-run equilibrium without international capital movement in order to clarify the working of the model, in particular, the mechanism for the rental rate to be determined. We focus on the situation where the amount of capital employed in each country is fixed at this moment. In the following analysis, we confine our attention to the situation where both the countries produce agricultural goods and open their goods markets.[1]

Let K and K^* be the amount of capital employed in the home and foreign countries, respectively. Note that K and K^* do not correspond to the capital owned by the home and foreign countries. From equation (6), we obtain the equilibrium number of varieties in the short-run as

$$n = K^{\frac{1}{1-\gamma}} \quad \text{and} \quad n^* = K^{*\frac{1}{1-\gamma}}. \tag{7}$$

Using equations (5) and (6), the profit for the firm producing variety i is organized as

$$\pi_i = p_i q_i - wB(n)q_i - rF(n) = p_i q_i - B(n)q_i - rF(n), \tag{8}$$

where q_i is the output for variety i, and w and r are the wage and rental rates, respectively, in the home country. From equation (8), the first-order condition for profit maximization is

$$p_i\left(1 - \frac{1}{\sigma}\right) - B(n) = 0. \tag{9}$$

Since free entry and exit are allowed, firms' profits are zero. Setting equation (8) to be zero, we have

$$rF(n) = \frac{1}{\sigma} p_i q_i. \tag{10}$$

Using equations (5), (7), and (9), we have

$$p_i = K^{-\frac{\beta}{1-\gamma}} \quad \text{and} \quad p_{i^*} = K^{*-\frac{\beta}{1-\gamma}}. \tag{11}$$

From equations (6), (7), and (11), the zero-profit condition yields

$$q_i = \sigma K^{\frac{\beta-\gamma}{1-\gamma}} r \quad \text{and} \quad q_{i^*} = \sigma K^{*\frac{\beta-\gamma}{1-\gamma}} r^*. \tag{12}$$

[1]Rigidly, the agricultural good is produced in both the countries if $\frac{\mu(\sigma-1)}{2(\sigma-\mu)} < 1$ is satisfied.

From demand functions (2) and (3), the world demand for variety i, d_i, is expressed as follows:

$$d_i = c_i + \tau c_i^* = \mu p_i^{-\sigma} \left(G^{\sigma-1} I + \tau^{1-\sigma} G^{*\sigma-1} I^* \right). \tag{13}$$

Now, let us introduce a parameter $\theta \equiv \frac{(\sigma-1)\beta+\gamma}{1-\gamma}$ in order to express the degree of external economies of scale. Notice that $\frac{\partial\theta}{\partial\beta} > 0$ and $\frac{\partial\theta}{\partial\gamma} > 0$ hold. Using the parameter θ and substituting equation (11) into equation (4), we have the price indices in the home and foreign countries as

$$G = (K^{1+\theta} + \tau^{1-\sigma} K^{*1+\theta})^{\frac{1}{1-\sigma}}, \tag{14}$$

$$G^* = (\tau^{1-\sigma} K^{1+\theta} + K^{*1+\theta})^{\frac{1}{1-\sigma}}. \tag{15}$$

Then, substituting equations (11), (14), and (15) into equation (13), the demands for varieties i and i^* are expressed by

$$d_i = \mu K^{\frac{\beta\sigma}{1-\gamma}} \left(\frac{I}{K^{1+\theta} + \tau^{1-\sigma} K^{*1+\theta}} + \frac{\tau^{1-\sigma} I^*}{\tau^{1-\sigma} K^{1+\theta} + K^{*1+\theta}} \right), \tag{16}$$

$$d_i^* = \mu K^{*\frac{\beta\sigma}{1-\gamma}} \left(\frac{\tau^{1-\sigma} I}{K^{1+\theta} + \tau^{1-\sigma} K^{*1+\theta}} + \frac{I^*}{\tau^{1-\sigma} K^{1+\theta} + K^{*1+\theta}} \right). \tag{17}$$

