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Unconventional Surface Impedance of a Normal-Metal Film Covering a Spin-Triplet Superconductor Due to Odd-Frequency Cooper Pairs

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We discuss the dynamic response of odd-frequency Cooper pairs to an electromagnetic field. By using the quasiclassical Green function method, we calculate the impedance \(Z = R - iX\) of a normal-metal thin film which covers a superconductor. In contrast with the standard relation (i.e., \(R \ll X\)), the impedance in spin-triplet proximity structures shows anomalous behavior (i.e., \(R > X\)) in the low frequency limit. This unusual relation is a result of the penetration of odd-frequency pairs into the normal metal and reflects the negative Cooper pair density.

Surface impedance \(Z = R - iX\) reflects the dynamic response of Cooper pairs to an electromagnetic field [18, 19]. The surface resistance, \(R\), corresponds to resistance due to normal electrons. In conventional superconductors, \(R\) drops exponentially with decreasing temperature below superconducting transition temperature \(T_c\). The reactance, \(X\), represents power loss of the electromagnetic field due to Cooper pairs. Typical temperature dependence of the surface impedance in a conventional superconductor is shown in Fig. 1(a). The positive amplitude of the Cooper pair density guarantees a robust relation \(R \ll X\) at low temperatures. The validity of the relation \(R < X\), however, is questionable for odd-\(\omega\) Cooper pairs because the odd-\(\omega\) symmetry and negative pair density are inseparable from each other according to the standard theory of superconductivity [20]. In the present Letter, we conclude that the unusual relation \(R > X\) can be observed in the presence of odd-\(\omega\) Cooper pairs at low frequency and at low temperature.

Let us consider a bilayer of a superconductor and a thin normal-metal film as shown in Fig. 1(b), where \(W\) is the...
size of the superconductor and $L$ is the thickness of the normal metal. We mainly consider spin-triplet $p$-wave symmetry in the superconductor and compare results with those in spin-singlet $s$- and $d$-wave symmetries. In all cases, the superconductor belongs to the conventional even-frequency symmetry class. The even- and odd-frequency pairs penetrate into the normal metal for the spin-singlet and spin-triplet cases, respectively [13,14,21]. We first solve the quasiclassical Usadel equation [22] in the standard $\theta$ parametrization [23], $\hbar D \nabla^2 \Psi(r, e) + 2ie e \sin \theta(r, e) = 0$, where $D$ is the diffusion constant of the normal metal and $e$ is the quasiparticle energy measured from the Fermi level. In what follows, we consider the normal-metal/superconductor (NS) junction in two dimensions as shown in Fig. 1(b). We choose $W \gg L$ so that we can neglect $y$ dependence of $\theta$. In the $p$-wave case, we assume that a spin-triplet Cooper pair consists of two electrons with opposite spin directions—this assumption does not break the generality of the following argument. The Usadel equation is supplemented by the boundary condition at the NS interface which depends on the pairing symmetry of the superconductor [13,24,25],

$$\frac{\partial_i \theta(x, e)}{\theta_0} = (R_D/R_B)((F/L)T_R)$$

where $\gamma$ is the incident angle of a quasiparticle measured from the $x$ axis, $\theta_0 = \theta(x = 0, e)$, and $R_D$ is the resistance of the normal metal. At the NS interface, a potential barrier described by $\nu_0 \delta(x)$ suppresses the transmission probability $T_R = \int_{-\pi/2}^{\pi/2} dy \gamma \cos \theta_0 - g_z \sin \theta_0) /

\langle F \rangle = \int_{-\pi/2}^{\pi/2} dy \gamma \cos \theta_0 - g_z \sin \theta_0) \frac{T_N \cos \gamma(f_s \cos \theta_0 - g_z \sin \theta_0)}{(2 - T_N) + T_N(g_s \cos \theta_0 + f_s \sin \theta_0)}

\frac{\sigma_1(x, e)}{\sigma_0} = \frac{1}{2\hbar \omega} \int_{-\infty}^{\infty} d[e][J(e + \hbar \omega) - J(e)]K_1,$

(2)

\frac{\sigma_2(x, e)}{\sigma_0} = \frac{1}{2\hbar \omega} \int_{-\infty}^{\infty} d[e][J(e + \hbar \omega)K_2 + J(e)K_3],

(3)

\begin{align*}
K_1 &= f_s(f_s + g_s) + g_s(f_s)g_s(e + \hbar \omega),
K_2 &= f_R(f_R + g_r) - g_s(f_s + g_s)g_s(e + \hbar \omega),
K_3 &= g_R(f_s + g_s) - g_s(f_s + g_s)g_s(e + \hbar \omega),
\end{align*}

(4)

(5)

with $J(e) = \tanh(\epsilon/2k_B T)$, $g_s(e) = \text{Re}[g(x, e)]$, $g_R(e) = \text{Im}[g(x, e)]$, $f_s(e) = \text{Re}[f(x, e)]$, and $f_R(e) = \text{Im}[f(x, e)]$. The local impedance in the normal metal is calculated from the complex conductivity as

$$Z_N(x, \omega) = R_N - iX_N = (1 - i) \sqrt{\frac{\hbar \omega}{\sigma_0} \frac{\sigma_0}{\sigma_0}} Z_0,$$

(7)

where $Z_0 = \sqrt{2\pi \Delta_0/\sigma_0 e^2 \hbar}$, $\Delta_0$ is the amplitude of pair potential at $T = 0$, and $\sigma_0$ is the Drude conductivity in the normal metal. The surface impedance in Eq. (1) is calculated from the above formula, taking the Green function of a uniform $s$-wave superconductor.

