Development of Numerical Analysis Method for Magnetic Separation of Magnetic Particle and Ion with Magnetic Chromatography

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***Abstract*—The magnetic chromatography is a very useful system for an ion and/or fine magnetic particle separation due to its strong magnetic field gradients in a very small flow channel. We have developed the magnetic chromatography system to separate the fine particles and ions. However, its numerical simulation is difficult, since the scale of the ions or fine particles in fluid is much different from the scale of the superconducting magnet generating the strong magnetic field. In order to accurately simulate the magnetic separation, it is necessary to develop the simulation code dealing with the multi-scale problem.**

***Index Terms*—Magnetic separation, magnetic chromatography, ferrohydorodynamics.**

I. INTRODUCTION

THE MAGNETIC separation is a very useful system for an ion and/or fine magnetic particle separation due to its strong magnetic field gradients in a very small flow channel. Therefore, the various systems for magnetic separation have been developed and presented. The high gradient magnetic separation technique, using superconducting magnets, was proposed and the development [1]–[3]. However, the high gradient magnetic separation technique is not useful in separating the magnetic particles that have very small radius (below 100nm). The reason is that the magnetic force of the particles is relatively smaller than their diffusion force. On the other hand, the magnetic chromatography (MC) system is a very useful device that uses strong magnetic field gradients for separating magnetic particles with different magnetic susceptibilities in a colloidal mixture [4]–[6].

We have developed the MC system to separate the ions and the fine particles [6],[8]. In the developed MC device, ferromagnetic wires that generated magnetic force are located at the wall [8]. Therefore, the particles with a large magnetic susceptibility are extracted in the radial direction of the flow channel and concentrated around the channel wall, and the flow velocity close to the channel wall is almost zero. The particles with a small magnetic susceptibility can go through the channel without attracting the channel wall. Thus, the distribution of concentration in radial direction would be different and the exhaustion time must be different as well.

In this paper, the numerical analysis method, which was developed coupling the fluid dynamics and the electromagnetics, is presented. The Navier-Stokes and the control volume equations are solved as fluid dynamics by the finite volume method [9],[10]. The electromagnetic field equation is solved by the magnetic moment method [11],[12], which is a kind of integration method. Our MC system has the millimeter-scale ferromagnetic wires and the meter-scale superconducting magnet, so the method of moment is employed to obtain an accurate simulation result. The numerical simulation results by the newly developed analysis method are compared the experimental results. And the influence of the ferromagnetic wire position is investigated.

II. NUMERICAL ANALYSIS METHOD

A. Fluid Dynamics

In this study, the numerical fluid dynamics analysis using the finite volume method [9] is carried out to simulate the magnetic separation. The governing equation of control volume method is following as

$$\frac{\partial}{\partial t}\left(\rho\phi\right)+\nabla\cdot\left(\rho\boldsymbol{v}\phi\right)=\nabla\cdot\left(\Gamma\,\mathrm{grad}\,\phi\right)+S\,, \tag{1}$$

where ρ is the fluid density, ϕ is the transported scalar variable, which is particle mass fraction, $\boldsymbol{v}$ is the fluid velocity, Γ is the scalar diffusion constant, and S is the scalar source term. By solving (1), the concentration distributions of magnetic particles or ions are obtained. The fluid velocity $\boldsymbol{v}$ is obtained by solving the Navier-Stokes equation,

$$\rho\frac{\partial\boldsymbol{v}}{\partial t}+\rho\boldsymbol{v}\cdot\nabla\boldsymbol{v}=-\nabla p^{*}+\mu_0\boldsymbol{M}\nabla\boldsymbol{H}\,, \tag{2}$$

where p^* is the composite pressure, μ_0 is the permeability in free space, $\boldsymbol{M}$ is the magnetization, and $\boldsymbol{H}$ is the magnetic field strength. In this simulation, it is supposed that the magnetization is parallel to the permeability, so the internal angular momentum of the magnetic particles can be ignored. Equation (2) is solved by the finite volume method with SIMPLEC method [13]. The magnetization and magnetic field strength are computed by the following method.