We now focus on consumers' income. From equation (10), the world capital income $rK + r^*K^*$ is equivalent to $\frac{1}{\sigma}$ of the value of the manufacturing aggregates. Since the total consumption of the manufacturing goods is given by $\mu(I + I^*)$, the following relation holds in the equilibrium:

$$rK + r^*K^* = \frac{\mu}{\sigma}(I + I^*). \tag{18}$$

Total income consists of labor income and capital income, *i.e.*, $I + I^* = wL + w^*L^* + rK + r^*K^*$. Then, using equation (18), we have

$$I + I^* = \frac{\sigma(L + L^*)}{\sigma - \mu}. \tag{19}$$

Substituting equation (19) back into equation (18), the total capital income becomes a function of the labor income:

$$rK + r^*K^* = \frac{\mu(L + L^*)}{\sigma - \mu}. \tag{20}$$

Let λ be the share of the home capital. Thus, $K = \lambda\bar{K}$ and $K^* = (1-\lambda)\bar{K}$. Using equation (18), the capital income in each country is then expressed as

$$r\lambda\frac{\bar{K}}{2} + r^*(1-\lambda)\frac{\bar{K}}{2} = \frac{\mu(L + L^*)}{2(\sigma - \mu)}. \tag{21}$$

From equation (21), we obtain the home and foreign incomes as follows:

$$I = L + \frac{\mu(L + L^*)}{2(\sigma - \mu)} \quad \text{and} \quad I^* = L^* + \frac{\mu(L + L^*)}{2(\sigma - \mu)}. \tag{22}$$

Since the two countries are symmetric, $L = L^*$. Therefore, equation (22) shows that $I = I^* = \mu L/(\sigma - \mu)$.

Finally, we derive the equilibrium rental rates. From equations (12), (16), and (17), we have the equilibrium rental rates as

$$r = \frac{\mu\rho}{\sigma}\lambda^\theta\left(\frac{1}{\lambda^{1+\theta} + \tau^{1-\sigma}(1-\lambda)^{1+\theta}} + \frac{\tau^{1-\sigma}}{\tau^{1-\sigma}\lambda^{1+\theta} + (1-\lambda)^{1+\theta}}\right), \tag{23}$$

$$r^* = \frac{\mu\rho}{\sigma}(1-\lambda)^\theta\left(\frac{\tau^{1-\sigma}}{\lambda^{1+\theta} + \tau^{1-\sigma}(1-\lambda)^{1+\theta}} + \frac{1}{\tau^{1-\sigma}\lambda^{1+\theta} + (1-\lambda)^{1+\theta}}\right), \tag{24}$$

where $\rho \equiv I/\bar{K}$.

Now, let us clarify the agglomerative and dispersive forces in our model. We focus on two features of the model: external economies of scale and market-crowding effect.

(i) External economies of scale

An increase in the variety through capital inflow leads to a decline in the marginal labor requirement, $B(n)$, and the fixed capital requirement, $F(n)$, in the manufacturing sector. Such declines affect the levels of the equilibrium rental rates derived in equations (23) and (24). From equation (10) and the market equilibrium condition $q_i = d_i$, we have

$$r = \frac{p_i d_i}{\sigma F(n)}. \tag{25}$$

Then, a decrease in $F(n)$ raises the rental rate, supposing that price and demand are fixed. Further, a decrease in $B(n)$ raises the numerator in equation (25) and thus raises the rental rate. From equation (9), a decrease in $B(n)$ lowers p_i. However, since the price elasticity of demand σ is greater than one, the decline in price increases the revenue $p_i d_i$. A decrease in $B(n)$ thus raises the rental rate.

In sum, the rental rate becomes higher in a country as more capital is employed (*i.e.*, λ is greater) for given price indices.[2] Therefore, the external economies of scale works as an agglomerative force.

(ii) Market-crowding effect

Since transport cost works as a trade barrier, competition is partly localized. When a firm moves from the foreign country to the home country through the movement of capital, competition in home country becomes severe, while that in the foreign country becomes less severe. Transport cost reduces this anti-competitive effect in the foreign country, and the anti-competitive effect in the foreign country does not perfectly offset the pro-competitive effect in the home country. Then, capital flow from the foreign country to the home country shifts the home demand function for each variety downward through the increase in the number of varieties in the home

[2] The effect through the external economies of scale corresponds to λ^θ and $(1-\lambda)^\theta$ in equations (23) and (24).

country.[3]

Without external economies of scale, such a downward-shift in demand lowers the relative rental rate.[4] Therefore, the market-crowding effect works as a dispersive force.

4 Long-run Equilibrium

In this section, we analyze the long-run equilibrium entailing international capital movement. In particular, we focus on the effect of a reduction in trade cost on industrial location and welfare in the long-run equilibrium.

