Dynamics of a paired optical vortex generated by second-harmonic generation

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Abstract: We study the dynamics of a paired optical vortex (OV) generated by second-harmonic generation (SHG) using sub-picosecond pulses. By changing the position of a thin nonlinear crystal along the propagation direction, we observe a rotation of two vortices with changing separation distance. The dynamics is well explained by SHG with a beam walk-off, which introduces a contamination of zero-order Laguerre-Gaussian beam (LG0) together with topological charge doubling. The quantitative analysis indicates that the rotation angle of the OVs manifests the Gouy phase while the splitting provides the walk-off angle of the crystal. We also show that the subtraction of LG0 is realized by the superposition of LG0 with an anti-balanced phase in the pump.

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References and links

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1. Introduction

Optical vortices (OVs) have been studied extensively in recent years owing to their various potential applications, such as laser trapping [1], spatially-resolved imaging [2], spectroscopy [3] and information processing [4]. The OVs are associated with a series of Laguerre-Gaussian (LG) modes $LG_{\ell}^p$ of the paraxial wave equation, where $\ell$ and $p$ are integer. The index $\ell$ is so-called topological charge or orbital angular momentum (OAM), given by the winding number of phase on the wavefront around the vortex [5]. The phase singularity also characterizes a null intensity of the beam cross section. Another parameter $p$ denotes the number of nodal rings about the beam axis. In the paper, we will focus our attention on singly-ringed $LG_{0}^\ell$ modes, and hereafter describe as $LG_{\ell}$. Since vortices make an important role in various branches of physics, such as fluid mechanics, condensed matter physics, and astrophysics, the study of OVs provides a universal viewpoint in optical physics and thus has attracted fundamental interest as well [6, 7].

Of the various fundamental studies, the propagation/interaction dynamics induced by the superposition of the OVs with different charges (composite OVs) have been intensively investigated. In general, the superposition of coherent light fields produces a phase distribution different from that of the component fields. In the case of OVs, owing to the topological phase singularities, composite vortices show characteristic changes along with propagation, such as creation and annihilation of a vortex, attraction, repulsion and rotation of vortex pairs. Since around 1990, numerous theoretical/experimental studies have established a comprehensive framework for analyzing the composite OVs [8, 9, 10, 11, 12, 13]. One of the experimental foundations was laid in [8], where the authors showed that the splitting of a vortex with a charge $|\ell|(|\ell| > 1)$ into $|\ell|$ vortices with a single charge is a common property and that it is reversible by superposition with a Hermite-Gaussian (HG$_{00}$ = $LG_{0}$) beam. They also demonstrated the second-harmonic generation (SHG) with a paired single-charge OV ($\ell = 1 + 1$) from the OV beam with $\ell = 1$, suggesting the SHG formed by $LG_{0}$+ $LG_{2}$. In principle, the OAM is transformed by SHG (in the particular case of [14] from a $\ell = 1$ beam to a $\ell = 2$ beam). Therefore the perturbation of $LG_{0}$ can be attributed to the breakdown of the OAM symmetry (azimuthal symmetry) associated with the SHG process, such as beam walk-off (birefringence) in nonlinear crystals [15, 16].

In this paper, we investigate an interaction of a paired OV generated by SHG in a thin BBO ($\beta$-Barium Borate) crystal. By changing the crystal position along the propagation direction,
a rotational dynamics of the paired OV including creation and collapse of pairing is observed and quantitatively analyzed. Each dynamics observed in the experiment is well reproduced by the theoretical simulation based on the beam walk-off, which produces the changes in relative amplitude and phase of the composite OVs (LG0+LG2) according to the position of the nonlinear crystal. The experiment is simple but provides an accurate evolution of the vortices without changing neither the propagation distance nor crystal length. We also demonstrate the compensation of the walk-off effect by superimposing the LG0 beam in the pump.

2. Second-harmonic vortices with a beam walk-off

In nonlinear wave-conversion/mixing, the crystal asymmetry easily breaks the azimuthal symmetry for OAM conversion. One of the major contributions is given by the walk-off of the nonlinear crystal [15, 16]. The pump depletion (back conversion from the second-harmonic beam to the fundamental beam) also plays an important role in the SHG pumped by a vortex beam [17]. We neglect this process assuming the SHG with low conversion efficiency. In the experimental section, we will show the dynamics of a paired OV as a function of the crystal position. The purpose of this section is to see how the beam walk-off affects the relative amplitude and phase of the second-harmonic OVs according to the crystal position.