B. Magnetic Field Simulation

In this study, the magnetic field simulation is solved by the magnetic moment method [12],[13]. The governing equation of the magnetic field is following as

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$$-\frac{1}{4\pi}\int_V \left(\nabla'\cdot \boldsymbol{M}(\boldsymbol{r}')\right)\frac{\boldsymbol{r}-\boldsymbol{r}'}{\left|\boldsymbol{r}-\boldsymbol{r}'\right|^3}\mathrm{d}V' + \frac{1}{4\pi}\int_S \left(\boldsymbol{M}(\boldsymbol{r}')\cdot \boldsymbol{n}\right)\frac{\boldsymbol{r}-\boldsymbol{r}'}{\left|\boldsymbol{r}-\boldsymbol{r}'\right|^3}\mathrm{d}S'$$
$$-\frac{1}{\chi_m(\boldsymbol{r})}\boldsymbol{M}(\boldsymbol{r}) = -\boldsymbol{B}_F(\boldsymbol{r}), \quad (3)$$

where $\boldsymbol{r}$ and $\boldsymbol{r}'$ are the position vector, χ_m is the susceptibility, and B_F is the magnetic field generated by the superconducting magnet. In this research, the magnetic moment method is more accurate than the domain method, such as the finite element method, because only the magnetic objects (ferromagnetic wire and the magnetic particle) are taken into account.

The magnetic property of the magnetic particle is given by [10]

$$M = \phi M_S L\left(\frac{\mu_0 mH}{kT}\right), \quad (4)$$

where M_S is the saturation magnetization, L is the Langevin function, k is the Boltzmann constant, T is the absolute temperature, and $m = V_P M_S$. V_P is the volume of one magnetic particle.

C. Simulation Algorithm

The flow of the simulation combining the fluid dynamics and the magnetic field is shown in Fig. 1. At first, the magnetization of the magnetic particles is obtained by solving (3). Substituting the obtained magnetization into (2), the fluid velocity and the composite pressure are computed. Then, the concentration of the magnetic particles is computed by solving (1) with the obtained fluid velocity. A series of simulations are repeated till the analysis time ends.

III. Experiment Condition

Fig. 2 shows the outline of the MC experimental systems. The solution including magnetic particles injects by the injector and flow in magnetic column with ferromagnetic wires located in the room temperature bore (180 mm in diameter) of the superconducting magnet, and the connection of magnetic particles in the solution is measured by the detector. Fig. 3 shows the schematic view of magnetic column consisted of flow channel and ferromagnetic wires. The height and width flow channel are 170 μm and 10mm, respectively. The ferromagnetic wires with width of 200 μm are vertically and alternately arranged against the flow direction with 200 μm apart. The detail of the experiments were presented in [6],[8].

IV. Simulation Results

A. Ion Flow through Tube

Using the developed simulation code, the ion flow through the simple tube with 0.5 mm diameter and 0.5 m length is simulated without the external magnetic field. The ions (nickel chloride, $NiSO_4$) are injected into the tube for 1.0 s. The simulated and experimental time variations of concentrations of ions at the outlet are shown in Fig. 4. The simulated times of starting flow-out of ions almost corresponds to the experimental ones, but the flow-out duration is different between the simulation and the experiment. The difference may be caused by the 2D simulation. The diameter of tube is too small that the 2D simulation can't represent the 3D phenomenon, such as velocity at wall.

