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IRREVERSIBLE INVESTMENT, OPERATING FLEXIBILITY, AND TIME LAGS

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We consider an optimal investment problem when a firm such as an electric power company has the operational flexibility to expand and contract capacity with fixed cost. This problem is formulated as an impulse control problem combined with optimal stopping. Consequently, we obtain optimal investment timing, optimal capacity expansion and contraction timing, and the investment value. We also show investment, capacity expansion and contraction rule are influenced by the price volatility and the initial capacity is also influenced by the ratio between base-load plant and peak-load plant. In addition, we investigate how time lag between investment and operation influences the investment rule.

Keywords: Real options; investment and operation under uncertainty; time lag.

1. Introduction

Throughout the last two decades, analysis of investment and evaluation of actual investment project under uncertainty have been studied by many research groups. Especially, projects which the firm have such as investment, operation, and abandonment (i.e., a life cycle of the project) in various industries have been analyzed by means of real options approach (Dixit and Pindyck, 1994; Trigeorgis, 1996). The

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firm has the investment option before the operation time, and sequential investment option such as time-to-build in construction time (Majd and Pindyck, 1987; Bar-Ilan and Strange, 1998). Furthermore, in operation time, the firm has the option to start-up and shut-down the project (Dixit, 1989; Dixit and Pindyck, 1994), or to expand and contract the capacity (Dixit and Pindyck, 2000; Guo and Pham, 2005; Goto, Takashima, and Tsujimura, 2006). Thus, these studies could be divided roughly into two categories of investment and operation. The optimal investment rule problem is one of major topics in real options theory, and is formulated as an optimal stopping problem. On the other hand, optimal operation problems include entry and exit of project, and the capacity expansion and contraction, and are formulated as singular stochastic control (Guo and Pham, 2005) and stochastic impulse control problems (Vollert, 2003; Goto, Takashima, and Tsujimura, 2007).

In electric power industry, both optimal investment and operation rules can be important issues. It takes more than five years to construct a power plant (IEA/OECD-NEA, 2005). Then, the determination of investment timing is a significant management strategy. In operation period, the firm has to adopt the start and stop strategies for the power plant to change the generation capacity with the demand and fuel cost trends. Therefore, investment and operation problems for the power plant have attracted growing attention, and there exist the following various researches: Pindyck (1993) evaluates investment of nuclear power plant under cost uncertainty. Thompson, Davison and Rasmussen (2004) analyze operation of hydroelectric and thermal power plants. Gollier et al. (2005) examine two investment projects with respect to a sequence of small nuclear power plants and a large nuclear power plant. These studies have not considered both problems of investment and operation simultaneously. Näsäkkälä and Fleten (2005) analyze the decision to upgrade a base load plant to a peak load plant under uncertainty of the spark spread which is a sum of short-term deviations and equilibrium price. Takashima et al. (2007) examine the optimal timing for decommissioning and refurbishment of nuclear power plants. Siddiqui and Maribu (2009) evaluate the investment and upgrade strategy, such as capacity and heat exchanger, for the microgrid's distributed generation.

Analyses of investment problem using stochastic impulse control combined with optimal stopping include the following previous works. Dixit and Pindyck (1994) and Brennan and Schwartz (1985) consider the operation problem such as entry-exit with abandonment of project. Although these studies provide the solution in specific case, Zervos (2003) solves completely this problem. To our knowledge, there exist no model like these for analysis of both investment and operation. As Zervos (2003) points out, the impulse control strategy for operation in this study is not the standard framework because each capacity of controlled variable is given. The solution of this optimal problem can be obtained due to this setting even if relatively complicated model is formulated.

The closest work to this paper includes those of Aguerrevere (2003) and Sødal (2006). Aguerrevere (2003) analyzes the optimal capacity choice taking into account investment option and operating flexibility [a], which implies that the unit of capacity can be started up or shut-down without cost. Sødal (2006) investigatesd entry and exit problem with construction, scrapping, and investment lags. In actual operation problem, the capacity may be expanded and contracted with fixed cost. We analyze the optimal investment and operation problems taking into account these costs, and additionally, the dependence of initial capacity on uncertainty and cost. We also consider the time lags between the time of investment and operation.

In this paper, we investigate the firm's optimal investment problem with operational flexibility under output price uncertainty. To solve the problem, we formulate it as an impulse control problem combined with optimal stopping. Consequently, we show the optimal investment rule and value with capacity expansion and contraction, so that the initial capacity in operation time is obtained. We also examine how the price volatility and investment cost influence the initial capacity. Furthermore, we investigate the time lag between the investment and the operation influences the initial capacity.

The paper is organized as follows. In the next section, we develop the basic model, and derive the solution. In Section 3, we describes the model taking into account the effect of time lag. Section 4 presents some comparative statics with regard to price volatility, capacity ratio of plant, and time lags. Finally, Section 5 summarizes the paper and provides a direction for future works.