Capital moves to a country which offers a higher reward. With respect to the movement of capital, we consider the following adjustment process:

$$\dot{\lambda} = \Lambda(r - r^*), \tag{26}$$

with $\Lambda(0) = 0$ and $\Lambda'(\cdot) > 0$.

4.1 Industrial location

First, we focus on industrial location. Since the two countries have common factors of production, a symmetric equilibrium must be obtained. Thus, the share of capital in each country is one-half. If some changes occur in the countries, for example, if some FDI-attracting policies are implimented in one of the countries, will this symmetric equilibrium remain stable? Examining the stability of the equilibrium, we can conjecture about the pattern of industrial location.

[3]This is also called the "local-competition effect." For details, see Baldwin, et al. (2003).

[4]This effect is expressed as τ in the numerator in equations (23) and (24).

Under adjustment process (26), we focus on the difference between rental rates in the home and foreign countries. Equations (23) and (24) derive

$$r - r^* = \frac{\mu\rho}{\sigma}\left(\frac{\lambda^\theta - \tau^{1-\sigma}(1-\lambda)^\theta}{\lambda^{1+\theta} + \tau^{1-\sigma}(1-\lambda)^{1+\theta}} + \frac{\tau^{1-\sigma}\lambda^\theta - (1-\lambda)^\theta}{\tau^{1-\sigma}\lambda^{1+\theta} + (1-\lambda)^{1+\theta}}\right). \tag{27}$$

From equation (27), we have the following result on the stability of the symmetric equilibrium.

Proposition 1

If the transport cost τ is greater than the threshold value of the transport cost $\bar{\tau}$, then the symmetric equilibrium is stable, otherwise the symmetric equilibrium is unstable, where

$$\bar{\tau} \equiv \left(\sqrt{1+\theta} - \sqrt{\theta}\right)^{\frac{2}{1-\sigma}}. \tag{28}$$

Proof. *See Appendix A.*

Proposition 1 states that the symmetric equilibrium may or may not be stable, depending on the level of transport cost. This implies that agglomeration occurs in the manufacturing sector as trade liberalization proceeds.

Figure 1 depicts the values of the difference in the rental rate, $r - r^*$, for the share of capital in the home country, λ. Figure 1 (a) is realized if the transport cost τ is higher than the threshold value $\bar{\tau}$, and the symmetric equilibrium (*i.e.*, $\lambda = 0.5$) is stable in this case. In contrast, Figure 1 (b) illustrates the situation where the transport cost τ is lower than $\bar{\tau}$, and the symmetric equilibrium is unstable. As we see from these figures, when τ is lower than $\bar{\tau}$, the manufacturing production is agglomerated to either of the countries; that is, all of the manufacturing productions are in one country (*i.e.*, an industrialized country) and the other country specializes in the agricultural production (*i.e.*, an agricultural country).

Let us explain the reason for the result using the two features of the model stated in section 3. Recall that the external economies of scale work as an agglomerative force, while the market-crowding effect works as a dispersive force. These two effects balance at $\tau = \bar{\tau}$. As the transport cost τ becomes lower, the market-crowding effect becomes smaller. If τ is less than $\bar{\tau}$, the agglomerative force via the external economies of scale outweighs the dispersive force via the market-crowding effect. Therefore, the symmetric equilibrium can be unstable.

4.2 Welfare

On the basis of industrial location, we examine the effects of the reduction in trade cost on welfare. In the following analysis, without loss of generality, we regard the home and foreign countries as the industrialized and agricultural countries, respectively.

The indirect utility function in the home country is given by $V = \mu^{\mu}(1-\mu)^{1-\mu}G^{-\mu}I$. As shown in equation (22), the home income is independent from λ and τ. Thus, it is sufficient to focus on the change in the price indices in order to clarify the effect of a decline in trade cost on welfare. Further, since we have $K = \lambda\bar{K}$ and $K^* = (1-\lambda)\bar{K}$, equations (14) and (15) are rewritten as

$$G = \left(\bar{K}^{1+\theta}v\right)^{\frac{1}{1-\sigma}} \quad \text{and} \quad G^* = \left(\bar{K}^{1+\theta}v^*\right)^{\frac{1}{1-\sigma}}, \tag{29}$$

where

$$v \equiv \lambda^{1+\theta} + \tau^{1-\sigma}(1-\lambda)^{1+\theta} \quad \text{and} \tag{30}$$

$$v^* \equiv \tau^{1-\sigma}\lambda^{1+\theta} + (1-\lambda)^{1+\theta}. \tag{31}$$

From equation (29), an increase in v leads to a decrease in G, and vice versa. The same mechanism also holds between v^* and G^*. We thus use v and v^* as indices expressing utility in the home and foreign countries, respectively.