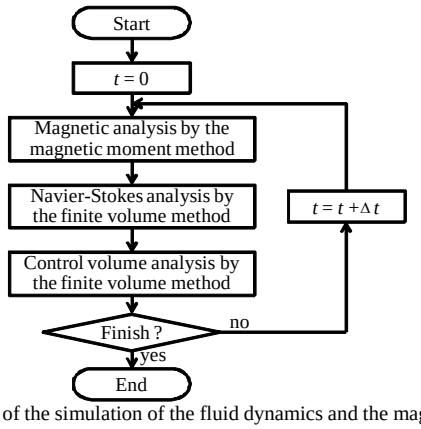

Fig. 1. Flow of the simulation of the fluid dynamics and the magnetic field.

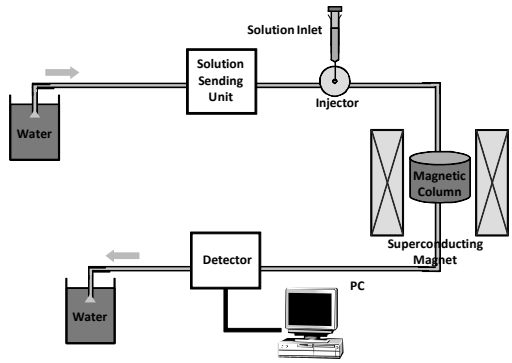

Fig. 2. Schematic draw of the magnetic chromatography system with magnetic column for magnetic particle separation.

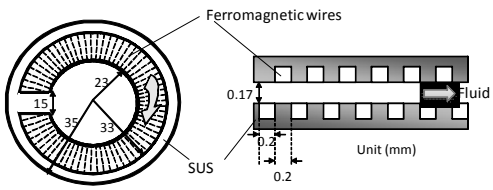

Fig. 3. Schematic view of the magnetic column and the flow channel with height of 0.17 mm and width of 10 mm. The ferromagnetic wires with width of 0.2 mm are vertically and alternately arranged against the flow direction with 200 μm.

B. Magnetic Particle Flow through Column

The simulation of the magnetic particles through the magnetic column mentioned above is performed, and compared with the experiments. The flow velocity is 5.0 ml/min, and the injection time of the magnetic particles is also 1.0 s. The used magnetic particle is Fe_3O_4 particle of 300 nm in diameter. The magnetic field applied by the superconducting magnet is 0.2 T.

The analytical and experimental results are shown in Fig. 5. The waveforms of the experiment and analysis don't correspond. The influence of the flow velocity at side wall becomes small because the channel is wide enough. One cause of differences is the tubes connecting from the injector to the magnetic column and from the magnetic column to the detector. The tube simulation has an error comparing with the experiment, mentioned above. We must developed 3D simulation code.

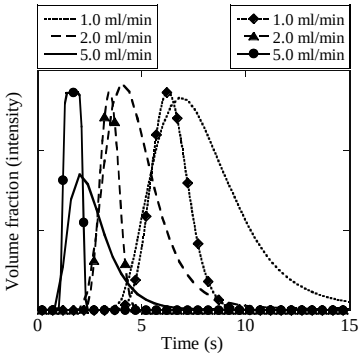

Fig. 4. The simulated and measured time variations of concentrations at the outlet. The flow velocity varies from 1.0 ml/min to 5.0 ml/min.

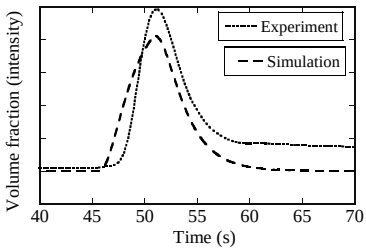

Fig. 5. The simulated and measured time variations of concentrations at the outlet through the magnetic column. The flow velocity is 5.0 ml/min.

C. *Investigation of Ferromagnetic Wire Position*

For investigation of the ferromagnetic wire position in magnetic column, the flows through the magnetic column with the different arrays of ferromagnetic wires were simulated. One is that the ferromagnetic wires are longitudinally alternatively arrayed as shown in Fig. 6(a), the other is that they are longitudinally same-positioned as shown in Fig. 6(b). In these simulations, the column length is 8 mm ($x = -0.004 \sim 0.004\,\mathrm{m}$) since the mesh is so small to observe the complicated flow in detail. The magnetic particle is also Fe_3O_4 particle of 300 nm in diameter and injected for 0.2 s, and the flow velocity is 5.0 ml/min. The applied magnetic field is 0.2 T.