2. Analytical Framework

In many actual situations like electric power plant projects, the firm first decides to investment project. Then, the firm operates the capacity of the plant. When the output price is higher than a level or lower than another level in operation period, the firm changes the capacity size with a fixed cost. In this paper, we assume that the firm switches between the larger size and the smaller size of the capacity. We therefore consider the optimal investment strategy with operational flexibility switching two given capacities.

We assume that output price follows the geometric Brownian motion:

$$dP_t = \mu P_t dt + \sigma P_t dW_t, \quad P_0 = p, \tag{2.1}$$

where μ is the expected price growth rate, σ is the volatility of price, and W_t is a standard Brownian motion adapted to a filtration $\mathcal{F}_t$ of a probability space $(\Omega, \mathcal{F}, P)$. Suppose that the firm invests at time $\tilde{T}$, and then begins the operation. In operation period, the capacity of plants can be changed to operation state $Z_t \in \{0, 1\}$ with the fixed costs. $Z_t = 0$ represents the small capacity and $Z_t = 1$ represents the large one. Then the firm's operation strategy v is defined by the sequence of the pair of the ith switching time τ_i and ith switching operation ζ_i:

[a] In addition to these characteristic, the capacity choice with time-to-build is also analyzed.

$v = (\tau_i, \zeta_i)_{i \geq 0}$. Note that $\zeta_i = Z_{\tau_i}$. Then the firm's investment and operation strategy ψ is defined as the following:

$$\psi = (\tilde{T}, v). \tag{2.2}$$

With investment and operation rule ψ, the firm's expected discounted value of profit is given by the following equation,

$$\begin{aligned} J(p;\psi) = \mathbb{E}\Bigg[\int_{\tilde{T}}^{\infty} & e^{-rt}\Big[\{QP_t - Q(\alpha C_b + (1-\alpha)C_p)\}\, Z_t \\ & + \alpha Q(P_t - C_b)(1 - Z_t)\Big]dt - e^{-r\tilde{T}}Q\left(\alpha I_b + (1-\alpha)I_p\right) \\ & - \sum_{i=1}^{\infty} e^{-r\tau_i} K(Z_{\tau_i -}, Z_{\tau_i})\mathbf{1}_{\{\tau_i < \infty\}}\Bigg], \end{aligned} \tag{2.3}$$

where Q is a total capacity of plant which the firm has, $0 < \alpha < 1$[b] is a capacity ratio of plant which can not be shut down (i.e., base-load plant), $1 - \alpha$ is a capacity ratio of plant which can be shut down (i.e., peak-load plant), αQ is a capacity of base-load plant, C_b is its operation cost, I_b is its construction cost, $(1-\alpha)Q$ is a capacity of peak-load plant, C_p is its operation cost, I_p is its construction cost,

$$K(0,0) = K(1,1) = 0, \tag{2.4}$$
$$K(0,1) = K_e, \tag{2.5}$$
$$K(1,0) = K_c \tag{2.6}$$

are fixed costs to change operation, and r is a discount rate. We must have $r > \mu$ in order to ensure that the firm value is finite.

The firm's problem is to maximize the expected discounted profit J by selecting an optimal investment and operation strategy. The value function is expressed by the following equation:

$$V(p) = \sup_{\psi \in \Psi} J(p;\psi) = J(p;\psi^*), \tag{2.7}$$

where Ψ is the collection of admissible controls and ψ^* is an optimal investment and operation strategy.

3. Investment And Operation Strategy

In this section, we solve the firm's problem (2.7). We can solve this investment problem by working backward, i.e., by first finding the value of the operation and then finding the value of the investment. Thus Eq.(2.7) is equivalent to the following calculation. To this end, we first solve the firm's operation problem. Next, we solve the firm's investment problem.

[b] Although it is theoretically possible to set $0 \leq \alpha \leq 1$, we do not consider the setting of $\alpha = 0, 1$ in this paper.

3.1. *Operation value*

We consider the operation value for two states of small and large capacities. The ordinary differential equation, which is satisfied by the operation value for a state of small capacity ϕ_0, is derived from Bellman's optimality principle, See, for example, Dixit and Pindyck (1994),

$$\frac{1}{2}\sigma^2 p^2 \phi_0'' + \mu p \phi_0' - r\phi_0 + \alpha Q(p - C_b) = 0. \tag{3.8}$$

The general solution of Eq.(3.8) is given by the following equation:

$$\phi_0(p) = A_1 p^{\lambda_1} + A_2 p^{\lambda_2} + \frac{\alpha Q p}{r-\mu} - \frac{\alpha Q C_b}{r}, \tag{3.9}$$

where A_1 and A_2 are constants to be determined and

$$\lambda_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1, \quad \lambda_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0. \tag{3.10}$$

