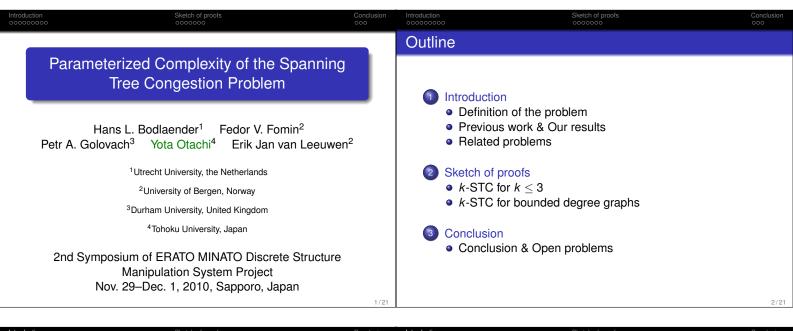


HOKKAIDO UNIVERSITY

Title	Parameterized Complexity of the Spanning Tree Congestion Problem
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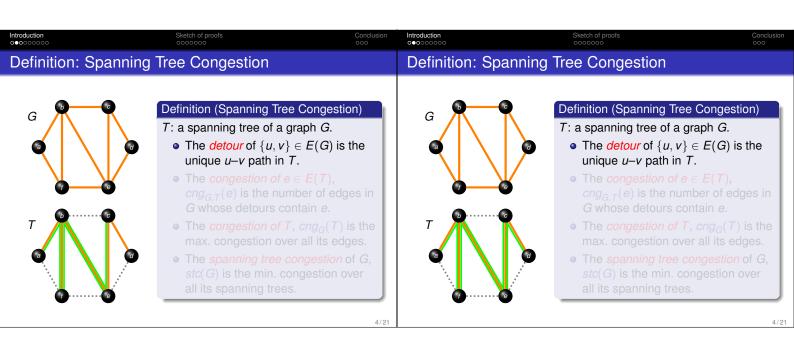


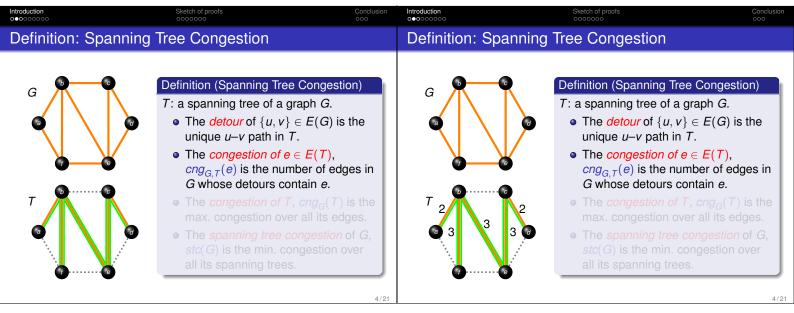
2010年度 ERATO湊離散構造処理系プロジェクト講究録



Introduction	Sketch of proofs 0 0000000	Conclusion 000	Introduction 00000000	Sketch of proofs	Conclusion 000
Outlir	ne		Definition: Spar	nning Tree Congestion	
1	Introduction Definition of the problem Previous work & Our results Related problems 		G	 Definition (Spanning Tree T: a spanning tree of a g The <i>detour</i> of {u, v} unique u-v path in the congestion of e 	raph G . $\in E(G)$ is the T.
2	 Sketch of proofs <i>k</i>-STC for <i>k</i> ≤ 3 <i>k</i>-STC for bounded degree graphs 		j ∂ —ĕ ₇ ♀ ⋯⋯♀	 The congestion of e cng_{G,T}(e) is the nun G whose detours co The congestion of T 	nber of edges in ntain <i>e</i> .
3	Conclusion Conclusion & Open problems			 The spanning tree of stc(G) is the min. co all its spanning trees 	er all its edges. ongestion of <i>G</i> , ingestion over
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Introduction ●●○○○○○○	Sketch of proofs	Conclusion 000	Introduction	Sketch of proofs	Conclusion 000
Definition: Spanning	g Tree Congestion		Definition: Sp	panning Tree Congestion	
	 Definition (Spanning Tr T: a spanning tree of a The detour of {u, v unique u-v path ir The congestion of cng_{G,T}(e) is the nu G whose detours of The congestion of max. congestion of max. congestion of max. congestion of all its spanning tree 	graph G. $Y_i \in E(G)$ is the T. $e \in E(T)$, unber of edges in contain e. T , $cng_G(T)$ is the ver all its edges. congestion of G, congestion over	G	 Definition (Spanning Tree T: a spanning tree of a g The detour of {u, v} unique u-v path in The congestion of e cong_{G,T}(e) is the num G whose detours of T max. congestion of T max. congestion ov The spanning tree of stc(G) is the min. congal its spanning tree 	graph <i>G</i> . $f \in E(G)$ is the <i>T</i> . $g \in E(T)$, mber of edges in ontain <i>e</i> . <i>T</i> , <i>cng</i> _G (<i>T</i>) is the er all its edges. <i>congestion</i> of <i>G</i> , ongestion over