From equations (30) and (31), the values of v and v^* at the symmetric equilibrium (*i.e.*, $\lambda = 0.5$) are given by

$$v|_{\lambda=0.5} = v^*|_{\lambda=0.5} = \frac{1+\tau^{1-\sigma}}{2^{1+\theta}}, \tag{32}$$

and those at the agglomeration (*i.e.*, $\lambda = 1$) are given by

$$v|_{\lambda=1} = 1 \quad \text{and} \tag{33}$$

$$v^*|_{\lambda=1} = \tau^{1-\sigma}. \tag{34}$$

From (33) and (34), we find that welfare in the industrialized country is unambiguously higher than that in the agricultural country. This is because the agricultural country must incur trade cost for all manufacturing products, and purchase them at a higher price than the industrialized country. Comparing equation (32) with equations (33) and (34), we have the following results.

Proposition 2

(i) Agglomeration unambiguously increases welfare in the industrialized country.

(ii) If $\theta < \bar{\theta}$ holds and τ is in the neighborhood of $\bar{\tau}$, then agglomeration decreases welfare in the agricultural country, while if $\theta \geqslant \bar{\theta}$ holds, then agglomeration increases welfare in the agricultural country, where $\bar{\theta}$ is the threshold degree of external economies of scale such that welfare in the agricultural country is unchanged before and after agglomeration.

Proof. *See Appendix B.*

Proposition 2 shows that external economies of scale is critical for the welfare effect of the reduction in trade cost. In particular, if external economies of scale are enough large, agglomeration makes both countries better off.

Figure 2 depicts the relationship between each country's utility level and transport cost. Figure 2 (a) illustrates the case for a small θ, while Figure 2 (b) illustrates the case for a large θ. Curves DE, AB, and AC express equations (32), (33), and (34), respectively. If $\tau > \bar{\tau}$, both countries' utility levels are shown by DE, and if $\tau \leqslant \bar{\tau}$, the industrialized home country's utility level is shown by AB, and the agricultural foreign country's utility level is shown by AC.

As the figure clearly indicates, the home utility level, and thus home welfare, is increased by agglomeration. Compared with the home country, foreign welfare is more complex. In the case of Figure 2 (a), the foreign country lowers its own utility level by specializing in agricultural production. In contrast, in the case of Figure 2 (b), the foreign country raises its own utility level by specializing in agricultural production.

The intuition behind the results is as follows. Agglomeration of manufacturing production lowers the price of the manufacturing goods and increases the number of varieties. This is a positive effect for both the countries, and, in particular, the manufacturing country enjoys the fruits of agglomeration. For the agricultural country, agglomeration provides another effect – the burden of transport cost. This is disadvantageous to the agricutural country, because it imports all manufacturing goods. Recall that θ is the parameter expressing the degree of external economies of scale, as stated in section 3. If θ is small, the negative effect from transport cost outweighs the positive effect from agglomeration, and vice versa. Therefore, if the degree of external economies of scale is lower (*resp.* higher), agglomeration deteriorates (*resp.* ameliorates) foreign utility and welfare.

Finally, we examine world welfare. We assume that world welfare is expressed as the Bentham type, *i.e.*, the sum of each country's welfare. Using equation (29), world welfare is then given by

$$V + V^* = \mu^\mu (1-\mu)^{1-\mu} \bar{K}^{-\frac{\mu(1+\theta)}{\sigma-1}} \left(v^{\frac{\mu}{\sigma-1}} + v^{*\frac{\mu}{\sigma-1}} \right) I. \tag{35}$$

As in the case of each country's welfare, we focus on v and v^* in the price indices, because the other parts in equation (35) are fixed under the given parameters. Let us denote the index expressing world welfare as

$$W \equiv v^{\frac{\mu}{\sigma-1}} + v^{*\frac{\mu}{\sigma-1}}. \tag{36}$$

As shown in Proposition 2, in most cases, agglomeration makes both countries better off. Further, agglomeration increases world welfare. In this case, agglomeration is Pareto-improving. The exception is the case where the transport cost is not very low and the degree of external economies of scale is quite low. In this case, the agricultural country becomes worse off by agglomeration. As we have shown in Appendix C, even in this case, agglomeration increases world welfare.