Using the similar methods discussed above, it is straightforward to exhibit that the operation value for large capacity ϕ_1 must satisfy the following ordinary differential equation:

$$\frac{1}{2}\sigma^2 p^2 \phi_1'' + \mu p \phi_1' - r\phi_1 + Qp - Q(\alpha C_b + (1-\alpha)C_p) = 0. \tag{3.11}$$

The general solution of Eq.(3.11) is given by

$$\phi_1(p) = B_1 p^{\lambda_1} + B_2 p^{\lambda_2} + \frac{Qp}{r-\mu} - \frac{Q(\alpha C_b + (1-\alpha)C_p)}{r}, \tag{3.12}$$

where B_1 and B_2 are constants to be determined. The operation value in Eqs. (3.9) and (3.12) must satisfy the following boundary conditions:

$$\phi_0(0) = -\frac{\alpha Q C_b}{r}, \qquad \lim_{p\to\infty}(B_1 p^{\lambda_1} + B_2 p^{\lambda_2}) = 0. \tag{3.13}$$

These imply that $A_2 = 0$ and $B_1 = 0$ in Eqs. (3.9) and (3.12), respectively. Moreover, at thresholds, p_e, of which the capacity is changed from small state to large one with a cost K_e, the operation value must satisfy the following boundary conditions,

$$\phi_0(p_e) = \phi_1(p_e) - K_e, \qquad \phi_0'(p_e) = \phi_1'(p_e). \tag{3.14}$$

These conditions are value-matching and smooth-pasting conditions, respectively. Likewise, at threshold p_c switching from large capacity to small one with a cost K_c, the conditions are:

$$\phi_1(p_c) = \phi_0(p_c) - K_c, \qquad \phi_1'(p_c) = \phi_0'(p_c). \tag{3.15}$$

Substituting Eqs (3.9) and (3.12) into Eqs (3.24) and (3.15), the equations are rewritten as follows:

$$A_1 p_e^{\lambda_1} + \frac{\alpha Q p_e}{r-\mu} - \frac{\alpha Q C_b}{r} = B_2 p_e^{\lambda_2} + \frac{Q p_e}{r-\mu} - \frac{Q(\alpha C_b + (1-\alpha)C_p)}{r} - K_e \tag{3.16}$$

$$A_1\lambda_1 p_e^{\lambda_1-1} + \frac{\alpha Q}{r-\mu} = B_2\lambda_2 p_e^{\lambda_2-1} + \frac{Q}{r-\mu} \tag{3.17}$$

$$B_2 p_c^{\lambda_2} + \frac{Qp_c}{r-\mu} - \frac{Q(\alpha C_b + (1-\alpha)C_p)}{r} = A_1 p_c^{\lambda_1} + \frac{\alpha Q p_c}{r-\mu} - \frac{\alpha Q C_b}{r} - K_c \tag{3.18}$$

$$B_2\lambda_2 p_c^{\lambda_2-1} + \frac{Q}{r-\mu} = A_1\lambda_1 p_c^{\lambda_1-1} + \frac{\alpha Q}{r-\mu} \tag{3.19}$$

These four equations provide simultaneous nonlinear equation system, which can be solved A_1, B_2, p_e, and p_c by means of numerical calculation such as Newton-Raphson method.

3.2. *Investment option*

Since the firm has the two operation values $\phi_0(p)$ and $\phi_1(p)$, we consider two investment options $F_0(p)$ and $F_1(p)$, respectively. The investment option F_j must satisfy the following ordinary differential equation:

$$\frac{1}{2}\sigma^2 p^2 F_j'' + \mu p F_j' - rF_j = 0, \tag{3.20}$$

where $j = 0, 1$. The operation starts in the initial state of small capacity when $j = 0$ and the large capacity when $j = 1$. The general solution of Eq. (3.20) is given by the following equation:

$$F_j(p) = C_1 p^{\lambda_1} + C_2 p^{\lambda_2}, \tag{3.21}$$

where C_1 is the unknown constant, $C_2 = 0$ due to boundary condition, $F_j(0) = 0$, and λ_1 and λ_2 are obtained by Eq. (3.10).

The initial capacity of the operation Q^* depends on the threshold value of investment p^* and the investment value at threshold $F_j(p^*)$. The dependence of the initial capacity on each threshold value such as p_e, p_c, and p^* can be represented as follows:

$$Q^* = \begin{cases} Q & (p_e < p^*), \\ Q \text{ or } \alpha Q & (p_c \le p^* \le p_e), \\ \alpha Q & (p_c > p^*). \end{cases} \tag{3.22}$$

Since the firm maximizes the value at p^* in the region of $p_c \le p^* \le p_e$, the initial capacity is determined by the following method:

$$Q^* = \begin{cases} Q & (F_1(p^*) \ge F_0(p^*)), \\ \alpha Q & (F_1(p^*) < F_0(p^*)). \end{cases} \tag{3.23}$$

