Estimation of Engel Curves from Survey Data with Zero Expenditures*

Hikaru Hasegawa†, Kazuhiro Ueda‡ and Kunie Mori§

†*Graduate School of Economics and Business Administration, Hokkaido University, Sapporo, Japan*
(e-mail: hasegawa@econ.hokudai.ac.jp)

‡*Faculty of Economics, Nihon Fukushi University, Okuda, Mihama-cho, Chita-gun, Aichi. 470-3295. Japan*
(e-mail: ueda@n-fukushi.ac.jp)

§*Faculty of Economics, Shimonoseki City University, Shimonoseki, Japan*
(e-mail: mori-k@shimonoseki-cu.ac.jp)

Abstract

The objective of this article is to propose a Bayesian method for estimating a system of Engel functions using survey data that includes zero expenditures. We deal explicitly with the problem of zero expenditures in the model and estimate a system of Engel functions that satisfy the adding-up condition. Furthermore, using MCMC, we estimate unobservable parameters, including consumption of commodities, total consumption and equivalence scale, and use their posterior distributions to calculate inequality measures and total consumption elasticities.

JEL Classification: **C11, D12**
Keywords: Bayesian method, generalised entropy measure, Gini coefficient, Markov chain Monte Carlo (MCMC), Working-Leser Engel curves

I. Introduction

*We are grateful to the Institute for Research on Household Economics for providing the micro data of the Japanese Panel Surveys of Consumption. Further, we appreciate the comments of two anonymous referees, professors Christopher Adam (the editor of this journal) and Noriko Hashimoto, and the participants of the seminars held at the Institute for Research on Household Economics and Doshisha University; these comments have significantly improved the article. The work of the first author was supported in part by a Grant-in-Aid for Scientific Research (No.16530137) from the JSPS, while that of the first and second authors was supported in part by a Grant-in-Aid for Scientific Research (No.14653004) from the MEXT.

Household budget surveys are a useful source of data for the estimation of consumer behaviour. However, when using micro data on household expenditure, we often find that expenditure is recorded as zero for certain commodities during the survey period. There are three possible causes for such zero expenditures.[1] Firstly, certain consumers may prefer not to purchase or consume certain commodities, for example alcohol and tobacco. In this case, consumers are at a corner solution. Secondly, although the commodity is actually consumed, purchases may not be recorded because the purchase interval is longer than the survey period. We refer to this second case as infrequency of purchase (IFP). Thirdly, for some reason, commodity purchases may not be recorded, although the purchases do actually occur. Such misreporting can lead to the recording of zero expenditures. In this article, we only deal with the case of IFP.

Numerous articles have been devoted to solving the problem of zero expenditures generated by IFP: Deaton and Irish(1984), Kay *et al.* (1984), Keen (1986), Blundell and Meghir (1987), Pudney (1989, 1990), Griffiths and Valenzuela (1998), *etc.* Deaton and Irish(1984) present a p-tobit model that extends the tobit specification to model zero expenditures. Recorded data for expenditure on commodities is $1/p$ times consumption during the survey period, where p denotes the ratio of the survey period to the purchase period. This is applicable when goods are consumed during the survey period; however, expenditures are only observed with a probability p because of infrequent purchasing (Deaton and Irish, 1984, p.63). Kay *et al.* (1984) extend the model proposed by Deaton and Irish (1984) by providing a stochastic relationship between expenditure and consumption in a sophisticated manner. Keen (1986) estimates a system of linear Engel functions that satisfy the adding-up condition and derives a consistent estimator based on the instrumental variables method. While the purchasing probabilities are constant parameters in Deaton and Irish (1984), Kay *et al.* (1984) and Keen (1986), Blundell and Meghir (1987) propose a model with probit-type purchasing probabilities. Griffiths and Valenzuela (1998) estimate a system of linear Engel functions and equivalence scales using a Bayesian method, and Pudney (1989, 1990) reviews numerous theoretical aspects associated with zero expenditures.

It is important to note the difference between expenditure and consumption when addressing the problem of zero expenditures. While expenditure is observable, consumption is not. Therefore, we can utilize data only for expenditures, which may include zero expenditures, and not for the true consumption of commodities. However, consumers derive utility from the consumption of commodities, not from expenditure. Thus, we have to introduce a stochastic relationship between expenditure and consumption in the model.

In this article, we propose a Bayesian approach for the estimation of a system of Engel functions using survey data that includes zero expenditures due to IFP. We explicitly introduce the stochastic relationship between expenditure and consumption in our Bayesian model. Using the Bayesian method, Hasegawa and Kozumi (2001) estimate a system of Working-Leser Engel functions that includes measurement errors. In this article, we employ the system used in Hasegawa and Kozumi (2001) and estimate the unobserved total consumption using the Bayesian framework. The posterior distributions of parameters play a central role in the Bayesian analysis; however, in complicated models, there exist

[1] See Keen (1986, p.277).

cases where it is impossible to derive these distributions analytically. However, recent developments in the Markov chain Monte Carlo (MCMC) method permit us to sample unobservable parameters from their posterior distributions.

There are two advantages in our Bayesian approach. Firstly, although both models deal with the zero expenditure problem and satisfy the adding-up condition, the Bayesian approach enables us to estimate not only linear expenditure systems but also more flexible expenditure systems.

Secondly, we can estimate unobserved commodity consumption and total consumption. Thereafter, using unobserved total consumption, we can calculate inequality measures and total consumption elasticities.[2] Since there exists a variation in the size of the households, the demographic aspects of households must be considered when measuring inequalities. Therefore, we introduce an equivalence scale in the estimation of Engel functions and calculate inequality measures based on the total consumption evaluated by this equivalence scale.

Estimation of demand and expenditure models using data with zero expenditures due to IFP are applied for varied purposes. Meghir and Robin (1992) use an infrequency of purchase model and estimate a demand system. Kimhi (1999) shows a model combining the double-hurdle and infrequency of purchase models to estimate household demand for tobacco. Newman *et al.* (2001) choose the double-hurdle and infrequency of purchase model for each meat product appropriately and estimate their expenditure equations. Madden (2000) estimates the expenditure elasticities of various items to calculate the poverty line. These constitute only a part of many empirical studies related to zero expenditures. This indicates that our Bayesian methods are applicable for diverse empirical analyses using data including them. However, it should be noted that zero expenditures on which we focus arise from IFP and not from corner solutions. Wales and Woodland (1983) propose the Kuhn-Tucker approach to estimate a demand system, and Fry *et al.* (2000, 2001) apply a compositional data analysis (CODA) for it, when the data include zero expenditures generated by corner solutions.

The article is organized as follows. In Section 2, we introduce a stochastic relationship between expenditure and consumption that deals with zero expenditures due to IFP and present a Bayesian model for estimating the system of Engel functions. In Section 3, we define the total consumption elasticity, the Gini coefficient and generalized entropy measures. In Section 4, we provide an empirical application of our approach to real data. In Section 5, we present concluding remarks and a few extensions of our approach.

II. The Bayesian model

Expenditure and consumption

Firstly, we introduce the following binary variables D_{hi}:

$$D_{hi} = \begin{cases} 1 & \text{if household } h \text{ purchases good } i \text{ over the interview period} \\ 0 & \text{otherwise} \end{cases}$$

[2]Hasegawa and Kozumi (2003) and Hasegawa *et al.* (2003) apply the MCMC method in order to estimate inequality measures.

for $h = 1, \cdots, H$ and $i = 1, \cdots, M$, where H is the number of households and M is the number of goods. We assume that D_{hi} are Bernoulli *i.i.d.* random variables with $\Pr(D_{hi} = 1) = p_i$, where p_i is the probability that a household will purchase good i during the interview period. Next, we define y_{hi} and c_{hi} as observed expenditure and unobserved consumption for good i and household h, respectively. Following Kay *et al.* (1984) and Keen (1986), we assume that

$$y_{hi} = \begin{cases} \dfrac{1}{p_i}(c_{hi} + v_{hi}) & \text{if } D_{hi} = 1 \\ 0 & \text{if } D_{hi} = 0 \end{cases}, \quad h = 1, \cdots, H, \; i = 1, \cdots, M, \tag{1}$$

where v_{hi} are normally distributed *i.i.d.* disturbances, $v_{hi} \sim \mathrm{N}(0, \omega)$, and D_{hi} and v_{hi} are assumed to be independently distributed. Since we do not have observations for y_{hi} when $D_{hi} = 0$, we introduce the following latent variable y^*_{hi}:[3]

$$y^*_{hi} = \frac{1}{p_i}(c_{hi} + v_{hi}), \quad \text{if } D_{hi} = 0. \tag{2}$$

Using y_{hi} and y^*_{hi}, we define y^{**}_{hi} as follows:

$$y^{**}_{hi} = \begin{cases} y_{hi} & \text{if } D_{hi} = 1 \\ y^*_{hi} & \text{if } D_{hi} = 0. \end{cases} \tag{3}$$