In the paper, we restrict our consideration to the case of type I phase-matching between the ordinary fundamental (o o) and extraordinary second-harmonic(e) beams . We also assume the crystal thickness is thin enough to satisfy the weak walk-off regime. The comprehensive analysis for various phase-matching conditions can be found elsewhere [15].

We start considering the SHG process with a walk-off [15]. Since the walk-off breaks the azimuthal symmetry, we rewrite the factor $a_{\ell}$ in Cartesian coordinates as

$$a_{\ell}(x,y,z) = \frac{w_0}{w(z)} \left( \frac{x \pm iy}{w(z)} \right)^{|\ell|} e^{\frac{i}{2}(x^2+y^2)/w(z)^2} e^{-\rho_G^e(z)}.$$  

Here we neglect the wavefront curvature. The Gouy phase $\Phi_G^e$ and beam size $w(z)$ are given by

$$\Phi_G^e(z) = (|\ell| + 1) \tan^{-1} \left( \frac{z}{z_R} \right), \quad w(z) = w_0 \sqrt{1 + \left( \frac{z}{z_R} \right)^2},$$

with the beam waist $w_0$ and Rayleigh range $z_R = kw_0^2/2$.

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where $x + iy (x - iy)$ is taken for positive (negative) $\ell$. From the coupled wave equations in a quadratic nonlinear crystal, the ordinary polarized fundamental and extraordinary second-harmonic field amplitudes ($E^F$ and $E^S$) follow the equations

$$\frac{\partial E^S}{\partial z} = -\alpha \frac{\partial E^S}{\partial x} + gE^F E^F, \quad \frac{\partial E^F}{\partial z} = 0,$$
where $\alpha$ is the walk-off angle of the extraordinary second-harmonic beam, and $g$ is the coupling coefficient. We note again that the pump depletion is neglected owing to the low conversion efficiency. Since we consider a thin crystal (thickness $L$), the beam size can be fixed to be constant throughout the crystal. At the center position of the crystal $z'$, the coupled amplitude equation becomes

$$\frac{\partial E^S}{\partial z} = -\alpha \frac{\partial E^S}{\partial x} + gE^F(x,y,z')E^F(x,y,z').$$

(6)

We note that the envelope function of $E^F$ is given by Eq. (4). For $x_1 = x - \alpha z$, we obtain

$$E^S(x_1,y,z') = g \int_0^L E^F(x_1,y,z')E^F(x_1,y,z')dz.$$  

(7)

To simplify the equation, we choose the normalization of the coordinates, $\xi = x/w(z'), \eta = y/w(z'), t = z/L$ and assume $\gamma = \alpha L/w(z')$. $F = (w_0/w(z'))^2E^S/(gL)$. The second-harmonic envelope at the crystal position $z'$ in Eq. (7) can be transformed to

$$F(\xi, \eta) = e^{-2\eta^2} \int_0^1 (\xi - \gamma t + i\eta)^2 ei -2(\xi - \eta)^2 dt.$$  

(8)

We now consider the simple case of the fundamental beams with $\ell = 1$. Taking into account the assumption that the crystal is sufficiently thin, Eq. (8) gives

$$F(\xi, \eta) = e^{-2\eta^2} \int_0^1 (\xi - \gamma t + i\eta)^2 e^{-2(\xi - \eta)^2} dt 
\approx \left[ \xi - \frac{\gamma}{2} + i \left( \eta - \frac{\gamma}{2\sqrt{3}} \right) \right] \left[ \xi - \frac{\gamma}{2} + i \left( \eta + \frac{\gamma}{2\sqrt{3}} \right) \right].$$

(9)