The theory basically includes three junction parameters: the thickness of the normal metal $L$, strength of the potential barrier $z_0$, and the diffusion constant in the normal metal $D$, which are converted to $R_D/R_B$, $E_{Th} = \hbar D/L^2$, and $z_0$ with $E_{Th}$ being the Thouless energy. As we discuss later on, the results shown below depend on these parameters only quantitatively. Thus we fix them at $R_D/R_B = 1$, $E_{Th} = 0.5\Delta_0$, and $z_0 = 2.5$ throughout this Letter, which leads to $T_B = 0.1$ and $L = 2.7\xi_T$ with $\xi_T = \sqrt{\hbar D/2\pi k_B T_c}$ being the coherence length. We add a small positive imaginary part $\delta e$ to energy to provide the retarded causality and to obtain reasonable convergence of the integration in Eqs. (2) and (3). In real junctions, $\delta e$ stems from inelastic scatterings of quasiparticles. We fix $\delta e$ at 0.01$\Delta_0$.

First, we compare the retarded Green functions at the surface of the normal metal ($x = -L$) for the $s$-wave symmetry in Figs. 2(a) and 2(b) with those for the chiral $p$ wave in (d) and (e). The real part of the normal Green function $g_R(e)$ is the quasiparticle density of states normalized by the normal density of states at the Fermi level. The normal Green function always satisfies $g(-e) = [g(e)]^*$ irrespective of pairing symmetry as shown in (a) and (d). In the $s$-wave case, $g_R(e)$ shows a minigap for $|e| < \Delta_0 - 0.15\Delta_0$. In the chiral $p$-wave case, $g_R$ in (d) shows a large peak around $e = 0$, which can be understood
As the penetration of midgap Andreev resonant states [27] into the normal metal. The penetration of such states into a normal metal is possible only for spin-triplet junctions [13,15]. The feature of the anomalous function $f_R(e)$ depends on the pairing symmetry of the superconductor. The results in (b) show that $f_R(e)$ is an even function of $e$ and $f_I(e)$ is an odd function of $e$ (i.e., $f(-e) = [f(e)]^*$). This relation means the presence of the even-frequency Cooper pairs in the normal metal and always holds for spin-singlet NS structures. The results in (e), however, show $f(-e) = -[f(e)]^*$ [21]. This relation represents the penetration of odd-frequency Cooper pairs into the normal metal and holds for spin-triplet NS structures.

Second, we show the local impedance at the surface of the normal metal $Z_N(-L, \omega)$ as a function of temperature at a small $\hbar \omega$ fixed at 0.01$\Delta_0$ in Fig. 3. In the $s$-wave (a) and $d_{x^2-y^2}$ (b) symmetry, the results show conventional behavior totally consistent with those in Fig. 1(a). Namely, $R_N$ and $X_N$ satisfy the relation $R_N \ll X_N$. For $d_{xy}$ and $p_z$ symmetries, a relation $R = X$ always holds irrespective of $\omega$ and $T$ (results are not shown). This reflects the skin effect of the normal metal because there is no proximity effect in these symmetries [25,28]. In the chiral $p$-wave symmetry in (c), on the other hand, $R_N$ becomes larger than $X_N$ for $T < T_c$. A similar behavior can be seen also in the $p_\sigma$-wave symmetry for $T < T^* \sim 0.4T_c$ as shown in (d). The impedance clearly exhibits different characteristic behaviors depending on the pairing symmetry of the superconductor. In Figs. (c) and (d), $\sigma_2$ changes its sign to negative. It is easy to confirm that the sign change of $\sigma_2$ results in $R_N > X_N$ in Eq. (7). Thus $R_N > X_N$ is a robust and universal property of odd-frequency Cooper pairs.