Equations (1), (2) and (3) are summarized as

$$\begin{pmatrix} p_1 & & & \mathbf{O} \\ & \ddots & & \\ & & p_m & \\ \mathbf{O} & & & p_M \end{pmatrix} \begin{pmatrix} y^{**}_{h1} \\ \vdots \\ y^{**}_{hm} \\ y^{**}_{hM} \end{pmatrix} = \begin{pmatrix} c_{h1} \\ \vdots \\ c_{hm} \\ c_{hM} \end{pmatrix} + \begin{pmatrix} v_{h1} \\ \vdots \\ v_{hm} \\ v_{hM} \end{pmatrix},$$

where $m = M - 1$. Or, more compactly,

$$\begin{pmatrix} \boldsymbol{P}_m & \mathbf{0} \\ \mathbf{0}' & p_M \end{pmatrix} \begin{pmatrix} \boldsymbol{y}^{**}_h \\ y^{**}_{hM} \end{pmatrix} = \begin{pmatrix} \boldsymbol{c}_h \\ c_{hM} \end{pmatrix} + \begin{pmatrix} \boldsymbol{v}_h \\ v_{hM} \end{pmatrix}, \tag{4}$$

where

$$\boldsymbol{P}_m = \mathrm{diag}(p_1, \cdots, p_m) \text{ (diagonal matrix)}$$

$$\boldsymbol{y}^{**}_h = \begin{pmatrix} y^{**}_{h1} \\ \vdots \\ y^{**}_{hm} \end{pmatrix}, \; \boldsymbol{c}_h = \begin{pmatrix} c_{h1} \\ \vdots \\ c_{hm} \end{pmatrix}, \; \boldsymbol{v}_h = \begin{pmatrix} v_{h1} \\ \vdots \\ v_{hm} \end{pmatrix}.$$

From the adding-up condition, $x_h = \boldsymbol{\iota}'_m \boldsymbol{c}_h + c_{hM}$ holds, where x_h is the total consumption and $\boldsymbol{\iota}_m = (\underbrace{1, \cdots, 1}_{m})'$. Substituting the adding-up condition into (4), we have

$$\begin{pmatrix} \boldsymbol{P}_m & \mathbf{0} \\ \mathbf{0}' & p_M \end{pmatrix} \begin{pmatrix} \boldsymbol{y}^{**}_h \\ y^{**}_{hM} \end{pmatrix} = \begin{pmatrix} \boldsymbol{I}_m & \mathbf{0} \\ -\boldsymbol{\iota}'_m & 1 \end{pmatrix} \begin{pmatrix} \boldsymbol{c}_h \\ x_h \end{pmatrix} + \begin{pmatrix} \boldsymbol{v}_h \\ v_{hM} \end{pmatrix}, \; h = 1, \cdots, H. \tag{5}$$

[3]See Griffiths and Valenzuela (1998).

Since $v_{hi} \sim \mathrm{N}(0, \omega)$, we have the following joint distribution of $\boldsymbol{y}_h^{**}, y_{hM}^{**}$ in (5):

$$p(\boldsymbol{y}_h^{**}, y_{hM}^{**}|\cdot) \propto \left(\prod_{i=1}^{M} p_i\right) \omega^{-M/2} \exp\left[-\frac{1}{2\omega}\begin{pmatrix}\boldsymbol{v}_h \\ v_{hM}\end{pmatrix}'\begin{pmatrix}\boldsymbol{v}_h \\ v_{hM}\end{pmatrix}\right],\ h = 1, \cdots, H.$$

Bayesian model for Engel curves

We consider the following Working-Leser Engel curves:[4]

$$\boldsymbol{w}_h = \boldsymbol{\alpha} + \boldsymbol{\beta} \log x_h + \boldsymbol{u}_h,\ \boldsymbol{u}_h \sim \mathrm{N}(\boldsymbol{0}, \boldsymbol{\Sigma}),\ h = 1, \cdots, H, \tag{6}$$

where $\boldsymbol{w}_h = \boldsymbol{c}_h/x_h$ is an $m \times 1$ share vector, $\boldsymbol{\alpha} = (\alpha_1, \cdots, \alpha_m)'$ and $\boldsymbol{\beta} = (\beta_1, \cdots, \beta_m)'$ are parameter vectors, and $\boldsymbol{u}_h$ is an $m \times 1$ normally distributed error vector. Following Lancaster *et al.* (1999, p.459), we introduce an equivalence scale defined as

$$m_h = n_h^a + \boldsymbol{\eta}' \boldsymbol{z}_h, \tag{7}$$

where n_h^a is the number of adults in household h, $\boldsymbol{\eta}$ is a $K \times 1$ scale parameter vector and $\boldsymbol{z}_h$ is a $K \times 1$ vector of demographic variables.[5] Banks and Johnson (1994) indicate the importance of the relativity between the weight of children in a household and inequality. In this specification of the equivalence scale, it is simple to incorporate various demographic differences into the Engel curves. The results of their estimation for various age groups are referred to in 4.3.

Deflating the total consumption in (6) by the equivalence scale (7), we have

$$\boldsymbol{w}_h = \boldsymbol{\alpha} + \boldsymbol{\beta} \log \frac{x_h}{m_h} + \boldsymbol{u}_h,\ \boldsymbol{u}_h \sim \mathrm{N}(\boldsymbol{0}, \boldsymbol{\Sigma}),\ h = 1, \cdots, H. \tag{8}$$

Defining $\boldsymbol{\Gamma} = (\boldsymbol{\alpha}, \boldsymbol{\beta})'$ and $x_h^* = x_h/m_h$, (8) can be written as

$$\boldsymbol{w}_h = \boldsymbol{\Gamma}' \begin{pmatrix} 1 \\ \log x_h^* \end{pmatrix} + \boldsymbol{u}_h,\ h = 1, \cdots, H.$$

For the Bayesian analysis, we consider the following prior information with regard to the hierarchical structure,[6]

$$\begin{cases} \log x_h \sim \mathrm{N}(\mu, \tau),\ \mu \sim \mathrm{N}(\mu_*, \kappa_*),\ \tau^{-1} \sim \mathrm{Gam}(a_*, b_*) \\ \boldsymbol{\gamma} \sim \mathrm{N}(\boldsymbol{\gamma}_*, \boldsymbol{G}_*), \boldsymbol{\Sigma}^{-1} \sim \mathrm{W}(\lambda_*, \boldsymbol{F}_*^{-1}),\ \omega^{-1} \sim \mathrm{Gam}(c_*, d_*) \\ p_i \sim \mathrm{Beta}(g_*, q_*),\ \boldsymbol{\eta} \sim \mathrm{N}(\boldsymbol{\eta}_*, \boldsymbol{A}_*), \end{cases} \tag{9}$$

where $\boldsymbol{\gamma} = \mathrm{vec}\,\boldsymbol{\Gamma}$, $\mathrm{Gam}(a, b)$ denotes a gamma distribution with a shape parameter a and scale parameter b, $\mathrm{W}(a, \boldsymbol{A})$ denotes a Wishart distribution with degrees of freedom a and a scale matrix $\boldsymbol{A}$ and $\mathrm{Beta}(a, b)$ denotes a beta distribution with parameters a and b. The prior distribution of $\boldsymbol{c}_h$ can be derived

[4]See Working (1943) and Leser (1963). We delete a share equation from the system using the adding-up condition. That is $\sum_{i=1}^{M} \alpha_i = 1, \sum_{i=1}^{M} \beta_i = 0$.

[5]See Ray (1983), Lancaster and Ray (1998) and Lancaster *et al.* (1999) for details on this definition of equivalence scale.

[6]These prior distributions are often used in Bayesian analyses. See Hasegawa and Kozumi (2001).

from the system of Engel functions (8). Further, we assume that the prior information of the latent variables y^*_{hi} is noninformative. Given (3), (5), (8) and (9), we can obtain the full conditional distributions (FCDs) of $\boldsymbol{\gamma}$, $\boldsymbol{\Sigma}^{-1}$, μ, τ^{-1}, ω^{-1}, $\boldsymbol{c}_h$ $(h = 1, \cdots, H)$ and y^*_{hi} $(h = 1, \cdots, H,\ i = 1, \cdots, M)$. Using Gibbs sampling, we can easily sample these parameters from their FCDs. However, the closed forms of FCDs for other parameters cannot be derived because the FCDs of x_h $(h = 1, \cdots, H)$, p_i $(i = 1, \cdots, M)$ and $\boldsymbol{\eta}$ are complicated functions. Therefore, we simulate these parameters using the Metropolis-Hastings (M-H) algorithm. The details of sampling algorithms are provided in the Appendix.

III. Inequality measures and total consumption elasticities

We use the expenditure data of households, including zero expenditures, for estimating the Engel functions. Expenditure is observable; however, in general, consumption is unobservable. As we mentioned in the introduction, one of the advantages of our Bayesian approach is that we can estimate unobserved consumption c_{hi} and total consumption x_h based on observable expenditure. The estimated posterior results provide information on the hidden true expenditure and consumption. Using these estimates enables us to calculate inequality measures and total consumption elasticities more accurately.