The latter approximation holds for $|\xi|, |\eta| \ll 1$, which is consistent with the weak walk-off condition. $F(\xi, \eta) = 0$ results in two vortices at $(\xi_0, \eta_0) = (\gamma/2, \pm \gamma/(2\sqrt{3}))$. Here, it should be noted that, while the usual walk-off between fundamental and second-harmonic waves occur in $x$-direction, another type of walk-off, that is, singular-point splitting arises in $y$-direction. In this case, the envelope function of SHG amplitude can be described by a product of two vortices,

$$u_s^x(x,y,z') = ga_1(x-s_1,y-s_2,z')a_1(x-s_1,y+s_2,z')e^{-2i\Phi_0^s(z')},$$

(10)

where $s_1 = \alpha L/2$ and $s_2 = \alpha L/(2\sqrt{3})$. The splitting of the vortices occurs in the $y$-direction (crystal axis) with a separation of $2s_2 = \alpha L/\sqrt{3}$. Equation (10) follows

$$u_s^x(x,y,z') = ge^{-2\xi^2/w(z')^2} \left( \frac{w_0}{w(z')} \right)^2 \frac{1}{w(z')^2} e^{-2(x-s_1)^2 + s_2^2} e^{-2i\Phi_0^s(z')} \left[ u_2(x-s_1,y,z')e^{i\Phi_0^s(z')} + \left( \frac{s_2}{w(z')} \right)^2 u_0(x-s_1,y,z')e^{i\Phi_0^s(z')} \right],$$

(11)

showing a superposition of LG0 and LG2 with different amplitude and phase. The amplitude of the second-harmonic beam consisting of a product of two vortices thus can be transformed to a series of OVs with different charges. It should be emphasized here that a simple SHG process with a walk-off introduces a contamination of $\ell = 0$ beam together with topological charge doubling. In addition, both the relative amplitude $\sim 1/w^2$ and phase difference $\Phi_0^s - \Phi_0^s$ depends on $z'$. As a result, we can evaluate the interaction dynamics of the paired vortex as a function of the crystal position in $z$-direction.
3. Experimental

We investigate the interaction dynamics of the second-harmonic OVs generated by LG1 pump. Figure 1 (a) shows the schematic of the experimental setup. A mode-locked Ti:sapphire laser with a repetition rate of $\sim 90$ MHz and a center wavelength of $\sim 800$ nm was used for the light source. The LG1 pulse (fundamental pulse for the SHG) was created from the HG00 pulse from the laser by passing through a spatial phase shifter (SPS) and an astigmatic mode converter (AMC) [18]. The SPS consists of two thin glass plates and inserted into the beam to create a HG10 with a relative phase shift of $\pi$ between the left and right halves of the HG00 beam [19]. The HG10 pulse is then fed to an astigmatic lens system consisting of two spherical and cylindrical lens.

Figure 1 (b) shows the CCD images of the output of AMC (left), which produces a dark-hole at the center of the beam. To check the topological charge of the pulsed beam, we also demonstrate the interferogram (right in Fig. 1 (b)) with a tilted reference HG00 pulse without delay. The fork-like fringe pattern clearly indicates the LG with a topological charge of unity (LG1) is dominant at this stage. The temporal chirp of the pulse was compensated by a pair of chirp mirrors just after the laser source. The average power of the LG1 was $\sim 10$ mW and its pulse duration evaluated by autocorrelation measurements was $\sim 100$ fs.

The LG1 was focused onto a BBO crystal (CASIX, thickness 0.1 mm) using a lens ($f=100$ mm), and the output SHG with type I phase matching was detected by a charge-coupled device (CCD) camera after passing through a UV pass filter. The BBO has a large walk-off angle ($\sim 3.9^\circ$ at 800 nm pump) [20], introducing an azimuthal asymmetry of the second-harmonic...
where \( z \) is efficient at \( z = 0 \) and decreases symmetrically with respect to \( z = 0 \), indicating that the perturbation of LG0 is efficient at \( z = 0 \) and decreases as increasing \( |z| \). These \( z \)-dependent splitting suggests the walk-off origin, which will be confirmed below.

We now compare the experimental data with the predictions in Sect.2. The intensity of the vortex field is \( I = |w_0^2|^2 \), where \( w_0^2 \) is given by in Eq. (11). In the calculations, only \( s_2 \) is a variable parameter to be optimized. We used \( s_2 = 4.1 \) \( \mu \)m, the validity of which will be discussed later. The intensity distributions of the composite OVs are shown in the upper part of Fig. 4, each
of which reproduces well the paired OV dynamics in Fig. 2. In the lower part, we also plot the phase distributions in order to clarify the positions of the vortices. Within the focal region, a distinct paired vortex exists owing to the intentional contamination of LG 0 \( \propto \frac{1}{w^2} \). When moving out of focal region, the pair rotates according to the relative Gouy rotation \( \Delta \Phi_G = \Phi_G^2 - \Phi_G^0 \), and its separation decreases as \( \text{LG}_0 \) decreases.