For $F_1(p^*) \ge F_0(p^*)$, from value-matching and smooth-pasting conditions,

$$F_1(p^*) = \phi_1(p^*) - Q\left(\alpha I_b + (1-\alpha)I_p\right), \qquad F_1'(p^*) = \phi_1'(p^*), \tag{3.24}$$

the equations can be written as

$$C_1 {p^*}^{\lambda_1} = B_2 {p^*}^{\lambda_2} + \frac{Qp^*}{r-\mu} - \frac{Q(\alpha C_b + (1-\alpha)C_p)}{r} - Q\left(\alpha I_b + (1-\alpha)I_p\right), \tag{3.25}$$

$$C_1\lambda_1 {p^*}^{\lambda_1-1} = B_2\lambda_2 {p^*}^{\lambda_2-1} + \frac{Q}{r-\mu}. \tag{3.26}$$

Eliminating C_1 from Eqs. (3.25) and (3.26), the nonlinear equation for p^* can be described as

$$(\lambda_1-\lambda_2)B_2{p^*}^{\lambda_2} + (\lambda_1-1)\frac{Q}{r-\mu}p^* - \frac{Q(\alpha C_b+(1-\alpha)C_p)\lambda_1}{r} \\ -Q\left(\alpha I_b+(1-\alpha)I_p\right)\lambda_1 = 0. \tag{3.27}$$

Equation (3.27) has no analytical solution, therefore it must be solved numerically.

Similarly, for $F_1(p^*) < F_0(p^*)$, we obtain the following equations,

$$C_1{p^*}^{\lambda_1} = A_1{p^*}^{\lambda_1} + \frac{\alpha Q p^*}{r-\mu} - \frac{\alpha Q C_b}{r} - Q\left(\alpha I_b+(1-\alpha)I_p\right), \tag{3.28}$$

$$C_1\lambda_1{p^*}^{\lambda_1-1} = A_1\lambda_1{p^*}^{\lambda_1-1} + \frac{\alpha Q}{r-\mu}. \tag{3.29}$$

Solving Eqs. (3.28) and (3.29) for p^* and C_1, respectively, gives

$$p^* = \frac{r-\mu}{\alpha Q}\frac{\lambda_1}{\lambda_1-1}\left[\frac{\alpha Q C_b}{r} + Q\left(\alpha I_b+(1-\alpha)I_p\right)\right], \tag{3.30}$$

$$C_1 = A_1 + \frac{1}{\lambda_1}\frac{\alpha Q}{r-\mu}{p^*}^{1-\lambda_1}. \tag{3.31}$$

Unlike in the case of $F_1(p^*) \geq F_0(p^*)$, we can obtain the analytical solutions for p^* and C_1.

4. Time Lags

In the previous section, we have considered the investment and operation strategy in which the firm begins the operation as soon as the construction investment. However, in realistic situations, there exists a time lag between the investment decision and the beginning time of the operation. Thus, in this section, we analyze the investment and operation problems with a time lag. We denote the investment lag or time to build by $\delta > 0$. When the profit flow of the operation for each capacity at time t is $\pi_t^j (j = 0, 1)$, the expected value of operation for each capacity is given by the following equation:

$$\begin{aligned}\Xi_j &= \mathbb{E}\left[\int_\delta^\infty e^{-rt}\ \pi_t^j\ dt\right] \\ &= e^{-r\delta}\ \mathbb{E}\left[\int_0^\infty e^{-rt}\ \pi_{t+\delta}^j\ dt\right] \\ &= e^{-r\delta}\ \mathbb{E}\left[\Pi_{t+\delta}^j\right],\end{aligned} \tag{4.32}$$

where $j = 0, 1$ denote the state of small and large capacity, respectively. In the state of small capacity, $\mathbb{E}[\cdot]$ in Eq. (4.32) can be written as

$$\begin{aligned}\mathbb{E}\left[\Pi^0_{t+\delta}\right] &= \int_0^{p_e}\left[A_1 p_{t+\delta}^{\lambda_1} + \frac{\alpha Q p_{t+\delta}}{r-\mu} - \frac{\alpha Q C_b}{r}\right] f(p_{t+\delta})\; dp_{t+\delta} \\ &\quad + \int_{p_e}^{\infty}\left[B_2 p_{t+\delta}^{\lambda_2} + \frac{Q p_{t+\delta}}{r-\mu} - \frac{Q(\alpha C_b + (1-\alpha)C_p)}{r} - K_e\right] f(p_{t+\delta}) \\ &\quad \times dp_{t+\delta}, \end{aligned} \tag{4.33}$$

where $f(\cdot)$ is the probability density function of output price. Substituting Eq. (4.33) into Eq. (4.32) and calculating the expected values in Eq. (4.32) (See, for example, in Aguerrevere (2003); Bar-Ilan and Strange (1996) for more detail), the operation value for small capacity with time lag is given by the following equation:

$$\begin{aligned}\Xi_0 &= e^{-r\delta}\Bigg[A_1\Phi(\nu_0 - \lambda_1\sigma\sqrt{\delta})\mathbb{E}\left[p_{t+\delta}^{\lambda_1}\right] + \Phi(\nu_0 - \sigma\sqrt{\delta})\frac{\alpha Q}{r-\mu}\mathbb{E}\left[p_{t+\delta}\right] \\ &\quad -\Phi(\nu_0)\frac{\alpha Q C_b}{r} + B_2\left(1 - \Phi(\nu_0 - \lambda_2\sigma\sqrt{\delta})\;\right)\mathbb{E}\left[p_{t+\delta}^{\lambda_2}\right] \\ &\quad + \left(1 - \Phi(\nu_0 - \sigma\sqrt{\delta})\;\right)\frac{Q}{r-\mu}\mathbb{E}\left[p_{t+\delta}\right] \\ &\quad - (1-\Phi(\nu_0))\frac{Q(\alpha C_b + (1-\alpha)C_p)}{r} - (1-\Phi(\nu_0))\,K_e\Bigg] \\ &= A_1\Phi(\nu_0 - \lambda_1\sigma\sqrt{\delta})p^{\lambda_1} + \Phi(\nu_0 - \sigma\sqrt{\delta})\frac{\alpha Q p}{r-\mu}e^{-(r-\mu)\delta} \\ &\quad -\Phi(\nu_0)\frac{\alpha Q C_b}{r}e^{-r\delta} + B_2\left(1 - \Phi(\nu_0 - u\sigma\sqrt{\delta})\;\right)p^{\lambda_2} \\ &\quad + \left(1 - \Phi(\nu_0 - \sigma\sqrt{\delta})\;\right)\frac{Qp}{r-\mu}e^{-(r-\mu)\delta} \\ &\quad - (1-\Phi(\nu_0))\frac{Q(\alpha C_b + (1-\alpha)C_p)}{r}e^{-r\delta} - (1-\Phi(\nu_0))\,K_e e^{-r\delta}, \end{aligned} \tag{4.34}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function and $\nu_0 = \left(\ln p_e - \ln p - \left(\mu - \frac{\sigma^2}{2}\right)\delta\right) / \left(\sigma\sqrt{\delta}\right)$.

Similarly, in the state of large capacity, $\mathbb{E}[\cdot]$ can be written as

$$\begin{aligned}\mathbb{E}\left[\Pi^1_{t+\delta}\right] &= \int_0^{p_c}\left[A_1 p_{t+\delta}^{\lambda_1} + \frac{\alpha Q p_{t+\delta}}{r-\mu} - \frac{\alpha Q C_b}{r} - K_c\right] f(p_{t+\delta})\; dp_{t+\delta} \\ &\quad + \int_{p_c}^{\infty}\left[B_2 p_{t+\delta}^{\lambda_2} + \frac{Q p_{t+\delta}}{r-\mu} - \frac{Q(\alpha C_b + (1-\alpha)C_p)}{r}\right] f(p_{t+\delta}) \\ &\quad \times dp_{t+\delta}. \end{aligned} \tag{4.35}$$

Thus, the operation value for large capacity is given by

$$\begin{aligned}\Xi_1 = {} & A_1\Phi(\nu_1 - \lambda_1\sigma\sqrt{\delta})p^{\lambda_1} + \Phi(\nu_1) - \sigma\sqrt{\delta})\frac{\alpha Qp}{r-\mu}e^{-(r-\mu)\delta} \\ & -\Phi(\nu_1)\frac{\alpha QC_b}{r}e^{-r\delta} - \Phi(\nu_1)K_c e^{-r\delta} + B_2\left(1 - \Phi(\nu_1 - u\sigma\sqrt{\delta})\right)p^{\lambda_2} \\ & + \left(1 - \Phi(\nu_1 - \sigma\sqrt{\delta})\right)\frac{Qp}{r-\mu}e^{-(r-\mu)\delta} \\ & - (1 - \Phi(\nu_1))\frac{Q(\alpha C_b + (1-\alpha)C_p)}{r}e^{-r\delta}, \end{aligned} \tag{4.36}$$

where $\nu_1 = \left(\ln p_c - \ln p - \left(\mu - \frac{\sigma^2}{2}\right)\delta\right) / \left(\sigma\sqrt{\delta}\right)$. From Eqs (3.21), (4.34), and (4.36), the optimal investment strategy and the project value with time lag can be obtained by using value-matching and smooth-pasting conditions.

5. Numerical Example

From the model described above, the investment value, optimal investment and operation strategy can be shown. In the following section, we present the numerical examples of the investment option value and the dependence of investment and operation strategy on uncertainty and plant ratio. In addition, we show the effect of time lag between investment and operation on the investment strategy.