We use the Gini coefficient and the generalised entropy measure for measuring inequality. The Gini coefficient has played a central role in inequality literature and has many practical advantages (Cowell, 2000, pp.111–112). However, while the Gini coefficient does not satisfy population decomposability (Amiel and Cowell, 1999, p.138), the generalised entropy measure does. Therefore, we use the generalized entropy measure as well as the Gini coefficient.

The Gini coefficient and the generalised entropy measure are defined as follows:

$$I_{Gini} = \frac{1}{2H^2\bar{x}} \sum_{h=1}^{H} \sum_{k=1}^{H} |x_h - x_k| \tag{10}$$

and

$$I_{GE}(\theta) = \frac{1}{\theta^2 - \theta} \left[\frac{1}{H} \sum_{h=1}^{H} \left(\frac{x_h}{\bar{x}} \right)^{\theta} - 1 \right], \tag{11}$$

where θ is a real parameter. The generalised entropy measure is closely related to the Atkinson measure and two types of Theil measures, for $\theta = 0, 1$. We use the generalised entropy measure for cases of $\theta = -1, 0, 1, 2$.

The total consumption elasticity of good i is defined as follows:

$$\left. \frac{\partial \log c_i}{\partial \log x} \right|_{c_i = \bar{c}_i, x = \bar{x}} = 1 + \frac{\beta_i}{\bar{w}_i}, \quad \text{where} \quad \bar{w}_i = \bar{c}_i / \bar{x}. \tag{12}$$

The total consumption elasticities are calculated by using the sample means of x and c_i — $\bar{x}$ and $\bar{c}_i$ — over the individual x_h and c_{hi}.

IV. Application to real data

Data

The data used for our empirical analysis are the micro-level survey data based on the Japanese Panel Survey of Consumers (JPSC) by the Institute for Research on Household Economics.

JPSC data provides details on consumption pertaining to young women in Japan. The survey began in 1993 and young women aged 24 to 34 years were the surveyed respondents. Later in 1997, the women aged 24 to 27 years were also included and the survey is continuing ever since.

In this article, we use Panel 8 of JPSC data, which is based on the survey in 2000. From the panel data we select, for the analysis, data pertaining to those households where there are no other adults except a husband and a wife. The number of households selected is 692. The women are aged between 27 and 41 years, and the average age is 34.1 years. The men are aged between 23 and 61 years, and the average age is 36.7 years.

JPSC data contains the expenditure data of households for the month of September. The average total expenditure of households in that month is 272,000 yen and the standard deviation is 121,000 yen.[7]

We divide the expense items of the data into ten items as follows: 'Food', 'Housing', 'Fuel, light and water charges', 'Furniture and household utensils', 'Clothes and footwear', 'Medical care', 'Transportation and communication', 'Education', 'Reading and recreation', and 'Miscellaneous'. Eight items are created from these ten items: `Food`, `Housing`, `Fuel`, `Furniture`, `Clothing`, `Medical`, `Transport` and `Others`.[8]

In the following empirical analysis, we set the dimension of demographic variable (z_h) $K = 1$ in (7), and use z_h to denote the number of children. We determine the values of hyperparameters for simulation as follows:

$$\begin{aligned}
&\boldsymbol{\gamma} \sim \mathrm{N}(\mathbf{0}, 100 \times \boldsymbol{I}_{2m}),\ \boldsymbol{\Sigma}^{-1} \sim \mathrm{W}(m+1, 20 \times \boldsymbol{I}_m),\ \mu \sim \mathrm{N}(0, 100) \\
&\tau^{-1} \sim \mathrm{Gam}(2, 0.05),\ \omega^{-1} \sim \mathrm{Gam}(2, 0.05),\ p_i \sim \mathrm{Beta}(1.5, 1.5) \\
&\eta \sim \mathrm{N}(0, 100).
\end{aligned}$$

These hyperparameter values correspond to a less informative specification. The MCMC simulation was run for 30,000 iterations and the first 10,000 samples were discarded as a burn-in period. The posterior results obtained thereafter are generated using the Ox version 4.02 (Doornik, 2006).

Posterior results

The empirical results are summarized in Tables 1 to 4. Several aspects of the results are described as follows.

Table 1 presents the number of zero expenditures in each item. The number of zeros in `Housing`, `Furniture`, `Clothing` and `Medical` are significantly higher

[7] The other descriptive statistics of the total expenditure of households are as follows. The median, the highest and the lowest expenditures are 250,000, 1,178,000, and 25,000 yen, respectively.

[8] `Food`='Food', `Housing`='Housing', `Fuel`='Fuel, light and water charges', `Furniture`='Furniture and household utensils', `Clothing`='Clothes and footwear', `Medical`='Medical care', `Transport`='Transportation and communication', and `Others`='Education'+'Reading and recreation'+'Miscellaneous'.

than those in other items. The large number of zeros in `Housing` is a result of ownership of houses. Although the households that own their houses account for a certain ratio of the respondents, the expenditure on `Housing` in the data rules out imputed rent. The large number of zeros in `Furniture`, `Clothing` and `Medical` are due to IFS. There exist seven zeros in `Food` and eleven zeros in `Others`. It is unlikely that there exist households without any expenditure on food or the goods included in `Others` over a certain period; however, it has been observed in certain households. The reason why zeros are recorded in `Food` and `Others` is unclear. However, this may be possible due to the probability that certain respondents of the survey do not maintain their exact household accounts, because JPSC data are collected through a questionnaire.

Table 2 presents the posterior results for parameters. Calculating the ratios of the posterior mean to the posterior standard deviation (mean/sd) of αs and βs, we find that their absolute values are higher than two, with the exception of a few parameters. p_i's values, which denote the probability of purchasing a commodity during the interview period, are lower in `Housing`, `Furniture`, `Clothing` and `Medical` than those in other items. Further, the order of sizes in the probabilities of `Furniture`, `Housing`, `Medical` and `Clothing` corresponds to that of the frequencies of zero expenditures in these items, as shown in Table 1. The posterior mean of η is 0.3695. The value of the equivalence scale for a reference household is 2.0 because the reference household comprises a husband and a wife with no children. Thus, we expect that η lies in the interval $(0, 1)$; indeed, we have $\hat{\eta} = 0.3695 \in (0, 1)$.

Table 3 presents the total consumption elasticities as calculated from the posterior results. This table demonstrates that `Furniture`, `Transport` and `Others` are luxury goods and that `Food`, `Housing`, `Fuel`, `Clothing` and `Medical` are necessary goods. The estimated elasticity of `Clothing` in our study is approximately one, and is lower than in many Japanese empirical studies where the elasticities exceed one.[9] However, according to recent studies, the estimate is concluded to be proper. In fact, Ogawa and Okamura (2001) show that the elasticity of clothing is less than one and that it exhibits a declining trend after 1989 in Japan.

Table 4 presents the posterior results for inequality measures. The values in the columns 'raw', 'p.c.', 'e.s.' and 'mean' denote inequality measures based on total expenditure, per capita total expenditure, total expenditure deflated by the posterior equivalence scale and posterior mean of total consumption deflated by the equivalence scale, respectively. According to this table, the values for 'p.c.' are higher than those for 'raw' and 'e.s.' for each inequality measure. Furthermore, the values of 'e.s.' are higher than those of 'mean'. Thus, the inequality measures calculated using the posterior mean are lower than those using other data. It is known in general that the values of inequality measures based on data that includes measurement errors are higher than those for unobservable data that do not include them.[10] This is consistent with our results and supports our Bayesian method for estimating inequality measures.

Comparisons of posterior results

[9]In Table 9, the estimate of the elasticity exceeds one when the number of goods and age groups increase. This aspect will be referred to in the next section.

[10]See Chakravarty and Eichhorn (1994), and Cowell (2000, p.137).

In this section, we provide the results of two directions of sensitivity analyses. One of them deals with the case in which the number of goods (M) changes. The other treats these cases where the number of demographic variables (K) changes.

Table 5 presents the categories used in the estimated models with $M = 5, 6, 7$ and 8. This table also provides the number of zero expenditures of the items in the models. Table 6 presents the summary for the number of children in our data ($K = 1, 2$ and 3).[11] The case of $K = 1$, described in the previous section, uses the age group of children from 0 to 18 years (η_1)—the right column in Table 6. We divide children into two age groups—children aged 0 to 12 years (η_1) and those aged 13 to 18 years (η_2)—for the case of $K = 2$ and into three age groups—children aged 0 to 6 years (η_1), 7 to 12 years (η_2) and 13 to 18 years (η_3)—for the case of $K = 3$.

Table 7 presents the posterior means of the purchasing probabilities p_i. There are no substantial changes in the values of p_i in the cases of $K = 1, 2$ and 3. The estimated probabilities are also similar to the actual relative frequencies denoted by 'data' in the table. Therefore, the posterior results for the purchasing probabilities p_i are robust.