The quantitative comparisons can be realized in Fig. 5, where we re-plot the experimental data of Fig. 3. In Fig. 5 (b), the rotation angle \( \phi_v \), which reflects the relative phase described by \( \Delta \Phi_G = 2 \tan^{-1}(z/z_R) \), is consistent with the experimental data. Our simple experiment thus provides an evaluation of the the Gouy phase with a fixed detection geometry. On the other hand, the splitting \( r_v \) in Fig. 5 (c) also shows good agreement with experimental results. We note that \( r_v \) is determined by the ratio of the amplitude between \( \text{LG}_0 \) and \( \text{LG}_2 \), which is given by \( (s_2/w)^2 \) in Eq. (11). From the experimental data \( r_v \) at \( z' = 0 \), \( s_2 = r_v/2 \) is evaluated to be 4.1 \( \mu \)m. Within the assumption that the beam walk-off is weak, \( s_2 \) can be described by \( \alpha'L/(2\sqrt{3}) \), where \( \alpha' \) is the external walk-off angle in the air which satisfies \( \sin(\alpha'/\sqrt{3}) = n_e(400 \text{ nm}) \sin(\alpha/\sqrt{3}) \) from the Snell’s law. In the present case, we obtained the internal walk-off angle \( \alpha = 5.1^\circ \), which is almost consistent with the typical value of BBO for the type-I phase-matching (**3.9**° at 800 nm pump) [20].

The analysis mentioned above successfully demonstrates that the charge splitting due to beam walk-off can be expanded into the composite OVs including \( \text{LG}_0 \). Therefore we can
remove the splitting by introducing LG\textsubscript{0} (HG\textsubscript{00}) as another fundamental source. Although such cancellation of LG\textsubscript{0} perturbation using coherent beam addition has already been reported in [8], we employ this technique to compensate the incomplete OAM transfer in the nonlinear conversion process. Since the imperfect OAM restricts the feasibility of OVs, the compensation demonstrated here will contribute to the practical use of the nonlinear OV applications [24, 25].

To prepare such composite OVs (LG\textsubscript{1}+\textit{e}\textsuperscript{\textit{i}\textphi\textsubscript{\textit{p}}}LG\textsubscript{0}) in the pump, the laser pulses (HG\textsubscript{00}) were splitted into two optical paths, one of which transfers HG\textsubscript{00} to LG\textsubscript{1} using SPS and AMC. Another path carries LG\textsubscript{0} beam through the optical delay line, which consists of a retro-reflector mounted upon a translation stage and a mirror upon a piezoelectric transducer. The former accounts for the difference between two optical paths lengths within the pulse duration while the latter provides a fine position to determine the relative phase \textphi\textsubscript{\textit{p}} with respect to LG\textsubscript{1}. The relative amplitude between LG\textsubscript{1} and LG\textsubscript{0} was controlled by a neutral density filter. The second-harmonic conversion and its detection schemes are the same as in Fig. 1 (a).

The upper part of Fig. 6 shows a series of CCD images of the SHG obtained at various \textphi\textsubscript{\textit{p}} at a fixed crystal position \textit{z}' = 0. The OV pair changes its separation according to \textphi\textsubscript{\textit{p}}. A significant reduction of the splitting was observed in the anti-phase condition \textphi\textsubscript{\textit{p}} \sim \pi. For clarity, the SHG images obtained by introducing individual pump beams (LG\textsubscript{1} and HG\textsubscript{00}) are shown in the lower part. It is important to note that optimization of the relative amplitude is necessary to realize the absence of splitting. In addition, the optimized amplitude varies with \textit{z}', indicating that the splitting observed in our experiment does not arise from the fundamental beam but from the SHG.

4. Summary

In summary, we demonstrated an interaction of a paired second-harmonic OV by changing the position of a thin nonlinear crystal (BBO) along the propagation direction. The observed rotational dynamics including creation and collapse of pairing were well reproduced by the theoretical prediction based on the beam walk-off in the crystal. The quantitative analysis indicated that the rotation angle of the paired vortex manifests the Gouy phase while the splitting
provides the walk-off angle of the crystal. The analysis also mentioned that the charge splitting can be expanded into the composite OVs including LG₀, the subtraction of which was experimentally realized by the superposition of LG₀ with an anti-balanced phase in the fundamental beam.

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