5.1. *Investment strategy and initial capacity*

Figure 1 shows sample paths of output price and capacity. For these paths, the firm would invest in time of 7.2 because output price reaches the critical price of 15, p^*,

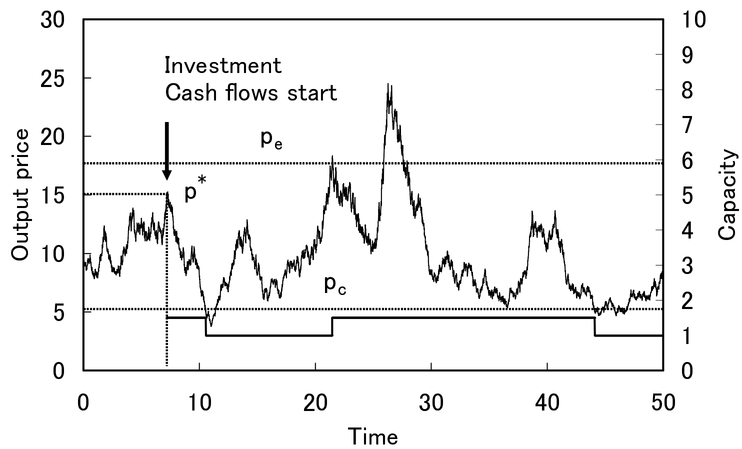

Fig. 1. Simulated path of investment and operation strategies. This figure shows the simulation of output price and capacity. The upper dashed line is threshold price of capacity expansion, and the lower dashed line is threshold price of capacity contraction.

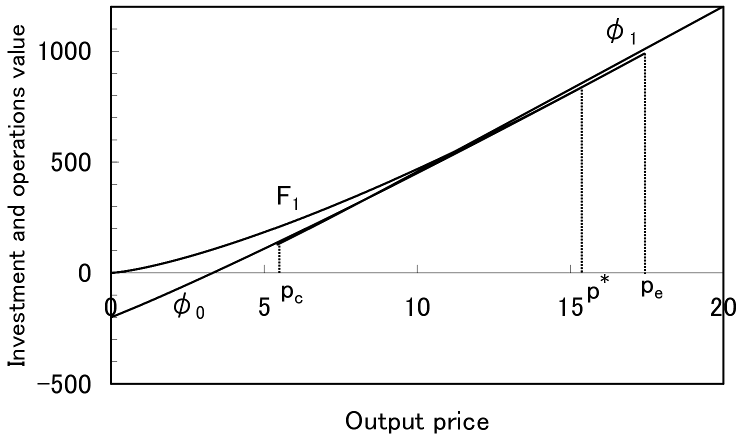

Fig. 2. Investment and operation values as a function of output price. Base case parameters are as follows: $\mu = 0.03$, $\sigma = 0.2$, $r = 0.05$, $Q = 1.5$, $\alpha = 0.67$, $C_b = 5$, $C_p = 10$, $K_e = 20$, $K_c = 10$, $I_b = 80$, and $I_p = 40$.

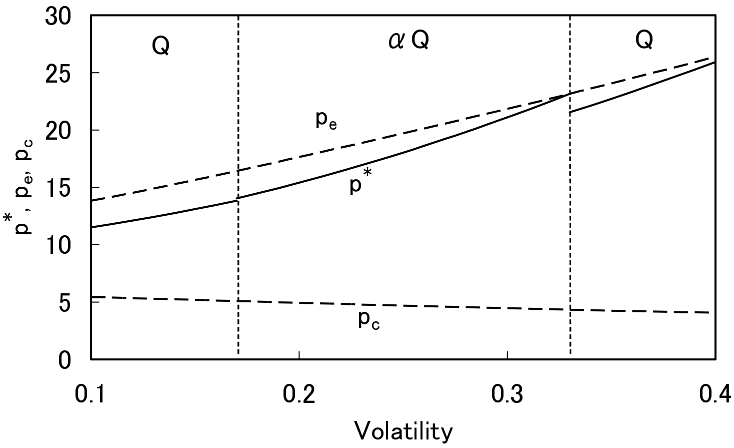

Fig. 3. Dependence of thresholds of investment, expansion, and contraction on price volatility. The upper and lower dashed line, and solid line show the threshold of expansion, contraction, and investment, respectively.

described as optimal investment rule. Then, the operation starts and the capacity can be changed with price level. In this figure, the capacity varies from large to small when price reaches the critical price of 5, p_c (e.g., in time of 10.5 and 44), and vice versa if price reaches the critical price of 17.5, p_e (e.g., in time of 21).