However, the equivalence scale is significantly influenced by the number of demographic variables and the number of categories in the model. Table 8 presents the posterior means of η_i that are used in the equivalence scale defined in (7). The value of η_1 in the case of $K = 1$ is greater than that in the cases of $K = 2$ and $K = 3$, except for the model with $M = 7$; in addition, the value of η_1 in the case of $K = 2$ is greater than that in the case of $K = 3$. Further, the value of η_1 in the case of $K = 2$ lies in the interval of (η_1, η_2) in the case of $K = 3$ for all M in Table 8. These findings for the values of η_i are consistent with the definitions of age groups. However, in the models with $M = 5, 6$ and 7, the value of η_2 in the case of $K = 2$ is greater than η_3 in the case of $K = 3$. Both η_2 in the case of $K = 2$ and η_3 in the case of $K = 3$ denote the demographic parameters for the group of children aged 13 to 18 years in (7).

Table 9 provides the posterior means of the total consumption elasticities. The estimated elasticities of `Food`, `Fuel`, `Furniture`, `Transportation` and `Others` are almost the same, except for certain cases. The elasticity of `Medical` is greater than one, except for one case. The elasticities of `Housing` and `Clothing` tend to become large as K and/or M increase. As a result, `Housing` and `Clothing` become luxury goods in certain cases. `Housing` does not include imputed rent. Further, the elasticity of `Housing` would become small if imputed rents were included in the data. The ages of women in our data are between 27 and 41 years, as described above. It is highly probable that they have children belonging to the younger age groups in the case of $K = 2$ or $K = 3$, according to Table 6. These households tend to be fashion conscious and highly concerned about their children's clothing. It is conceivable that their consumption of `Clothing` reacts to the movement of their total consumption. This would lead to the conclusion that the elasticity of `Clothing` is greater than one when $K = 3$ for $M = 7$ and $K = 2$ and 3 for $M = 8$. Similar reasons may hold true for these cases where the elasticities of `Medical` and `Housing` are

[11] We obtained the results that the posterior means of η_2 and/or η_3 in the equivalence scale are greater than one, when $K = 2$ and $K = 3$ in the models with $M = 9$ and 10. Since η should lie in the interval (0,1), we omit the posterior results of the models with $M = 9$ and 10.

greater than one.

Table 10 presents the values of inequality measures. The values do not change substantially according to the number of demographic variables (K) and the number of categories (M). Therefore, the posterior means of inequality measures are robust with regard to K and M.

V. Conclusions and extensions

Applying the Bayesian method, we estimated a system of Engel curves for Japanese households using JPSC micro data with zero expenditures and calculated inequality measures.

The Bayesian method enables us to estimate a model that overcomes the zero expenditure problem, satisfies the adding-up condition for a system of flexible Engel curves, introduces an equivalence scale and calculates inequality measures. As far as we know, few studies like ours have ever been attempted since it is not possible to estimate a model with the given conditions using non-Bayesian methods — this includes the maximum likelihood method.

We obtain the appropriate posterior results for the coefficients of a system of Engel functions and inequality measures. The sensitivity analyses — conducted by extending the categories of goods and demographic variables — indicate the robustness of our posterior results. The purchasing probabilities and inequality measures are particularly robust. The scale parameters of the equivalence scale result in appropriate values, although they are influenced by demographic variables. The consumption elasticities of goods are also valid depending on the categories of goods and demographic variables.

Our model can be extended as follows:

1. We can extend the Working-Leser Engel curves to a system of quadratic Engel curves.

2. By adding price variables to the model, we can estimate demand systems in order to use the advantages of panel data.

Firstly, instead of Equation (8), let us consider the following quadratic specification:

$$\boldsymbol{w}_h = \boldsymbol{\alpha} + \boldsymbol{\beta} \log \frac{x_h}{m_h} + \boldsymbol{\delta} \left(\log \frac{x_h}{m_h} \right)^2 + \boldsymbol{u}_h, \ \boldsymbol{u}_h \sim \mathrm{N}(\mathbf{0}, \boldsymbol{\Sigma}), \ h = 1, \cdots, H, \tag{13}$$

where $\boldsymbol{\delta} = (\delta_1, \cdots, \delta_m)'$. For this modification, the Bayesian estimation procedure is similar as in Section 2 and Appendix. The main changes aim to redefine $\boldsymbol{\gamma}$ as follows:[12]

$$\boldsymbol{\gamma} = \mathrm{vec}\, \boldsymbol{\Gamma}, \quad \text{such that} \quad \boldsymbol{\Gamma} = (\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\delta})'.$$

Next, we can extend the Working-Leser Engel curves (8) to the almost ideal demand system (Deaton and Muellbauer, 1980). The almost ideal demand

[12] See Hasegawa and Kozumi (2001).

system, which satisfies the adding-up, homogeneity and symmetry conditions, can be written as follows:

$$w_i = \alpha_i + \sum_{j=1}^{m} \psi_{ij} \log\left(\frac{\pi_j}{\pi_M}\right) + \beta_i \log\left(\frac{x^*}{a^*(\boldsymbol{\pi})}\right), \ i = 1, \cdots, m, \tag{14}$$

where w_i is the budget share of good i, π_i is the price of good i, x is the total expenditure $x^* = x/\pi_M$ and

$$\log a^*(\boldsymbol{\pi}) = \alpha_0 + \sum_{i=1}^{m} \alpha_i \log\left(\frac{\pi_i}{\pi_M}\right) + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} \psi_{ij} \log\left(\frac{\pi_i}{\pi_M}\right) \log\left(\frac{\pi_j}{\pi_M}\right). \tag{15}$$

Using the linear algebra, (14) and (15) can be written as follows:

$$\boldsymbol{w} = \boldsymbol{\alpha} + (\boldsymbol{\pi}' \otimes \boldsymbol{I}_m)\boldsymbol{D}\boldsymbol{\psi} + \boldsymbol{\beta} \log\left(\frac{x^*}{a^*(\boldsymbol{\pi})}\right) \tag{16}$$

$$\log a^*(\boldsymbol{\pi}) = \alpha_0 + \boldsymbol{\alpha}'\boldsymbol{\pi} + \frac{1}{2}(\mathrm{vec}\,\boldsymbol{\pi}\boldsymbol{\pi}')'\boldsymbol{D}\boldsymbol{\psi}, \tag{17}$$

where

$$\boldsymbol{w} = \begin{pmatrix} w_1 \\ \vdots \\ w_m \end{pmatrix}, \ \boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_m \end{pmatrix}, \ \boldsymbol{\beta} = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_m \end{pmatrix}, \ \boldsymbol{\pi} = \begin{pmatrix} \log(\pi_1/\pi_M) \\ \vdots \\ \log(\pi_m/\pi_M) \end{pmatrix},$$

$\boldsymbol{D}$ is a $m^2 \times \frac{1}{2}m(m+1)$ duplication matrix, and $\boldsymbol{\psi}$ is a $\frac{1}{2}m(m+1) \times 1$ vector that is obtained from $\mathrm{vec}\,\boldsymbol{\Psi}$ by eliminating all supradiagonal elements of $\boldsymbol{\Psi} = \begin{pmatrix} \psi_{11} & \cdots & \psi_{1m} \\ \vdots & \ddots & \vdots \\ \psi_{m1} & \cdots & \psi_{mm} \end{pmatrix}$.[13]

By using the demand system (16) and (17), for household h ($h = 1, \cdots, H$) at period t ($t = 1, \cdots, T$), we define the following econometric model:

$$\boldsymbol{w}_{ht} = \boldsymbol{\alpha} + (\boldsymbol{\pi}_t' \otimes \boldsymbol{I}_m)\boldsymbol{D}\boldsymbol{\psi} + \boldsymbol{\beta} \log\left(\frac{x^*_{ht}}{a^*(\boldsymbol{\pi}_t)}\right) + \boldsymbol{u}_{ht}, \tag{18}$$

$$\log a^*(\boldsymbol{\pi}_t) = \boldsymbol{\alpha}'\boldsymbol{\pi}_t + \frac{1}{2}(\mathrm{vec}\,\boldsymbol{\pi}_t\boldsymbol{\pi}_t')'\boldsymbol{D}\boldsymbol{\psi} \tag{19}$$

where $\boldsymbol{w}_{ht}$ is an $m \times 1$ share vector and $\boldsymbol{\pi}_t = \big(\log(\pi_{t1}/\pi_{tM}), \cdots, \log(\pi_{tm}/\pi_{tM})\big)'$, $x^*_{ht} = x_{ht}/(m_{ht}\pi_{tM})$, m_{ht} is an equivalence scale and $\boldsymbol{u}_{ht}$ is a vector of normally distributed disturbances.[14] We can introduce the similar structure for household's expenditure on good i at period t, say y^{**}_{hti} ($h = 1, \cdots, H,\ t = 1, \cdots, T,\ i = 1, \cdots, M$), as described in Section 2. Combining the structure for y^{**}_{hti} with the demand system (18) and (19) and noting that the demand system (18) and (19) are linear in coefficient parameters $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\psi})$ in terms of their FCDs, we can apply the method in this article to the estimation of the demand system with zero expenditures.