Definition: Spanning Tree CongestionDefinition: Spanning Tree Congestion $G \rightarrow G \rightarrow$	Introduction 00000000	Sketch of proofs ooooooo	Conclusion 000	Introduction	Sketch of proofs	Conclusion 000
T: a spanning tree of a graph G. T: a	Definition: Spanning	Tree Congestion		Definition: Spanni	ng Tree Congestion	
4/21 4/21		 T: a spanning tree of a graph G. The <i>detour</i> of {u, v} ∈ E(G) unique u-v path in T. The <i>congestion of e</i> ∈ E(T), cng_{G,T}(e) is the number of eraction G whose detours contain e. The <i>congestion of T</i>, cng_G(T max. congestion over all its eraction of C) and the spanning tree congestion stc(G) is the min. congestion 	is the dges in) is the edges. n of <i>G</i> , over		 <i>T</i>: a spanning tree of a gra The <i>detour</i> of {<i>u</i>, <i>v</i>} ∈ unique <i>u</i>−<i>v</i> path in <i>T</i>. The <i>congestion of e</i> ∈ <i>cng_{G,T}(e)</i> is the number <i>G</i> whose detours contate The <i>congestion of T</i>, <i>c</i> max. congestion over a The <i>spanning tree constc</i> of <i>stc</i>(<i>G</i>) is the min. congestion <i>stc</i>(<i>G</i>) is the min. <i>stc</i>(<i>G</i>) is the <i>stc</i>(<i>G</i>) is <i>tc</i>(<i>G</i>) is <i></i>	ph G. E(G) is the E(T), er of edges in ain e. $eng_G(T)$ is the all its edges. gestion of G,

troduction ⊙●○○○○○○	Sketch of proofs 0000000	Conclusion 000	Introduction 00000000	Sketch of proofs ocoooco	Conclusion 000
The problems			The problems		
Problem: STC			Problem: STC		
Instance: Co	nnected graph G, positive integer k.		Instance: Co	onnected graph G, positive integer k.	
Question: sto	$c(G) \leq k?$		Question: st	$c(G) \leq k?$	
Problem: k-STC	2		Problem: k-ST	C	
Instance: Co	nnected graph G.		Instance: Co	onnected graph G.	
Question: sto	$c(G) \leq k$?		Question: st	$c(G) \leq k?$	
Note: k is	s a fixed constant.		Note: k	is a fixed constant.	
We investigate t	the complexity of STC and <i>k</i> -STC.		We investigate	the complexity of STC and <i>k</i> -STC.	
(k-STC is a para	ameterized version of STC.)		(k-STC is a particular technology)	rameterized version of STC.)	
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Introduction	Sketch of proofs	Conclusion 000	Introduction	Sketch of proofs	Conclusion 000
Outline			Previous wor	k	
 D P R Sket <i>k</i>- 	duction efinition of the problem revious work & Our results elated problems ch of proofs -STC for $k \leq 3$ -STC for bounded degree graphs		History Simonson '87 1990's Ostrovskii '04 2008–2010	latively new graph parameter. 7 (implicitly) Bounds for outerplanar g (implicitly) in papers on Tree Spanner I named the parameter S.T.C. Bounds or exact values for some grap grids, complete <i>k</i> -partite graphs, and	problems
	clusion onclusion & Open problems	6/21	No complexit	y result (to the best of my knowledge)). 7/2

Introduction	Sketch of proofs	Conclusion 000	Introduction	Sketch of proofs	Conclusion
Previous work			Our results		
History Simonson '87 (impli 1990's (impli Ostrovskii '04 nam 2008–2010 Bound grids,	ly new graph parameter. plicitly) Bounds for outerplanar grap citly) in papers on Tree Spanner pro- ned the parameter S.T.C. ds or exact values for some graphs complete <i>k</i> -partite graphs, and hyp ult (to the best of my knowledge).	such as	k -STC is line $k \le 3$. $input$ gra $input$ gra $input$ graTheorem (Ne k -STC is NP $k \ge 8$, $input$ gra $input$ gra $input$ gra	par time solvable for each of the follow apply are apex-minor-free. apply a state apex-minor-free. apply have bounded maximum degree egative results) -complete even if the following condit apply are K_6 -minor-free, and apply have only one vertex of unbound complete for planar graphs. less $P = NP$.	o. ions hold:

roduction Sketch of prod >>>∞∞∞∞∞∞∞∞∞∞∞∞∞∞∞∞∞∞∞∞∞∞∞∞∞∞∞∞∞∞∞∞∞	fs Conclusion 000	Introduction	Sketch of proofs	Conclusion 000
pex-minor-free graphs		Outline		
Definition (Apex graphs) An <i>apex graph</i> is a graph that can be removal of a single vertex.	be made planar by the		ition of the problem	
Examples of apex graphs K_5 , $K_{3,n}$ for any n (and of course all	planar graphs).		ous work & Our results ed problems of proofs	
Definition (Apex-minor-free graphs)			C for $k \leq 3$	
A graph class is <i>apex-minor-free</i> if i graph as a minor.	t excludes a fixed apex	k-ST Conclus	C for bounded degree graphs ion	
Examples of apex-minor-free graph	s	 Concl 	lusion & Open problems	
Planar graphs, bounded genus grap treewidth.	ohs, and graphs of bounded			10/21

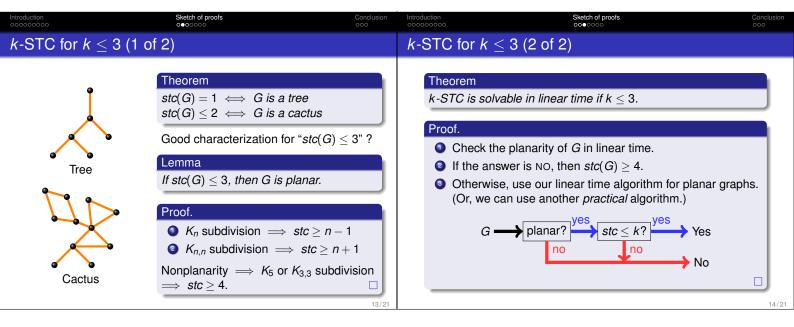
Introduction ○○○○○○○●	Sketch of proofs		Conclusion 000	Introduction	Sketch of proofs		Conclusion 000
Related problems				Related problems			
T a stretch = 2	Tree spanner Minimize the Bandwidth & Embed a gra linear arrange	problem stretch. Cutwidth prot bh on a line (d ement) so tha	or, find a good	T T T T T T T T T T T T T T T T T T T	Tree spanner Minimize the Bandwidth & Embed a grap linear arrange	r problem stretch. Cutwidth prol ph on a line (r ement) so tha	or, find a good
congestion = 3	is minimized.			congestion = 3	is minimized.		
stretch = 3 congestion = 4	stretch congestion		spanning tree Tree Spanner S.T.C.	a b o o d stretch = 3 congestion = 4	stretch congestion		spanning tree Tree Spanner S.T.C.
			11/21				11/21

Introduction ○○○○○○○○●	Sketch of proofs		Conclusion 000	Introduction	Sketch of proofs		Conclusion 000
Related problems				Related problems			
T stretch = 2 congestion = 3	Tree spanner Minimize the Bandwidth & Embed a gra linear arrange	problem stretch. Cutwidth prot oh on a line (o ement) so tha	or, find a good	7 stretch = 2 congestion = 3	Tree spanner Minimize the Bandwidth & Embed a gra linear arrange	problem stretch. Cutwidth pro ph on a line (ement) so tha	or, find a good
3 b 7 3 9 stretch = 3 congestion = 4	stretch congestion		spanning tree Tree Spanner S.T.C.	stretch = 3 congestion = 4	a stretch congestion		spanning tree Tree Spanner S.T.C.
			11/21				11/21

Introduction ○○○○○○○●	Sketch of proofs		Conclusion 000	Introduction ○○○○○○○●	Sketch of proofs		Conclusion ooo
Related problems				Related problems			
T stretch = 2 congestion = 3	Tree spanner Minimize the Bandwidth & Embed a gra linear arrange	problem stretch. Cutwidth prol ph on a line (ement) so tha	or, find a good	7 a stretch = 2 congestion = 3	Tree spanner Minimize the Bandwidth & Embed a gra linear arrange	problem stretch. Cutwidth pro ph on a line (ement) so tha	or, find a good
stretch = 3 congestion = 4	a stretch congestion		spanning tree Tree Spanner S.T.C.	a b b c b stretch = 3 congestion = 4	a stretch congestion		spanning tree Tree Spanner S.T.C.
			11/21				11/21