We can obtain the solution to optimal investment and operation strategy from

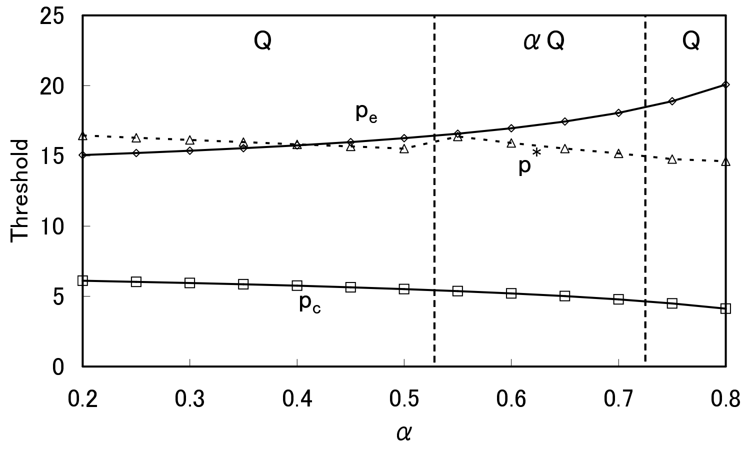

Fig. 4. Dependence of thresholds of investment, expansion, and contraction on ratio of base-load plant. Rhombus, triangle, and square represent thresholds of investment, expansion, and contraction, respectively.

the model presented in the previous section. Given two states of small and large capacities in operation period, Eqs. (3.27) and (3.31) give the optimal strategy of investment in each case. In this section, the investment and the operation values and the critical price at which investing and switching capacity are optimal are provided.

In Figure 2, the solutions for F_1, ϕ_1, and ϕ_0 and each critical price of p^*, p_e, and p_c for base case parameters, $\mu = 0.03$, $\sigma = 0.2$, $r = 0.05$, $Q = 1.5$, $\alpha = 0.67$, $C_b = 5$, $C_p = 10$, $K_e = 20$, $K_c = 10$, $I_b = 80$, and $I_p = 40$, are shown. The differences between values of ϕ_0 and ϕ_1 at each critical prices as p_c and p_e correspond to costs switching from one capacity state to another, respectively.

The dependence of thresholds of investment, expansion, and contraction on price volatility is illustrated in Figure 3. It is clear from this figure that as the price volatility increases, the capacity expansion p_e increase, while that of the capacity contraction p_c decreases. Although the thresholds of investment p^* increases with the the price volatility as the the capacity expansion, at $\sigma = 0.33$, p^* is decreasing. This implies the decrease of each opportunity, especially, in operation period, the probability of inaction becomes higher as results in Dixit and Pindyck (1994). As stated above, the initial capacity is determined from investment value at optimal timing $F(p^*)$. As a result, the curve of investment threshold p^* is discontinuous in σ (e.g., of 0.17 and 0.33). It can be seen that the initial capacity is the large state for small σ, and the small one for large σ. This result describes actual situations, and shows a standard characteristic of capacity choice, similar to earlier works as Pindyck (1988), Dixit and Pindyck (1994), and Aguerrevere (2003). However, the effect of entry and exit setting leads to a different result from previous works, which show that the initial capacity is large state for σ large enough. This is because that the firm wait to invest, and price increases sufficient to expand the capacity.

Figure 4 shows the dependence of thresholds of investment, expansion, and contraction on ratio of base-load plant. As can be seen from this figure, the threshold of expansion increases and that of contraction decreases as the ratio of base-load plant α becomes large. This is because the opportunities of capacity expansion and contraction decrease due to the reduction of the ratio of peak-load plant which has the flexible operation strategy as shut-down option. In addition, for small ratio of base-load plant, the initial capacity is large state Q because the threshold of capacity expansion becomes relatively small. On the other hand, when the ratio of base-load plant become large as 0.8, the initial capacity is large state Q because of the reduction of opportunity for shut-down. It is found that the initial capacity is small state αQ for the range of α from 0.52 to 0.72.

5.2. *Time lags*

In this section, we investigate the effect of time lag on investment rule for various volatilities and plant ratios by the model presented in Section 4.

In figure 5, the dependence of investment threshold on time-lag for various volatilities is shown. The effect of the time lag induces increment of investment opportunity by decreasing the investment option. The result of this analysis represents the actual situation as construction investment . Bar-Ilan and Strange (1996) and Vollert (2003) also show the similar results. Also it seems that, for large time lag, the probability of high output price is large at the start time of the operation. That is, the price level can be higher than threshold price of capacity expansion.