[13]For the details of duplication matrix, see Magnus and Neudecker (1999, pp.48–49).

[14]In (19), we have deleted α_0.

Furthermore, from these two extensions, our Bayesian approach can be applied to the quadratic almost ideal demand system (Banks *et al.*, 1997) as well as the almost ideal demand system.

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Appendix A. Sampling algorithms

A.1. Full conditional distributions of parameters

The FCDs of $\boldsymbol{\gamma}$, $\boldsymbol{\Sigma}^{-1}$, μ, τ^{-1}, ω^{-1}, $\boldsymbol{c}_h$ $(h = 1, \cdots, H)$ and y^*_{hi} $(h = 1, \cdots, H$, $i = 1, \cdots, M)$ are as follows:

- **FCD of $\boldsymbol{\gamma}$:**

$$\boldsymbol{\gamma}|\cdots \sim \mathrm{N}(\boldsymbol{\gamma}_{**}, \boldsymbol{G}_{**}) \tag{A.1}$$
$$\boldsymbol{G}_{**} = \left[\boldsymbol{G}_*^{-1} + (\boldsymbol{\Sigma}^{-1} \otimes \boldsymbol{X}'_*\boldsymbol{X}_*)\right]^{-1}$$
$$\boldsymbol{\gamma}_{**} = \boldsymbol{G}_{**}\left[\boldsymbol{G}_*^{-1}\boldsymbol{\gamma}_* + (\boldsymbol{\Sigma}^{-1} \otimes \boldsymbol{I}_2)\,\mathrm{vec}(\boldsymbol{X}'_*\boldsymbol{W})\right],$$

  where '$|\cdots$' denotes conditioning on the values of all other parameters and data,

$$\boldsymbol{X} = \begin{pmatrix} 1 & \log x_1^* \\ \vdots & \vdots \\ 1 & \log x_H^* \end{pmatrix}, \ \boldsymbol{W} = \begin{pmatrix} \boldsymbol{w}'_1 \\ \vdots \\ \boldsymbol{w}'_H \end{pmatrix}$$

  and $\otimes$ denotes a Kronecker product.

- **FCD of $\boldsymbol{\Sigma}^{-1}$:**

$$\boldsymbol{\Sigma}^{-1}|\cdots \sim \mathrm{W}(\lambda_{**}, \boldsymbol{F}_{**}^{-1}) \tag{A.2}$$
$$\lambda_{**} = \lambda_* + H, \ \boldsymbol{F}_{**} = \boldsymbol{F}_* + \sum_{h=1}^{H} \boldsymbol{u}_h\boldsymbol{u}'_h.$$

- **FCD of $\boldsymbol{\mu}$:**

$$\mu|\cdots \sim \mathrm{N}(\mu_{**}, \kappa_{**}) \tag{A.3}$$
$$\kappa_{**} = \left(\frac{1}{\kappa_*} + \frac{H}{\tau}\right)^{-1}, \ \mu_{**} = \kappa_{**}\left(\frac{\mu_*}{\kappa_*} + \frac{1}{\tau}\sum_{h=1}^{H}\log x_h\right).$$

- **FCD of $\boldsymbol{\tau^{-1}}$:**

$$\tau^{-1}|\cdots \sim \mathrm{Gam}(a_{**}, b_{**}) \tag{A.4}$$
$$a_{**} = a_* + \frac{H}{2},\ b_{**} = b_* + \frac{1}{2}\sum_{h=1}^{H}(\log x_h - \mu)^2.$$

- **FCD of $\boldsymbol{\omega^{-1}}$:**

$$\omega^{-1}|\cdots \sim \mathrm{Gam}(c_{**}, d_{**}),\ i = 1, \cdots, M \tag{A.5}$$
$$c_{**} = c_* + \frac{MH}{2},\ d_{**} = d_* + \frac{1}{2}\sum_{h=1}^{H}\begin{pmatrix}\boldsymbol{v}_h \\ v_{hM}\end{pmatrix}'\begin{pmatrix}\boldsymbol{v}_h \\ v_{hM}\end{pmatrix}.$$

- **FCD of $\boldsymbol{c}_h$:**

$$\boldsymbol{c}_h|\cdots \sim \mathrm{N}(\boldsymbol{c}_{h**}, \boldsymbol{\Sigma}_{h**}),\ h = 1, \cdots, H \tag{A.6}$$
$$\boldsymbol{\Sigma}_{h**} = \left[\frac{1}{x_h^2}\boldsymbol{\Sigma}^{-1} + \frac{1}{\omega}(\boldsymbol{I}_m + \boldsymbol{\iota}_m\boldsymbol{\iota}_m')\right]^{-1}$$
$$\boldsymbol{c}_{h**} = \boldsymbol{\Sigma}_{h**}\left[\frac{1}{x_h}\boldsymbol{\Sigma}^{-1}(\boldsymbol{\alpha} + \boldsymbol{\beta}\log x_h^*) + \frac{1}{\omega}\left\{(x_h - p_M y_{hM}^{**})\boldsymbol{\iota}_m + \boldsymbol{P}_m\boldsymbol{y}_h^{**}\right\}\right].$$

- **FCD of $\boldsymbol{y}_{hi}^*$:** When $D_{hi} = 0$,

$$y_{hi}^*|\cdots \sim \mathrm{N}\left(\frac{c_{hi}}{p_i}, \frac{\omega}{p_i^2}\right),\ h = 1, \cdots, H,\ i = 1, \cdots, M. \tag{A.7}$$

Using Gibbs sampling, we can easily simulate the above-mentioned parameters from their FCDs.

However, since the FCDs of x_h $(h = 1, \cdots, H)$, p_i $(i = 1, \cdots, M)$ and $\boldsymbol{\eta}$ are complicated functions, the closed forms of FCDs for the parameters cannot be derived. Therefore, we simulate these parameters using the Metropolis-Hasting (M-H) algorithm.[15]

A.2. Sampling of $\boldsymbol{x}_h$

Let $f(x)$ and $q(x', x)$ denote the target and proposal densities of a transition from x' to x, respectively. The Metropolis-Hastings (M-H) algorithm can be described as follows:

1. At the $(t+1)$th iteration, given the current sample $x^{(t)}$, sample x from the proposal density $q(x^{(t)}, x)$.

2. Generate $u \sim \mathrm{U}(0, 1)$, a uniform distribution on $(0, 1)$ and take

$$x^{(t+1)} = \begin{cases} x & \text{if}\ \ u < \min\left\{\dfrac{f(x)q(x, x^{(t)})}{f(x^{(t)})q(x^{(t)}, x)}, 1\right\} \\ x^{(t)} & \text{otherwise.}\end{cases}$$

[15] See, for example, Tierney (1994) and Chib and Greenberg (1995).

There are several suggestions for choosing the proposal density. In this article, we employ a tailored proposal density.[16]

Since the FCD of x_h is

$$\begin{aligned} p(x_h|\cdots) &\propto \frac{1}{x_h}\exp\left[-\frac{1}{2\tau}(\log x_h-\mu)^2\right]\left(\frac{1}{x_h}\right)^m \exp\left(-\frac{1}{2}\boldsymbol{u}_h'\boldsymbol{\Sigma}^{-1}\boldsymbol{u}_h\right)\exp\left(-\frac{1}{2\omega}v_{hM}^2\right) \\ &= \left(\frac{1}{x_h}\right)^{m+1}\exp\left[-\frac{1}{2}\left\{\boldsymbol{u}_h'\boldsymbol{\Sigma}^{-1}\boldsymbol{u}_h+\frac{1}{\omega}v_{hM}^2+\frac{1}{\tau}(\log x_h-\mu)^2\right\}\right], \end{aligned}$$

we have

$$\log p(x_h|\cdots) = \text{const.} - (m+1)\log x_h - \frac{1}{2}\left[\boldsymbol{u}_h'\boldsymbol{\Sigma}^{-1}\boldsymbol{u}_h+\frac{1}{\omega}v_{hM}^2+\frac{1}{\tau}(\log x_h-\mu)^2\right].$$