Introduction ○○○○○○○●	Sketch of proofs		Conclusion 000	Introduction	Sketch of proofs		Conclusion 000
Related problems				Related problems			
7 5 5 5 5 5 5 5 5 5 5	stretch = 2 linear arrangement) so that the stretch (bandwidth) or the congestion (cutwidth)		7 3 3 3 3 3 3 3 3 3 3	Tree spanner Minimize the Bandwidth & Embed a gra linear arrange	r problem stretch. Cutwidth prol ph on a line (ement) so tha	or, find a good	
		line	spanning tree			line	spanning tree
stretch = 3	stretch	Bandwidth	Tree Spanner	stretch = 3	stretch	Bandwidth	Tree Spanner
congestion = 4	congestion	Cutwidth		congestion = 4	congestion	Cutwidth	S.T.C.
			11/21				11/21

Introduction 000000000	Sketch of proofs ●oooooo	Conclusion 000	Introduction 00000000	Sketch of proofs o●ooooo	Conclusion 000
Outline			<i>k</i> -STC for $k \le 3$ (1 of 2)		
2 Sk • •	troductionDefinition of the problemPrevious work & Our resultsRelated problemssetch of proofs k -STC for $k \leq 3$ k -STC for bounded degree graphsonclusionConclusion & Open problems		Tree Tree Cactus	Theorem $stc(G) = 1 \iff G$ is a tree $stc(G) \le 2 \iff G$ is a cactusGood characterization for " $stc(G) \le$ LemmaIf $stc(G) \le 3$, then G is planar.Proof.If K_n subdivision $\implies stc \ge n - 1$ $K_{n,n}$ subdivision $\implies stc \ge n + 1$ Nonplanarity $\implies K_5$ or $K_{3,3}$ subdivision $\implies stc \ge 4$.	1
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Introduction 00000000		Conclusion 000	Introduction 00000000	Sketch of proofs ○○○○●○○	Conclusion 000
Outline			k-STC for gr	aphs of bounded degree (1 of	2)
 Introduction Definition of the prob Previous work & Our Related problems Sketch of proofs k-STC for k ≤ 3 k-STC for bounded of Conclusion Conclusion & Open for 	r results degree graphs	15/21	Lemma $tw(G) \le \Delta(G)$ Lemma k-STC is line Proof of the	e following two lemmas. $(\Delta(G)$ is the matrix $G)(stc(G) - 1)/2$. ear time solvable for graphs of bounded second lemma. be expressed in MSO logic. Courcelle's emma.	treewidth.

Introduction 00000000	Sketch of proofs ○○○○●○	Conclusion 000	Introduction 000000000	Sketch of proofs	Conclusion 000
MSO logic exp	pression for <i>k</i> -STC		k-STC for g	raphs of bounded degree (2 of 2)	
$Part(V_1, V_2, V_3) := Adj(v_1, v_2, E_1) := E_1 = Ind(V_1) := E_1 = Inc_v(E_1) := V_1 = Inc_v(E_1) := Conn(E_1) := BiConn(E_1) := BiConn(E_1) := Tree(E_1) := Tree(E_1) := Path(v_1, v_2, E_1) := SpnTree(E_1) := Detour(e_1, E_1) := Cong_k(e_0, E_0) := Cong_k(e_0, E$	$\begin{array}{l} (\exists e_{1} \in E_{1})(\forall e_{2} \in E_{1})(e_{1} = e_{2} \iff inc(v_{1}, e_{2}))\\ V_{2} \neq \emptyset \land V_{3} \neq \emptyset \land (V_{2} \cup V_{3} = V_{1}) \land (V_{2} \cap V_{3} = \emptyset)\\ v_{1} \neq v_{2} \land (\exists e_{1} \in E_{1})(inc(v_{1}, e) \land inc(v_{2}, e))\\ (\forall e_{1})(e_{1} \in E_{1} \iff (\exists v_{1}, v_{2} \in V_{1})(v_{1} \neq v_{2} \land inc(v_{1}, e_{1}) \land inc(v_{2}, v_{2}))\\ (\forall v_{1})(v_{1} \in V_{1} \iff (\exists e_{1} \in E_{1})(inc(v_{1}, e_{1}))))\\ (\forall v_{2}, V_{3})(Part(Incv(E_{1}), V_{2}, V_{3}) \implies (\exists v_{2} \in V_{2}, v_{3} \in V_{3})(Adj(v_{2}, v_{2}, v_{2} \in encv(E_{1}))(v_{1} \neq v_{1}) \land i < i < j \leq 3) \land (\forall v_{4})(Cont(E_{1} \land (\forall v_{1} \subseteq Incv(E_{1}))(v_{2} \neq v_{1})(1 \leq i < j \leq 3) \land (\forall v_{4})(Cont(E_{1} \land (\forall v_{1} \subseteq Incv(E_{1}))(-BiCont(Ind(V_{1}) \cap E_{1})))\\ Forest(E_{1}) \land Cont(E_{1})\\ Tree(E_{1}) \land (\forall v_{0} \in Incv(E_{1}))(Deg1(v_{3}, E_{1}) \iff v_{3} = v_{1} \lor v_{3} = v_{2}\\ Tree(E_{1}) \land (\forall v)(v \in Incv(E_{1}))\\ \neg (\exists e_{1}, \ldots, e_{k})((e_{i} \notin E_{0})(1 \leq i \leq k) \land e_{i} \neq e_{j}(1 \leq i < j \leq k) \land (\exists E_{i})(e_{0} \in E_{i} \land E_{i} \subseteq E_{0} \land Detour(e_{i}, E_{i}))(1 \leq i \leq i \leq k) \land (\exists E_{i})(e_{0} \in E_{i} \land E_{i} \subseteq E_{0} \land Detour(e_{i}, E_{i}))(1 \leq i \leq k) \land (\exists E_{0})(SpnTree(E_{0}) \land (\forall e_{0} \in E_{0})(Cong_{k}) \end{cases}$	<pre>v₃, E₁))) Inc_E(v₄)))) </pre>	Proof. ● Check ● If the at ● O.w., us G →	tw(G) $\leq d(k-1)/2$ in linear time. ($d := \Delta(C)$ nswer is NO, then $stc(G) > k$. se a linear time algorithm for bounded treewing $tw \leq d(k-1)/2$? No No No No No No No No No No No No No	۶)) dth. es