Proposition 3

Agglomeration unambiguously increases world welfare.

Proof. *See Appendix C.*

Proposition 3 shows that agglomeration is potentially Pareto-improving. That is, even if the agricultural country becomes worse off by agglomeration, some adequate income transfer can make both countries better off.

5 Concluding Remarks

We have investigated the effect of a reduction in trade cost on industrial location and welfare in an economy with external economies of scale. We propose an analytically-solvable model concerning industrial location without losing accumulative agglomerative force, and investigate the change in welfare when agglomeration occurs.

With respect to industrial location, we show that a reduction in trade cost is likely to lead to agglomeration. With respect to welfare, we demonstrate that agglomeration makes a country unambiguously better off, whereas a country without agglomeration may become better off or worse off, depending on the degree of external economies of scale. Finally, we find that regardless of whether a country without agglomeration becomes better or worse off, agglomeration unambiguously makes the overall economy better off. This result implies that agglomeration is potentially Pareto-improving.

In this paper, we focus on industrial location and welfare in two symmetric countries. Some asymmetries, such as differences in production costs and/or degree of external economies of scale, provide more realistic and interesting results. We would like to include these factors in our analysis in our future research.

Appendix A: Proof of Proposition 1

We find the value of τ such that the partial derivative of equation (27) with respect to λ is zero, evaluating at the symmetric equilibrium. Partially differentiating equation (27) with respect to λ, we have

$$\frac{\sigma}{\mu\rho}\frac{\partial(r-r^*)}{\partial\lambda} = \theta\left\{\frac{\lambda^{\theta-1}+\tau^{1-\sigma}(1-\lambda)^{\theta-1}}{\lambda^{1+\theta}+\tau^{1-\sigma}(1-\lambda)^{1+\theta}} + \frac{\tau^{1-\sigma}\lambda^{\theta-1}+(1-\lambda)^{\theta-1}}{\tau^{1-\sigma}\lambda^{1+\theta}+(1-\lambda)^{1+\theta}}\right\}$$
$$-(1+\theta)\left\{\left(\frac{\lambda^{\theta}-\tau^{1-\sigma}(1-\lambda)^{\theta}}{\lambda^{1+\theta}+\tau^{1-\sigma}(1-\lambda)^{1+\theta}}\right)^2 + \left(\frac{\tau^{1-\sigma}\lambda^{\theta}-(1-\lambda)^{\theta}}{\tau^{1-\sigma}\lambda^{1+\theta}+(1-\lambda)^{1+\theta}}\right)^2\right\}.$$

Evaluating it at the symmetric equilibrium, we have

$$\left.\frac{\sigma}{\mu\rho}\frac{\partial(r-r^*)}{\partial\lambda}\right|_{\lambda=0.5} = 4\left\{\theta-(1+\theta)\left(\frac{1-\tau^{1-\sigma}}{1+\tau^{1-\sigma}}\right)^2\right\}.$$

It follows that

$$\left.\frac{\sigma}{\mu\rho}\frac{\partial(r-r^*)}{\partial\lambda}\right|_{\lambda=0.5} \lesseqgtr 0 \quad\Leftrightarrow\quad \bar{\tau} \equiv \left(\sqrt{\theta}-\sqrt{1+\theta}\right)^{\frac{2}{1-\sigma}} \lesseqgtr \tau.$$

Then, if the transport cost τ goes below $\bar{\tau}$, the symmetric equilibrium becomes unstable, and, finally, manufacturing production is concentrated in the either of countries. ■

Appendix B: Proof of Proposition 2

Statement (i) on home utility is straightforward from equations (32) and (33) because $\tau > 1$ and $\sigma > 1$.

We now prove statement (ii) on foreign utility. As shown in Proposition 1, agglomeration (*resp.* dispersion) occurs if $\tau \leqslant \bar{\tau}$ (*resp.* $\tau > \bar{\tau}$).