The simulated time variations of concentrations at the outlet are shown in Fig. 7. All the waveforms are almost the same, but the flow-out in the case with the magnetic field is slightly earlier than that in the other two cases. The difference doesn't appear because the magnetic column is too short.

The distribution maps of the density of the magnetic particle and at 0.07, 0.17, 0.27 and 0.37 s in three cases are shown in Figs. 8, 9 and 10, where the dark color means the high density. In these figures, the flows are also represented by arrows. As shown in Fig. 10, in the case without the applied magnetic field, the flow is surely straight without turbulence.

In the case of the alternatively positioning, the flow in front of the collective magnetic particles meanders, as shown in Fig. 8(a). The meandering flow continues while the collective magnetic particles pass (Figs. 8(b) and (c)). On the other hand, the flow in front of the collective magnetic particles varies in longitudinal direction, in the case of the magnetic column of the longitudinal same-positioned magnetic wires, as shown in Fig. 9(a). As the time passes, the meandering flow is observed in little, but the flow varying in longitudinal direction continues

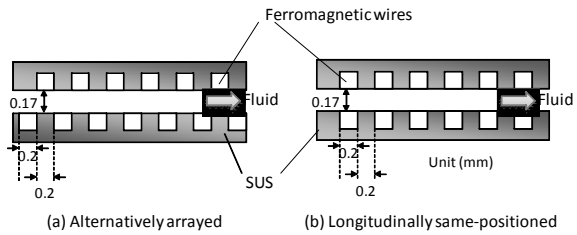

Fig. 6. Ferromagnetic wires are arrayed in the different ways. One is the longitudinally alternatively positioned ferromagnetic wires, the other is the longitudinally same-positioned ferromagnetic wires.

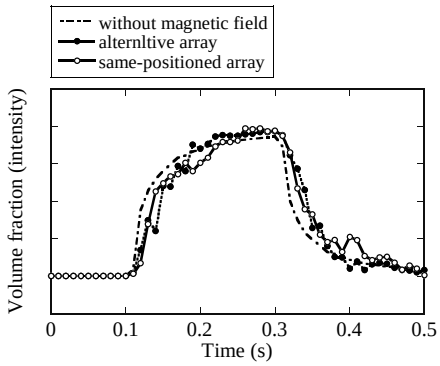

Fig. 7. The simulated time variations of concentrations at the outlet through the magnetic column. The results of three kind of column are shown; i) the magnetic wires longitudinally alternatively positioned, ii) the magnet wires longitudinally same-positioned, and iii) without applied magnetic field. The flow velocity is 5.0 ml/min.

(Figs. 9(b) and (c)).

For a magnetic separation, it is desired that magnetic particles attracts on wall by a force generated by magnetic field gradients. However, the magnetic particles don't go to the wall in the case of the magnetic column with the longitudinally same-positioned ferromagnetic wires. Therefore, the magnetic column with the longitudinally alternatively positioned ferromagnetic wires is effective in magnetic separation, due to the meandering flow.

V. Conclusion

In order to accurately simulate the magnetic separation, we have developed the simulation program dealing with the multi-scale problem. The simulation and experimental results were compared, but it is necessary to develop the 3D simulation program. The position of the ferromagnetic wires in magnetic column was investigated. Their position affects the flow of magnetic particles, but the longer magnetic column must be investigated in simulation.