To understand the characteristic feature of complex conductivity at small $\omega$, we analyze the spectral pair density defined by $K_s(e) = f_R(e)f_I(e) - g_R(e)g_I(e) = \text{Im} f^2(e)$ which appears in the integrand of $\sigma_2$ in Eq. (3) at very small $\omega$. The spectral pair density contains full information about the frequency symmetry of Cooper pairs. In the $s$-wave case, $K_s$ in Fig. 2(c) is mostly positive for $e > 0$ and has a positive peak around $e = \Delta_0$. Since $K_s$ is an odd function of $e$, we only show the results for $e < 0$. On the other hand in the chiral $p$-wave symmetry in Fig. 2(f), $K_s$ is mostly negative for $e > 0$ and has a large negative peak near $e = 0$ due to the odd-frequency symmetry. The Cooper pair density in the normal metal $n_i = \int_{-\infty}^{\infty} d\epsilon f_I(\epsilon)K_s(\epsilon)$ and $\sigma_2$ are positive for the $s$-wave case. On the other hand, they are negative for the chiral $p$-wave case. Therefore we conclude that the negative $\sigma_2$ is the direct consequence of the odd-frequency symmetry of Cooper pairs. This argument generally holds when $\hbar \omega$ is smaller than all energy scales relating to the proximity effect. The smallest energy scale at the present calculation is the peak width of the Green function shown in Fig. 2. We have confirmed that $T^*$ in Fig. 3(d) increases to $T_c$ with decreasing $\omega$. The conclusion remains unchanged under varying the junction parameters $R_B/R_D$, $E_{\text{Th}}$ and $c_0$, since they do not affect the frequency symmetry of Cooper pairs.

Next, we briefly discuss the physics behind the negative $\sigma_2$. The electric current in the normal metal may be described by two contributions: $j = j_N + j_s$ with the normal current $j_N = \sigma_0 E$ and the supercurrent $j_s = -\frac{e^2}{m} A = i\frac{e^2}{m} E$ with $m$ being the mass of electron [29]. The complex conductivity then becomes $\sigma_1 + i\sigma_2 = \sigma_0 + i\frac{e^2}{m} E$. The positive value of $n_i$ is indispensable for explaining the Meissner effect. This argument can be easily confirmed by solving the magnetic field.
with $\nabla \times \mathbf{H} = -\frac{\mu_0}{c} j_S$. In the case of odd-frequency pairs, however, $n_s$ becomes negative, which implies absence of the Meissner effect. This fact also significantly modifies the impedance of a NS bilayer defined by

$$Z_{NS} = R_{NS} - iX_{NS} = \frac{Z_S \cos k_n L - iZ_N \sin k_n L}{Z_N \cos k_n L - iZ_S \sin k_n L},$$

where $Z_S$ is the impedance of the superconductor shown in Fig. 1(a). Since $L$ being comparable to $\xi_T$, the conductivity depends on $x$ only slightly in the normal metal, which enables us to define spatially averaged values: the conductivity $\tilde{\sigma}_N = \int_0^L dx \sigma_N(x)/L$, the impedance $\tilde{Z}_N = -i\sqrt{4\pi i\omega/(c^2 \tilde{\sigma}_N)} = \tilde{R}_N - i\tilde{X}_N$, and the wave number of the electromagnetic field $k_n = \sqrt{4\pi i\omega \tilde{\sigma}_N/c^2}$. The sign change of $\sigma_2$ results in the real value of $k_n$ leading to the absence of the Meissner effect. In Fig. 3, we show the impedance of a NS bilayer for the $s$ wave in (e) and chiral $p$-wave NS junctions in (f), where we assume the resistivity of a normal metal $\rho = \sigma_0^{-1}$ is about $2 \times 10^{-3} \Omega \cdot cm$. In the $s$-wave symmetry, the impedance of the NS bilayer [e]] shows the conventional relation $R_{NS} < X_{NS}$. In the chiral $p$-wave symmetry, on the other hand, $R_{NS}$ goes beyond $X_{NS}$ at low temperature. In addition, the absence of the Meissner effect in the normal metal causes the sign change of $X_{NS}$. Using relations $\tilde{X}_N \ll \tilde{R}_N$ and $Z_S \approx -iX_S$, it is possible to show $X_{NS} \approx 0$ at $\tilde{R}_{NS}/\tilde{X}_S = \tan[\text{Re}(\tilde{k}_n L)]$.

Finally, we briefly discuss realistic structures for observing the unusual impedance. One possible example is a bilayer consisting of a normal metal and spin-triplet chiral $p$-wave superconductor Sr$_2$RuO$_4$ [30]. At the same time, the proposed effect can also be observed in superconductor/ferromagnet structures [3]. An example of this type is a bilayer of Nb/CrO$_2$, where only odd-frequency pairs can penetrate into the half-metallic ferromagnet CrO$_2$ [4–8].

In summary, we have theoretically studied the impedance ($Z = R - iX$) of a bilayer consisting of a normal metal and a superconductor. For spin-singlet superconducting proximity structures, we found the conventional and robust relation $R \ll X$. For spin-triplet ones, $R$ can be larger than $X$ and $X$ may change sign as a function of temperature at low $T$. On the basis of obtained results, we propose a method to analyze the spin symmetry of Cooper pairs in a superconductor and to detect odd-frequency superconductivity.

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[23] In a normal metal, the anomalous Green function \sin \theta corresponds to the $\tilde{\tau}_1 (\tilde{\tau}_2)$ component of Nambu space in spin-triplet (spin-singlet) NS junctions.