The first and second derivatives of $\log p(x_h|\cdots)$ with respect to x_h are as follows:[17]

$$\begin{aligned} \frac{\partial \log p(x_h|\cdots)}{\partial x_h} &= -\frac{m+1}{x_h} - \frac{x_h-(p_M y_{hM}^{**}+\boldsymbol{\iota}_m'\boldsymbol{c}_h)}{\omega} - \frac{\log x_h-\mu}{\tau x_h} - \frac{1}{2}\frac{\partial \boldsymbol{u}_h'\boldsymbol{\Sigma}^{-1}\boldsymbol{u}_h}{\partial x_h} \\ \frac{\partial^2 \log p(x_h|\cdots)}{\partial x_h^2} &= \frac{1}{x_h^2}\left[m+1+\frac{1}{\tau}(\log x_h-\mu-1)\right] - \frac{1}{\omega} - \frac{1}{2}\frac{\partial^2 \boldsymbol{u}_h'\boldsymbol{\Sigma}^{-1}\boldsymbol{u}_h}{\partial x_h^2}, \end{aligned}$$

where

$$\begin{aligned} \frac{\partial \boldsymbol{u}_h'\boldsymbol{\Sigma}^{-1}\boldsymbol{u}_h}{\partial x_h} &= 2\left(\frac{\partial \boldsymbol{u}_h}{\partial x_h}\right)'\boldsymbol{\Sigma}^{-1}\boldsymbol{u}_h \\ \frac{\partial^2 \boldsymbol{u}_h'\boldsymbol{\Sigma}^{-1}\boldsymbol{u}_h}{\partial x_h^2} &= 2\left(\frac{\partial \boldsymbol{u}_h}{\partial x_h}\right)'\boldsymbol{\Sigma}^{-1}\left(\frac{\partial \boldsymbol{u}_h}{\partial x_h}\right) + 2\boldsymbol{u}_h'\boldsymbol{\Sigma}^{-1}\left(\frac{\partial^2 \boldsymbol{u}_h}{\partial x_h^2}\right) \\ \frac{\partial \boldsymbol{u}_h}{\partial x_h} &= -\frac{1}{x_h}(\boldsymbol{w}_h+\boldsymbol{\beta}) \\ \frac{\partial^2 \boldsymbol{u}_h}{\partial x_h^2} &= \frac{1}{x_h^2}(2\boldsymbol{w}_h+\boldsymbol{\beta}). \end{aligned}$$

Then, employing the first and the second derivatives of $\log p(x_h|\cdots)$, we can obtain the mode of $\log p(x_h|\cdots)$, say $\hat{x}_h$. Now, define the tailored proposal density as follows:

$$q(x_h', x_h) = q(x_h) = f_t(x_h|\hat{x}_h, s_h^2, \nu) \propto \left[1+\frac{1}{\nu s_h^2}(x_h-\hat{x}_h)^2\right]^{-(\nu+1)/2},$$

where ν is an adjustable constant,[18] $f_t(\cdot|a,b,\nu)$ denotes a t density with ν degrees of freedom, location parameter a and scale parameter b, and

$$s_h^2 = \left[-\frac{\partial^2 \log p(x_h|\cdots)}{\partial x_h^2}\bigg|_{x_h=\hat{x}_h}\right]^{-1},$$

provided $s_h^2 > 0$. Thus, we have the following M-H algorithm with a tailored proposal density:

[16]See, for example, Chib *et al.* (1998).

[17]For the derivatives of vectors and matrices, see Lütkepohl (p.175, 1996).

[18]In Section 4, we set $\nu = 7$.

1. At the $(t+1)$th iteration, given the current sample $x_h^{(t)}$, sample x_h from the proposal density $q(x_h)$ until $x_h \geq \sum_{i=1}^m c_{hi}$.

2. Generate $u \sim \mathrm{U}(0,1)$, the uniform distribution on $(0,1)$ and take

$$x_h^{(t+1)} = \begin{cases} x_h & \text{if } \; u < \min\left\{ \dfrac{\mathrm{e}^{\log p(x_h|\cdots)}\, q(x_h^{(t)})}{\mathrm{e}^{\log p(x_h^{(t)}|\cdots)}\, q(x_h)}, 1 \right\} \\ x_h^{(t)} & \text{otherwise.} \end{cases}$$

A.3. Sampling of $\boldsymbol{p}$

Since

$$\begin{aligned} p(\boldsymbol{p}|\cdots) &\propto \prod_{i=1}^{M} p_i^{g_*-1}(1-p_i)^{q_*-1} p_i^{H+\sum_{h=1}^H D_{hi}}(1-p_i)^{H-\sum_{h=1}^H D_{hi}} \\ &\quad \times \exp\left[-\frac{1}{2\omega}\sum_{h=1}^{H}(p_i y_{hi}^{**}-c_{hi})^2\right] \\ &= \prod_{i=1}^{M} p_i^{g_*+H+\sum_{h=1}^H D_{hi}-1}(1-p_i)^{q_*+H-\sum_{h=1}^H D_{hi}-1} \exp\left[-\frac{1}{2\omega}\sum_{h=1}^{H}(p_i y_{hi}^{**}-c_{hi})^2\right], \end{aligned}$$

we have

$$\begin{aligned} \log p(\boldsymbol{p}|\cdots) &= \text{const.} + \sum_{i=1}^{M}\left(g_* + H + \sum_{h=1}^{H} D_{hi} - 1\right)\log p_i \\ &\quad + \sum_{i=1}^{M}\left(q_* + H - \sum_{h=1}^{H} D_{hi} - 1\right)\log(1-p_i) - \frac{1}{2\omega}\sum_{i=1}^{M}\sum_{h=1}^{H}(p_i y_{hi}^{**}-c_{hi})^2. \end{aligned}$$

The first and second derivatives of $\log p(\boldsymbol{p}|\cdots)$ with respect to p_i are as follows:

$$\begin{aligned} \frac{\partial \log p(\boldsymbol{p}|\cdots)}{\partial p_i} &= \frac{1}{p_i}\left(g_* + H + \sum_{h=1}^{H} D_{hi} - 1\right) \\ &\quad - \frac{1}{1-p_i}\left(q_* + H - \sum_{h=1}^{H} D_{hi} - 1\right) - \frac{1}{\omega}\sum_{h=1}^{H}(p_i y_{hi}^{**}-c_{hi})y_{hi}^{**} \\ \frac{\partial^2 \log p(\boldsymbol{p}|\cdots)}{\partial p_i^2} &= -\frac{1}{p_i^2}\left(g_* + H + \sum_{h=1}^{H} D_{hi} - 1\right) \\ &\quad - \frac{1}{(1-p_i)^2}\left(q_* + H - \sum_{h=1}^{H} D_{hi} - 1\right) - \frac{1}{\omega}\sum_{h=1}^{H} y_{hi}^{**2}. \end{aligned}$$

Then, using the first and the second derivatives of $\log p(\boldsymbol{p}|\cdots)$, we can obtain the mode of $\log p(\boldsymbol{p}|\cdots)$, say $\hat{\boldsymbol{p}}$. Now, define the tailored proposal density as follows:

$$q(\boldsymbol{p}', \boldsymbol{p}) = q(\boldsymbol{p}) = f_{Mt}(\boldsymbol{p}|\hat{\boldsymbol{p}}, \boldsymbol{S}, \nu) \propto \left[1 + \frac{1}{\nu}(\boldsymbol{p}-\hat{\boldsymbol{p}})'\boldsymbol{S}^{-1}(\boldsymbol{p}-\hat{\boldsymbol{p}})\right]^{-(\nu+M)/2},$$

where ν is an adjustable constant,[19] and $f_{Mt}(\cdot|\boldsymbol{a}, \boldsymbol{A}, \nu)$ denotes a multivariate t density with ν degrees of freedom, location parameter vector $\boldsymbol{a}$ and scale parameter matrix $\boldsymbol{A}$, and

$$\boldsymbol{S} = \left[-\frac{\partial^2 \log p(\boldsymbol{p}|\cdots)}{\partial \boldsymbol{p} \partial \boldsymbol{p}'} \bigg|_{\boldsymbol{p}=\hat{\boldsymbol{p}}} \right]^{-1},$$

provided $\boldsymbol{S}$ is positive definite. Thus, we have the following M-H algorithm with a tailored proposal density:

1. At the $(t+1)$th iteration, given the current sample $\boldsymbol{p}^{(t)}$, sample $\boldsymbol{p}$ from the proposal density $q(\boldsymbol{p})$.
2. Generate $u \sim \mathrm{U}(0,1)$, the uniform distribution on $(0,1)$ and take

$$\boldsymbol{p}^{(t+1)} = \begin{cases} \boldsymbol{p} & \text{if } \ u < \min\left\{ \dfrac{e^{\log p(\boldsymbol{p}|\cdots)}\, q(\boldsymbol{p}^{(t)})}{e^{\log p(\boldsymbol{p}^{(t)}|\cdots)}\, q(\boldsymbol{p})}, 1 \right\} \\ \boldsymbol{p}^{(t)} & \text{otherwise.} \end{cases}$$