Let v_0^* and v_1^* be the foreign utility levels under dipersion and agglomeration, respectively. Note that v_0^* and v_1^* are given by equations (32) and (34). We consider the difference between these utilities, $v_0^* - v_1^*$. Define $\tilde{\tau}$ as the trade cost level such that $v_0^* - v_1^* = 0$. Since $(v_0^* - v_1^*)|_{\tau=1} < 0$ and $\frac{\partial v_0^* - v_1^*}{\partial \tau} > 0$, we find that

$$\tau \lesseqqgtr \tilde{\tau} \quad \Leftrightarrow \quad v_0^* \lesseqqgtr v_1^*.$$

Whether agglomeration increases or decreases foreign utility levels depends on the levels of the transport costs τ and $\tilde{\tau}$. That is,

Case (a): For $\tilde{\tau} < \bar{\tau}$, $v_0^* > v_1^*$ if $\tilde{\tau} < \tau < \bar{\tau}$,

$v_0^* \leqslant v_1^*$ if $\tau \leqslant \tilde{\tau} < \bar{\tau}$, and

Case (b): For $\bar{\tau} \leqslant \tilde{\tau}$, $v_0^* \leqslant v_1^*$.

We thus see that v_0^* is greater than v_1^* only if $\tilde{\tau}$ is less than $\bar{\tau}$ and τ is in the neighborhood of $\bar{\tau}$.

We then look at the relationship between τ and θ. Letting $\phi \equiv \tau^{1-\sigma}$, then

$$\tilde{\tau} \lesseqqgtr \bar{\tau} \quad \Leftrightarrow \quad \tilde{\phi}(\theta) \gtreqqless \bar{\phi}(\theta), \tag{B1}$$

where $\tilde{\phi} \equiv \tilde{\tau}^{1-\sigma}$ and $\bar{\phi} \equiv \bar{\tau}^{1-\sigma}$. Equations (32) and (34) yield

$$\tilde{\phi}(\theta) = \frac{1}{2^{1+\theta} - 1}. \tag{B2}$$

Then, from equations (28) and (B2), we obtain

$$\tilde{\phi}(\theta) \lesseqqgtr \bar{\phi}(\theta) \quad \Leftrightarrow \quad \ln\left(\frac{1+\theta-\sqrt{\theta(1+\theta)}}{1+2\theta-2\sqrt{\theta(1+\theta)}}\right) \lesseqqgtr \theta \ln 2. \tag{B3}$$

Let us define the LHS in (B3) as $\psi(\theta)$, which is a strictly increasing and convex function satisfying $\lim_{\theta\to 0}\psi'(\theta) = \infty$ and $\lim_{\theta\to\infty}\psi'(\theta) = 0$. In this case, there exists a unique $\bar{\theta}$ satisfying (B3) with equality, and the following relationship holds:

$$\psi(\theta) \gtreqless \theta \ln 2 \quad \Leftrightarrow \quad \bar{\theta} \gtreqless \theta. \tag{B4}$$

From (B1) and (B4) , we obtain

$$\tilde{\tau} \lesseqgtr \bar{\tau} \quad \Leftrightarrow \quad \tilde{\phi}(\theta) \gtreqless \bar{\phi}(\theta) \quad \Leftrightarrow \quad \bar{\theta} \gtreqless \theta.$$

Therefore, Case (a) holds if $\theta < \bar{\theta}$ and Case (b) holds if $\bar{\theta} \leqslant \theta$. ■

Appendix C: Proof of Proposition 3

Let W_S and W_A be the level of W, which is given by equation (36), at the symmetric equilibrium (*i.e.*, $\lambda = 0.5$) and agglomaration (*i.e.*, $\lambda = 1$) evaluated at $\tau = \bar{\tau}$, respectively. Substituting equations (32), (33), and (34) into (36), we obtain the following values:

$$W_S(\theta) = 2\left(\frac{1+\bar{\phi}}{2^{1+\theta}}\right)^{\epsilon} \quad \text{and} \quad W_A(\theta) = 1 + \bar{\phi}^{\epsilon}, \tag{C1}$$

where $\epsilon \equiv \frac{\mu}{\sigma-1}$ and $\bar{\phi} \equiv \bar{\tau}^{1-\sigma} = \left(\sqrt{1+\theta} - \sqrt{\theta}\right)^2$ (see Appendix B). We define $J(\theta)$ as the difference between the indices of world welfare, *i.e.*,

$$J(\theta) \equiv W_A(\theta) - W_S(\theta). \tag{C2}$$

We need to prove that $J(\theta) > 0$ for $\theta < \bar{\theta}$. It is straightforward to see that $J(0) = 0$. Furthermore, differentiating (C2), we obtain