Introduction 00000000	Sketch af proofs 0000000	Conclusion ●oo	Introduction 00000000	Sketch of proofs 0000000	Conclusion o●o
Outline			Our results		
•	roduction Definition of the problem Previous work & Our results Related problems		 <i>k</i> ≤ 3. <i>input grap</i> 	itive results) Ir time solvable for each of the followin ohs are apex-minor-free. Ins have bounded maximum degree.	ng cases:
•	etch of proofs k -STC for $k \leq 3$ k-STC for bounded degree graphs		Theorem (Neg <i>k-STC is NP-c</i> • <i>k</i> > 8.	pative results) complete even if the following condition	ons hold:
	nclusion Conclusion & Open problems		 input grap 	ohs are K ₆ -minor-free, and ohs have only one vertex of unbounde	ed degree.
		19/21	STC is NP-cor No PTAS, unle	nplete for planar graphs. P = NP.	20/21

Introduction 00000000	Sketch of proofs	Conclusion ○○●	Introduction Sketch of proofs 00000000 000000	Conclusion ○○●
Open problems			Open problems	
 Complexity of <i>I</i> I think 4-ST Complexity of S Recent prographs, and Approximation. Constant fa Is STC ∈ FPT f If <i>k</i> = 1, i.e. 	k-STC for $k \in \{4, 5, 6, 7\}$. To might be NP-complete. STC and k-STC for some graph classes gress with Y. Okamoto, R. Uehara, and T. U complete for split graphs and chordal bipar d linear time solvable for trivially perfect gra ($O(\log n)$ -factor approximation?) ictor approximation might be NP-hard. for k-outerplanar graphs? (parameter is ., for outerplanar graphs, STC can be solved (with Bodlaender, Kozawa, Matsushima) <i>Thank you for your atter</i>	Jno: rtite phs. s <i>k</i>) ed in	 Complexity of <i>k</i>-STC for <i>k</i> ∈ {4,5,6,7}. I think 4-STC might be NP-complete. Complexity of STC and <i>k</i>-STC for some graph of a second progress with Y. Okamoto, R. Uehara, a STC is NP-complete for split graphs and chords graphs, and linear time solvable for trivially perfection. Approximation. (<i>O</i>(log <i>n</i>)-factor approximation?) Constant factor approximation might be NP-harm. Is STC ∈ FPT for <i>k</i>-outerplanar graphs? (parameter in the solvable of the solution of the second programmeter in the solvable of the second programmeter in the solvable of the second provided programmeter in the solvable of the second provided programmeter in the solvable of the second provided programmeter in the second programmeter in the second programmeter in the second programmeter in the second programmeter is t	and T. Uno: al bipartite ect graphs.) d. neter is k) e solved in nima)
		21/21		21/21