A.4. Sampling of $\boldsymbol{\eta}$

Since

$$\begin{aligned} p(\boldsymbol{\eta}|\cdots) &\propto \exp\left[-\frac{1}{2}(\boldsymbol{\eta} - \boldsymbol{\eta}_*)' \boldsymbol{A}_*^{-1} (\boldsymbol{\eta} - \boldsymbol{\eta}_*) \right] \exp\left(-\frac{1}{2} \sum_{h=1}^{H} \boldsymbol{u}_h' \boldsymbol{\Sigma}^{-1} \boldsymbol{u}_h \right) \\ &= \exp\left[-\frac{1}{2} \left\{ (\boldsymbol{\eta} - \boldsymbol{\eta}_*)' \boldsymbol{A}_*^{-1} (\boldsymbol{\eta} - \boldsymbol{\eta}_*) + \sum_{h=1}^{H} \boldsymbol{u}_h' \boldsymbol{\Sigma}^{-1} \boldsymbol{u}_h \right\} \right] \end{aligned}$$

we have

$$\log p(\boldsymbol{\eta}|\cdots) = \text{const.} - \frac{1}{2} \left[(\boldsymbol{\eta} - \boldsymbol{\eta}_*)' \boldsymbol{A}_*^{-1} (\boldsymbol{\eta} - \boldsymbol{\eta}_*) + \sum_{h=1}^{H} \boldsymbol{u}_h' \boldsymbol{\Sigma}^{-1} \boldsymbol{u}_h \right]$$

The first and second derivatives of $\log p(\boldsymbol{\eta}|\cdots)$ by $\boldsymbol{\eta}$ are as follows:[20]

$$\begin{aligned} \frac{\partial \log p(\boldsymbol{\eta}|\cdots)}{\partial \boldsymbol{\eta}} &= -\boldsymbol{A}_*^{-1}(\boldsymbol{\eta} - \boldsymbol{\eta}_*) - \frac{1}{2} \sum_{h=1}^{H} \frac{\partial \boldsymbol{u}_h' \boldsymbol{\Sigma}^{-1} \boldsymbol{u}_h}{\partial \boldsymbol{\eta}} \\ \frac{\partial^2 \log p(\boldsymbol{\eta}|\cdots)}{\partial \boldsymbol{\eta} \partial \boldsymbol{\eta}'} &= -\boldsymbol{A}_*^{-1} - \frac{1}{2} \sum_{h=1}^{H} \frac{\partial^2 \boldsymbol{u}_h' \boldsymbol{\Sigma}^{-1} \boldsymbol{u}_h}{\partial \boldsymbol{\eta} \partial \boldsymbol{\eta}'}, \end{aligned}$$

[19] In Section 4, we set $\nu = 7$.
[20] See Lütkepohl (p.175, 1996).

where

$$
\begin{aligned}
\frac{\partial \boldsymbol{u}_h' \boldsymbol{\Sigma}^{-1} \boldsymbol{u}_h}{\partial \boldsymbol{\eta}} &= 2\left(\frac{\partial \boldsymbol{u}_h}{\partial \boldsymbol{\eta}'}\right)' \boldsymbol{\Sigma}^{-1} \boldsymbol{u}_h \\
\frac{\partial^2 \boldsymbol{u}_h' \boldsymbol{\Sigma}^{-1} \boldsymbol{u}_h}{\partial \boldsymbol{\eta} \partial \boldsymbol{\eta}'} &= 2\left(\frac{\partial \boldsymbol{u}_h}{\partial \boldsymbol{\eta}'}\right)' \boldsymbol{\Sigma}^{-1} \frac{\partial \boldsymbol{u}_h}{\partial \boldsymbol{\eta}'} + 2(\boldsymbol{u}_h' \boldsymbol{\Sigma}^{-1} \otimes \boldsymbol{I}_K) \frac{\partial}{\partial \boldsymbol{\eta}'}\left[\mathrm{vec}\left(\frac{\partial \boldsymbol{u}_h'}{\partial \boldsymbol{\eta}}\right)\right] \\
\frac{\partial \boldsymbol{u}_h}{\partial \boldsymbol{\eta}'} &= \frac{\boldsymbol{\beta} \boldsymbol{z}_h'}{m_h} \\
\frac{\partial}{\partial \boldsymbol{\eta}'}\left[\mathrm{vec}\left(\frac{\partial \boldsymbol{u}_h'}{\partial \boldsymbol{\eta}}\right)\right] &= \frac{\partial}{\partial \boldsymbol{\eta}'}\left[\mathrm{vec}(\boldsymbol{z}_h \boldsymbol{\beta}') \frac{1}{m_h}\right] = -\frac{1}{m_h^2} \mathrm{vec}(\boldsymbol{z}_h \boldsymbol{\beta}') \boldsymbol{z}_h'.
\end{aligned}
$$

Then, using the first and the second derivatives of $\log p(\boldsymbol{\eta}|\cdots)$, we can obtain the mode of $\log p(\boldsymbol{\eta}|\cdots)$, say $\hat{\boldsymbol{\eta}}$. Now, define the tailored proposal density as follows:

$$
q(\boldsymbol{\eta}', \boldsymbol{\eta}) = q(\boldsymbol{\eta}) = f_{Mt}(\boldsymbol{\eta}|\hat{\boldsymbol{\eta}}, \boldsymbol{S}, \nu) \propto \left[1 + \frac{1}{\nu}(\boldsymbol{\eta} - \hat{\boldsymbol{\eta}})' \boldsymbol{S}^{-1} (\boldsymbol{\eta} - \hat{\boldsymbol{\eta}})\right]^{-(\nu+K)/2}.
$$

where ν is adjustable constant[21] and

$$
\boldsymbol{S} = \left[-\frac{\partial^2 \log p(\boldsymbol{\eta}|\cdots)}{\partial \boldsymbol{\eta} \partial \boldsymbol{\eta}'}\bigg|_{\boldsymbol{\eta}=\hat{\boldsymbol{\eta}}}\right]^{-1}
$$

provided $\boldsymbol{S}$ is positive definite. Thus, we have the following M-H algorithm with the tailored proposal density:

1. At the $(t+1)$th iteration, given the current sample $\boldsymbol{\eta}^{(t)}$, sample $\boldsymbol{\eta}$ from the proposal density $q(\boldsymbol{\eta})$.
2. Generate $u \sim \mathrm{U}(0,1)$, the uniform distribution on $(0,1)$ and take

$$
\boldsymbol{\eta}^{(t+1)} = \begin{cases} \boldsymbol{\eta} & \text{if } \; u < \min\left\{\dfrac{\mathrm{e}^{\log p(\boldsymbol{\eta}|\cdots)}\, q(\boldsymbol{\eta}^{(t)})}{\mathrm{e}^{\log p(\boldsymbol{\eta}^{(t)}|\cdots)}\, q(\boldsymbol{\eta})}, 1\right\} \\ \boldsymbol{\eta}^{(t)} & \text{otherwise.} \end{cases}
$$

[21] In Section 4, we set $\nu = 7$.

TABLE 1
Number of zero expenditures

Number of households	692
(1) Food	7
(2) Housing	278
(3) Fuel	8
(4) Furniture	359
(5) Clothing	200
(6) Medical	253
(7) Transport	7
(8) Others	11

TABLE 2
Posterior results for parameters

	mean	sd		mean	sd
α_1	0.56762	0.04249	σ_{11}	0.0077473	0.0004278
α_2	0.21000	0.05159	σ_{21}	−0.0022755	0.0003394
α_3	0.31146	0.01720	σ_{31}	0.0002357	0.0001129
α_4	0.00771	0.01416	σ_{41}	−0.0001544	0.0001045
α_5	0.04690	0.01910	σ_{51}	−0.0002563	0.0001277
α_6	0.03701	0.01564	σ_{61}	−0.0002448	0.0001096
α_7	0.09855	0.03524	σ_{71}	−0.0009417	0.0002488
α_8	−0.27925	0.07585	σ_{22}	0.0089570	0.0005241
β_1	−0.07066	0.00925	σ_{32}	−0.0003286	0.0001245
β_2	−0.01635	0.01113	σ_{42}	0.0003557	0.0001041
β_3	−0.04969	0.00358	σ_{52}	−0.0000398	0.0001294
β_4	0.00487	0.00309	σ_{62}	0.0001045	0.0001127
β_5	−0.00040	0.00413	σ_{72}	−0.0006200	0.0002661
β_6	−0.00148	0.00339	σ_{33}	0.0010774	0.0000597
β_7	0.00153	0.00765	σ_{43}	0.0000586	0.0000389
β_8	0.13218	0.01580	σ_{53}	0.0000868	0.0000477
μ	5.53810	0.01371	σ_{63}	0.0000045	0.0000389
τ	0.12885	0.00697	σ_{73}	−0.0000986	0.0000916
ω	0.02396	0.01441	σ_{44}	0.0006862	0.0000495
p_1	0.97847	0.00542	σ_{54}	0.0000552	0.0000405
p_2	0.67823	0.01340	σ_{64}	0.0000292	0.0000341
p_3	0.98663	0.00472	σ_{74}	−0.0000807	0.0000789
p_4	0.53905	0.01992	σ_{55}	0.0012141	0.0000816
p_5	0.73317	0.01064	σ_{65}	0.0000980	0.0000440
p_6	0.67793	0.02160	σ_{75}	−0.0001046	0.0001033
p_7	0.98663	0.00274	σ_{66}	0.0008488	0.0000669
p_8	0.93285	0.00349	σ_{76}	0.0001404	0.0000836
η_1	0.36949	0.08494	σ_{77}	0.0053424	0.0002893

Notes: 'mean' and 'sd' denote the posterior mean and posterior standard deviation.