$$
\begin{aligned}
J'(\theta) &= 2\epsilon \left(\frac{1+\bar{\phi}}{2^{1+\theta}} \right)^{\epsilon} \ln 2 - \left\{ \bar{\phi}^{\epsilon-1} - \left(\frac{1+\bar{\phi}}{2^{1+\theta}} \right)^{\epsilon} \frac{2}{1+\bar{\phi}} \right\} \frac{\epsilon \bar{\phi}}{\sqrt{\theta(1+\theta)}} \\
&> 2\epsilon \bar{\phi}^{\epsilon} \ln 2 - \left(\bar{\phi}^{\epsilon-1} - \bar{\phi}^{\epsilon} \frac{2}{1+\bar{\phi}} \right) \frac{\epsilon \bar{\phi}}{\sqrt{\theta(1+\theta)}} \\
&= 2\epsilon \bar{\phi}^{\epsilon} \left\{ \ln 2 - \frac{1}{2(1+\bar{\phi})} \right\} > 0.
\end{aligned}
$$

Therefore, $J(\theta) > 0$ for $\theta < \bar{\theta}$. That is, agglomeration makes both countries better off. ■

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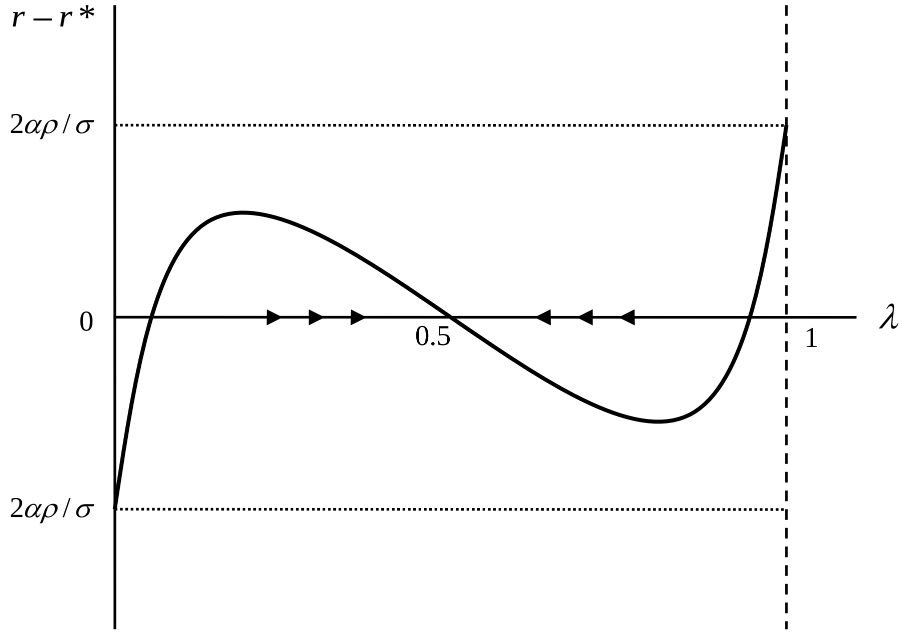