We set small range of the time lag as $0 - 0.05$ in order to investigate the effect of

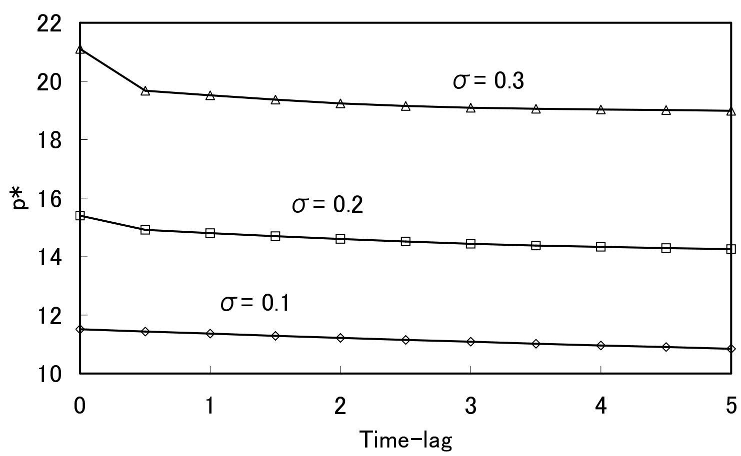

Fig. 5. Dependence of investment threshold on time-lag, for various volatilities (σ = 0.1, 0.2, 0.3).

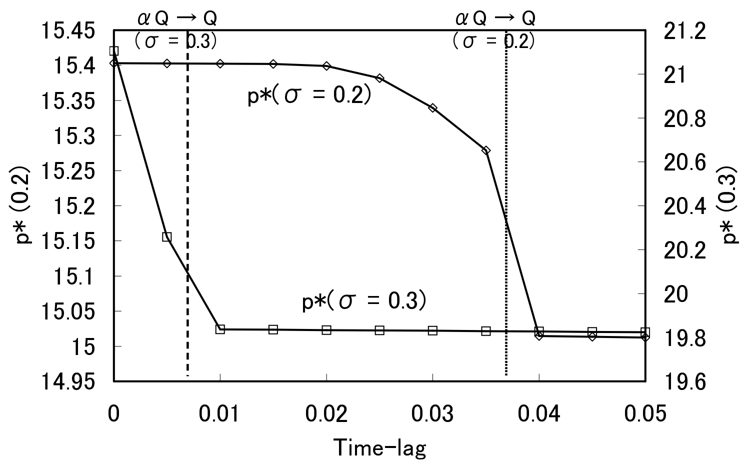

Fig. 6. Dependence of investment threshold on time-lag, for various volatilities. Range of time-lag level is 0 – 0.05.

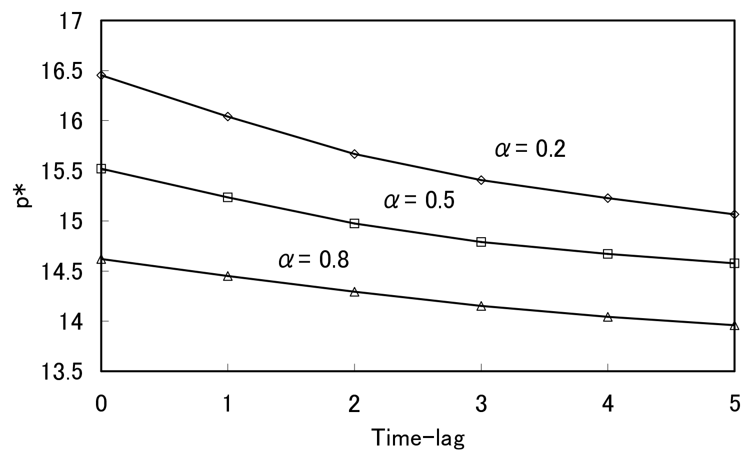

Fig. 7. Dependence of thresholds of investment on time-lag, for various plant ratio ($\alpha = 0.2$, 0.5, and 0.8).

time lag on initial capacity. Figure 6 shows the dependence of investment threshold on time-lag for $\sigma = 0.2, 0.3$, in this range of time lag. It turns out that the transition of initial capacity from small state to large state occurs in smaller level of time lag as the volatility becomes large.

Figure 7 shows the dependence of thresholds of investment on time-lag, for various plant ratio. This function of time lag have more sharp gradient for small ratio of base-load plant as 0.2. This indicates strong dependence of the threshold for small α, which means that there exist many operational options, on time lag. This is because the effect of time lag leads to decrease the option value not only of investment but also of operation.

6. Conclusions

We have proposed the model for analyzing optimal investment and operation strategy under output price uncertainty. The model was formulated as an impulse control problem combined with optimal stopping. We have shown that as price volatility increases, the opportunities of investment, capacity expansion, and contraction decrease. We also showed the dependence of initial capacity at the beginning of the operation on the price volatility and the capacity ratio of plant by calculating investment values in each case. If the price volatility of output produced from the project becomes too large, the initial capacity was found to be large state due to increment of investment threshold. Consequently, it turns out that the effect of entry and exit setting leads to a different result from previous works. The effect of the capcity ratio, which varies total construction cost and operational cost, on the investment and operation strategy are shown. Additionally, we calculated the optimal investment rule and the value with time lag between the investment decision and the beginning time of the operation.

Future work will include the applications to electric power industry such as new entry and plants operation for IPPs. Furthermore, we will construct a model for analyzing not only the investment timing and the operating flexibility but also the capacity choice for the firm.

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