



Title	正四面体と他のプラトン立体の間の共通の（辺）展開図に関する研究
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Citation	2010年度科学技術振興機構ERATO湊離散構造処理系プロジェクト講究録. p.456-457.
Issue Date	2011-06
Doc URL	http://hdl.handle.net/2115/48338
Type	conference presentation
Note	ERATO湊離散構造処理系プロジェクト：2010年度初冬のワークショップ（ERATO合宿）．2010年11月29日（月）～12月1日（水）．札幌北広島クラッセホテル．
File Information	18.uehara.pdf



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正四面体と他のプラトン立体の間の共通の (辺)展開図に関する研究

上原隆平(JAIST) with
堀山貴史 (埼玉大学)
白川俊博 (ジー・モード)

ERATO合宿 2010/11/29-12/1

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Folding & Unfolding

Tiling

Puzzle

隆平(JAIST) with
堀山貴史 (埼玉大学)
白川俊博 (ジー・モード)

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Introduction

- Open problem 25.6
(by M. Demaine, F. Hurtado, E. Pegg)

- Can any **Platonic solid** be cut open and unfolded to a polygon that may be refolded to a **different Platonic solid**?

For ex., may a cube be so dissected to a tetrahedron?



Tetrahedron Cube (Hexahedron) Octahedron Dodecahedron Icosahedron

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Introduction

- Open problem 25.6

Are there polygons that can be folded to two (or more) different Platonic solids?

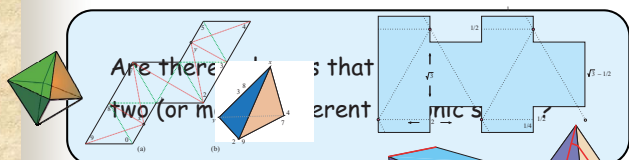
For ex., may a cube be so dissected to a tetrahedron?



Tetrahedron Cube (Hexahedron) Octahedron Dodecahedron Icosahedron

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Introduction



Close call ! [O'Rourke]

Regular Octahedron
⇔ Tetramonohedron
(tetrahedron all of whose
faces are congruent)

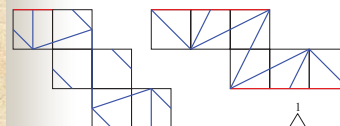
Close call !! [Hirata]

Regular Tetrahedron
⇔ Box $1 \times 1 \times 1.232$

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Are there polygons that can be folded to two (or more) different Platonic solids?

Close calls



Cube (Regular hexahedron)
⇔ Tetramonohedron
1 : 0.972 : 0.972

Regular Octahedron
⇔ Tetramonohedron
1.0072 : 0.9965 : 0.9965

Regular Icosahedron
⇔ Tetramonohedron 1 : 1.145 : 1.25

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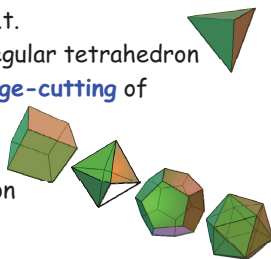
Are there polygons that can be folded to two (or more) different Platonic solids ?

Our Results

There exists no polygon P s.t.

- (1) P is a **development** of a regular tetrahedron
- (2) P is a **development by edge-cutting** of

- a regular hexahedron
- a regular octahedron
- a regular dodecahedron
- a regular icosahedron



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The Key Theorem

- Theorem [Akiyama 2007, Akiyama Nara 2007]

Let P be a development of a **regular tetrahedron**.

Then, P is a **tiling**. That is, P **fills a plane**.



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The Key Theorem

- Theorem
 P is a development of a **regular tetrahedron** iff
 - (1) P has a **p2 tiling**, i.e., tiling by 180° rotations
 - (2) **4** of the **rotation centers** define the triangular lattice
 - (3) no two of the 4 rotation centers belong to the same equivalent class on the tiling



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Conclusion

Open problem:

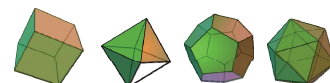
Are there polygons that can be folded to two (or more) different Platonic solids ?

Development of



vs

Development of



Future Work: 1.

by edge-cutting

2.

Relation among these 4

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