Figure 1 (a) : the case of $\tau > \overline{\tau}$

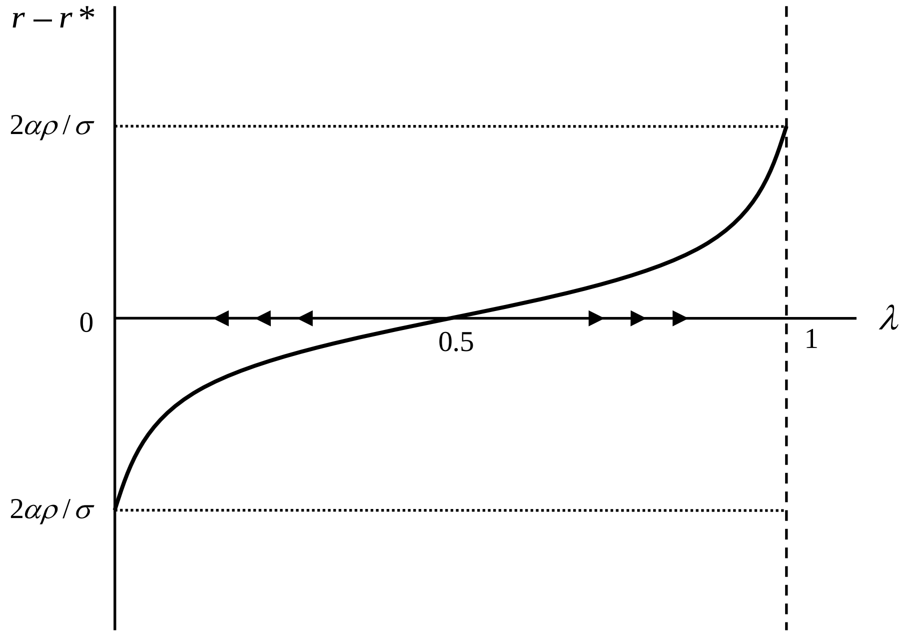

Figure 1 (b) : the case of $\tau \le \overline{\tau}$

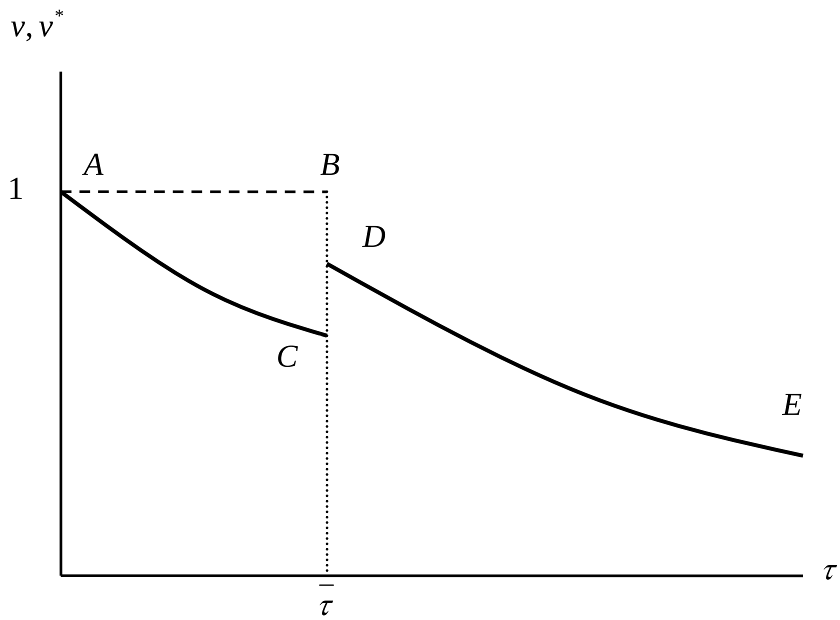

Figure 2 (a) : the case of $\theta < \bar{\theta}$

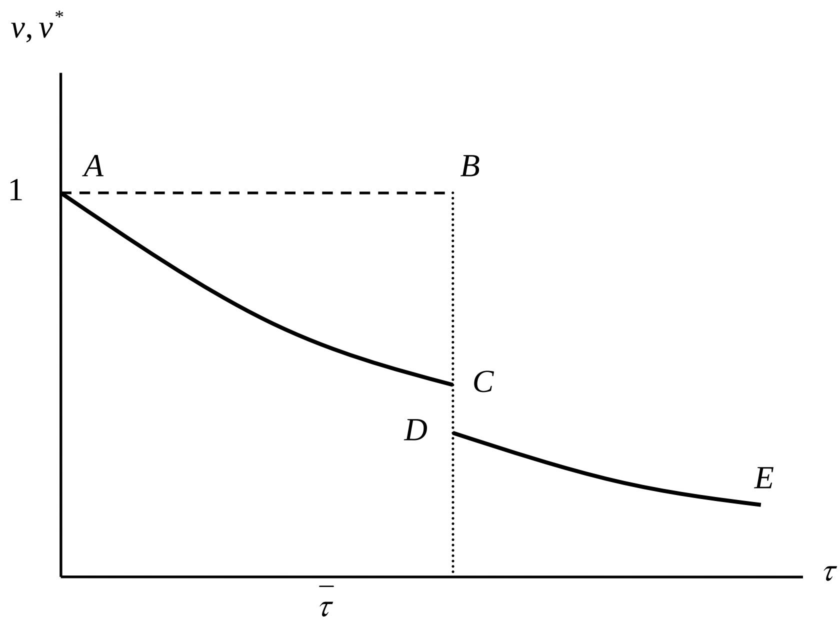

Figure 2 (b) : the case of $\bar{\theta} \leq \theta$