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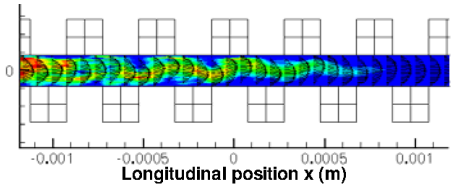
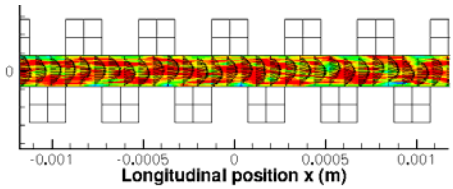
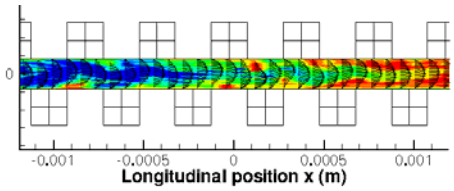


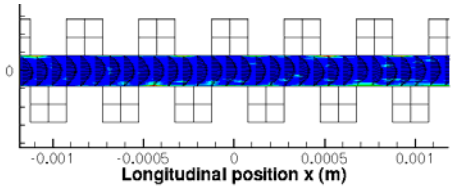
(a) t = 0.07 s



(b) t = 0.17 s

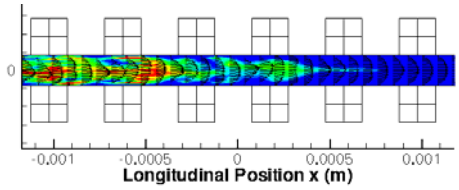


(c) t = 0.27 s

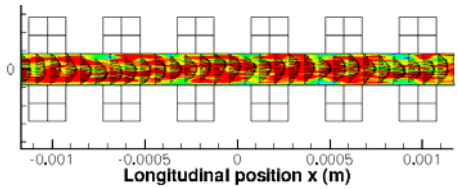


(d) t = 0.37 s

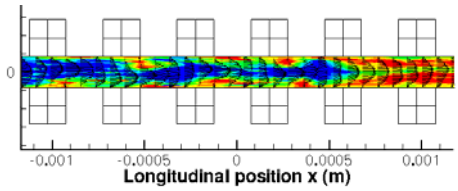
Fig. 8. The distribution maps of volume fraction and the flow represented by arrows in the case of the longitudinally alternatively positioned ferromagnetic wires.



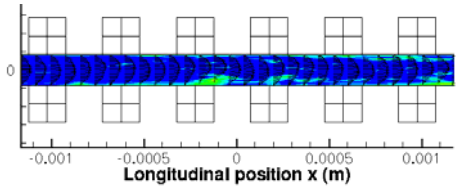
(a) t = 0.07 s



(b) t = 0.17 s

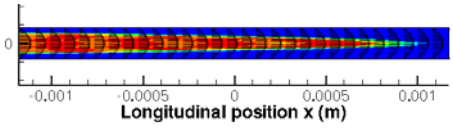


(c) t = 0.27 s

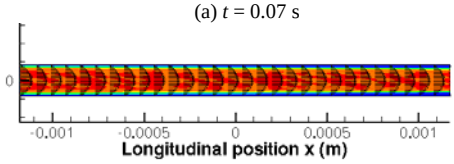


(d) t = 0.37 s

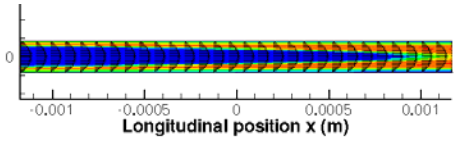
Fig. 9. The distribution maps of volume fraction and the flow represented by arrows in the case of the longitudinally same-positioned ferromagnetic wires.



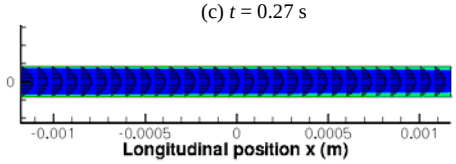
(a) t = 0.07 s



(b) t = 0.17 s



(c) t = 0.27 s



(d) t = 0.37 s

Fig. 10 The distribution maps of volume fraction and the flow represented by arrows in the case without applied magnetic field.

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