TABLE 3

Posterior results for elasticities

	mean	sd
Food	0.6982	0.0396
Housing	0.8742	0.0857
Fuel	0.3554	0.0463
Furniture	1.1588	0.1006
Clothing	0.9912	0.0929
Medical	0.9504	0.1131
Transport	1.0142	0.0712
Others	1.3820	0.0457

Notes: 'mean' and 'sd' denote the posterior mean and posterior standard deviation.

TABLE 4

Posterior results for inequality measures

	data			posterior	
	raw	p.c.	e.s.	mean	sd
I_{Gini}	0.2214	0.2640	0.2271	0.2097	0.00377
$I_{GE}(-1)$	0.0916	0.1273	0.0956	0.0714	0.00259
$I_{GE}(0)$	0.0825	0.1135	0.0858	0.0705	0.00237
$I_{GE}(1)$	0.0850	0.1168	0.0882	0.0751	0.00239
$I_{GE}(2)$	0.0993	0.1364	0.1022	0.0878	0.00266

Notes: 'raw,' 'p.c.' and 'e.s.' denote the inequality measures based on the total expenditure, the per capita total expenditure, and the total expenditure deflated by posterior equivalence scale, respectively. 'mean' and 'sd' denote the posterior mean and posterior standard deviation.

TABLE 5

*Classification of goods and number of zero expenditures**

$M = 5$	$M = 6$	$M = 7$	$M = 8$
Food 7	Food 7	Food 7	Food 7
Housing 148	Housing 278	Housing 278	Housing 278
	Furniture 359	Furniture 359	Furniture 359
Fuel 8	Fuel 8	Fuel 8	Fuel 8
Clothing 200	Clothing 200	Clothing 200	Clothing 200
Others 1	Others 1	Medical 253	Medical 253
		Others 1	Transport 7
			Others 11

Notes: *The figures in lower row denote the number of zero expenditures in the item.

TABLE 6

Number of children and age grouping

# of children\age	$0 \sim 6$	$7 \sim 12$	$0 \sim 12$	$13 \sim 18$	$0 \sim 18$
0	312^*	368	129	571	99
1	236	201	223	82	173
2	133	113	259	35	296
3	11	9	73	3	105
4	0	1	7	1	17
5	0	0	1	0	2
sum	$535^\dagger$	458	993	165	$1,158$

Notes: '# of children' denotes the number of children in a household. 'age' denotes the age group of children. For example, '$0 \sim 6$'means the age group of children aged 0 to 6 years. The figure of * denotes the number of households. The figure of † denotes the number of children corresponding to the age group in the data.

TABLE 7

Posterior means of purchasing probabilities

	$M=5$				$M=6$			
	$K=1$	$K=2$	$K=3$	data	$K=1$	$K=2$	$K=3$	data
Food	0.991	0.986	0.989	0.990	0.986	0.986	0.985	0.990
Housing	0.771	0.786	0.788	0.786	0.615	0.637	0.602	0.598
Fuel	0.988	0.987	0.987	0.988	0.987	0.988	0.988	0.988
Furniture					0.536	0.538	0.543	0.481
Clothing	0.741	0.733	0.748	0.711	0.724	0.738	0.735	0.711
Medical								
Transport								
Others	0.969	0.987	0.964	0.999	0.978	0.970	0.949	0.999

	$M=7$				$M=8$			
	$K=1$	$K=2$	$K=3$	data	$K=1$	$K=2$	$K=3$	data
Food	0.987	0.984	0.981	0.990	0.978	0.983	0.978	0.990
Housing	0.668	0.683	0.663	0.598	0.678	0.668	0.642	0.598
Fuel	0.987	0.988	0.985	0.988	0.987	0.988	0.988	0.988
Furniture	0.550	0.548	0.533	0.481	0.539	0.540	0.530	0.481
Clothing	0.729	0.720	0.739	0.711	0.733	0.729	0.725	0.711
Medical	0.678	0.678	0.701	0.634	0.678	0.679	0.662	0.634
Transport					0.987	0.989	0.981	0.990
Others	0.927	0.953	0.953	0.999	0.933	0.936	0.949	0.984

Notes: 'data' denotes $1 - \frac{1}{H}\sum D_{hi}$.

TABLE 8

Posterior means of $\boldsymbol{\eta}$ in equivalence scales

	$M=5$			$M=6$		
	$K=1$	$K=2$	$K=3$	$K=1$	$K=2$	$K=3$
η_1	0.4094	0.3641	0.2152	0.3320	0.2799	0.1823
η_2		0.9459	0.5235		0.7240	0.3815
η_3			0.8558			0.6857
	$M=7$			$M=8$		
	$K=1$	$K=2$	$K=3$	$K=1$	$K=2$	$K=3$
η_1	0.3415	0.3440	0.2013	0.3695	0.3010	0.2319
η_2		0.8799	0.4073		0.8242	0.4902
η_3			0.7520			0.8689

TABLE 9

Posterior means of elasticities

	$M=5$			$M=6$		
	$K=1$	$K=2$	$K=3$	$K=1$	$K=2$	$K=3$
Food	0.6705	0.6206	0.5984	0.6843	0.6522	0.6213
Housing	0.9667	1.0725	1.0979	0.8250	0.8208	0.8642
Fuel	0.3442	0.3453	0.3478	0.3293	0.3283	0.3149
Furniture				0.9858	1.0842	1.1140
Clothing	0.8398	0.8503	0.8700	0.8500	0.9107	0.9262
Medical						
Transport						
Others	1.2903	1.2758	1.2843	1.3163	1.3250	1.3339
	$M=7$			$M=8$		
	$K=1$	$K=2$	$K=3$	$K=1$	$K=2$	$K=3$
Food	0.6955	0.6584	0.6254	0.6982	0.6455	0.6081
Housing	0.8547	0.9501	0.9183	0.8742	0.9401	1.0452
Fuel	0.3409	0.3483	0.3274	0.3554	0.3460	0.3503
Furniture	1.1686	1.0164	1.1152	1.1588	1.1142	1.1199
Clothing	0.9446	0.9591	1.0036	0.9912	1.0219	1.0742
Medical	1.0472	1.0829	1.0504	0.9504	1.0620	1.0881
Transport				1.0142	1.0183	1.0179
Others	1.3090	1.2996	1.3190	1.3820	1.3815	1.3544

TABLE 10

Inequality measures for goods and age groups

			I_{Gini}	$I_{GE}(-1)$	$I_{GE}(0)$	$I_{GE}(1)$	$I_{GE}(2)$
		raw	0.2214	0.0916	0.0825	0.0850	0.0993
		p.c.	0.2640	0.1273	0.1135	0.1168	0.1364
	$K=1$	e.s.	0.2289	0.0971	0.0871	0.0895	0.1037
		mean	0.2153	0.0759	0.0745	0.0792	0.0926
$M=5$	$K=2$	e.s.	0.2294	0.0959	0.0867	0.0893	0.1035
		mean	0.2175	0.0767	0.0754	0.0800	0.0931
	$K=3$	e.s.	0.2291	0.0966	0.0869	0.0892	0.1027
		mean	0.2135	0.0746	0.0732	0.0776	0.0900
	$K=1$	e.s.	0.2254	0.0944	0.0847	0.0871	0.1010
		mean	0.2144	0.0758	0.0742	0.0790	0.0926
$M=6$	$K=2$	e.s.	0.2238	0.0917	0.0830	0.0856	0.0992
		mean	0.2098	0.0716	0.0708	0.0759	0.0898
	$K=3$	e.s.	0.2230	0.0916	0.0827	0.0851	0.0983
		mean	0.2072	0.0700	0.0690	0.0735	0.0853
	$K=1$	e.s.	0.2258	0.0947	0.0850	0.0874	0.1013
		mean	0.2115	0.0726	0.0717	0.0765	0.0896
$M=7$	$K=2$	e.s.	0.2278	0.0947	0.0857	0.0883	0.1023
		mean	0.2122	0.0731	0.0720	0.0767	0.0897
	$K=3$	e.s.	0.2246	0.0927	0.0837	0.0861	0.0994
		mean	0.2087	0.0707	0.0702	0.0753	0.0887
	$K=1$	e.s.	0.2271	0.0956	0.0858	0.0882	0.1022
		mean	0.2097	0.0714	0.0705	0.0751	0.0878
$M=8$	$K=2$	e.s.	0.2256	0.0929	0.0841	0.0867	0.1005
		mean	0.2079	0.0691	0.0689	0.0738	0.0864
	$K=3$	e.s.	0.2284	0.0959	0.0864	0.0887	0.1023
		mean	0.2115	0.0725	0.0716	0.0763	0.0889

Notes: 'raw,' 'p.c.' and 'e.s.' denote the inequality measures based on the total expenditure, the per capita total expenditure, and the total expenditure deflated by posterior equivalence scale, respectively. 'mean' denotes the posterior mean.