Psychophysics of time-perception and intertemporal choice models.

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Abstract

Intertemporal choice and psychophysics of time-perception have been attracting attention in econophysics and neuroeconomics. Several models have been proposed for intertemporal choice; exponential discounting, general hyperbolic discounting (exponential discounting with logarithmic time-perception of Weber-Fechner law, a q-exponential discount model based on Tsallis' statistics), simple hyperbolic discounting, and Stevens' power law-exponential discounting (exponential discounting with Stevens' power time-perception). In order to examine the fitness of the models to behavioral data, we estimated the parameters and AICc (Akaike Information Criterion with small sample correction) of the intertemporal choice models by assessing the points of subjective equality (indifference points) at seven delays. Our results have shown that the orders of the goodness-of-fit for both group and individual data were [Weber-Fechner discounting (General hyperbola) > Stevens' power law-discounting > Simple hyperbolic discounting > Exponential discounting], indicating that human time-perception in intertemporal choice may follow Weber-Fechner law. Indications of the results for neuropsychopharmacological treatments of addiction and biophysical processing underlying temporal discounting and time-perception are discussed.

Keywords: intertemporal choice, discounting, time-perception, impulsivity, neuroeconomics, psychophysics

1. Introduction

Studies in psychopharmacology, neuroscience, and behavioral economics have revealed that humans and non-human animals prefer sooner smaller rewards to larger later ones (“delay discounting”, [1-20]). Substance misuse and addiction are associated with impulsivity in intertemporal choice (a large time-discount rate) [1-5, 9-12, 15-17]. Furthermore, recent psychophysical studies suggested that addicts and impulsive psychiatric patients such as ADHDs (attention deficit hyperactivity disorder patients) and orbitofrontal lesion patients have impaired time-perception, which may result in difficulty in patience/self-control in intertemporal choice[2, 3, 15-17, 21]. Impulsivity in intertemporal choice is defined as strong preference for small immediate rewards over large delayed ones. For instance, subject A who prefers "one apple available one year later" to "two apples available one year and a week later" is more impulsive than subject B who prefers "two apples available one year and a week later" to "one apple available one year later" (most people may behave as the subject B, in this example). In classical economic theory, it has been assumed that subjects have consistent intertemporal choice behavior (a constant temporal discounting rate), based on the assumption of human rationality and/or preference for simplicity of economic theory [6]. However, recent behavioral and neuroeconomic evidence indicates that human and animal discounting rates are changeable, more specifically, decreasing towards the future (decreasing impatience), resulting in preference reversal over time [1,4,6]. Suppose the above example again. If the same subject B prefers "one apple available today" to "two apples available one week later" (again, most people may make decisions in this way), her intertemporal choice is inconsistent, in that delays from the sooner rewards to later ones are the same (i.e., a week) in the two

intertemporal choice problems. As shown in the examples, if subjects prefer larger later rewards in intertemporal choice in the distant future, but prefer smaller sooner rewards in intertemporal choice in the near future, their intertemproal choices are "dynamically inconsistent" (in economics' sense), because their preferences reverse as time passes. On the contrary, if someone prefers smaller sooner rewards in both examples; i.e. "one apple available one year later" and "one apple available today", her intertemporal choice is impulsive but dynamically consistent ("rational" in economics' sense), because preference does not reverse over time.

Several studies have reported that neuropsychiatric and neurological patients (such as addicts, psychostimulant abusers, orbitofrontal brain lesion patients, and sleep-deprived subjects as well) are associated with impaired time-perception [2,3,14-17, 21-22]. In other words, "short temporal horizon" (future myopia) due to overestimation of delay lengths in intertemporal choice may partly account for the augmented discounting in the neuropsychiatric patients. No study to date, however, has examined psychophysical effects of time-perception on intertemporal choice in a quantitatively rigorous and systematic manner. A recent neuroimaging study [20] demonstrated that time-perception in intertemporal choice is represented in dopaminergic neural circuits (e.g., the caudate nucleus) which are known to be modified in drug-dependent patients [23]. Therefore, it is important to examine the fitness of intertemporal choice models which incorporate psychophysical effects on time-perception into time-discounting behavior, to human behavioral data, for a better understanding of neuropsychopharmacological and biophysical basis of impulsive behavior. In the present study, we examined the fitness of several types of intertemporal choice models with different types of psychophysical effects on time-perception, i.e., no psychophysical effects (i.e., exponential discounting and simple

hyperbolic discounting), the Weber-Fechner law (general hyperbolic discounting), and Stevens' power law. Furthermore, biophysical simulation studies in nonlinear physics indicate that psychophysical laws of sensation (e.g., Stevens' power law and Weber-Fechner law, introduced later) are due to non-synaptic electrical coupling between sensory signal-processing neurons [25].

1.1 Exponential discounting

Consistent temporal discounting (simple exponential discounting), which is typically assumed in neoclassical economic theory including theory of rational addiction, follows the exponential equation [4,6,15]:

$$V(D) = A\exp(-kD) \qquad \text{(Equation 1)}$$

where V is the subjective value of a reward, A is the (objective) amount of the reward, and D is the objective (physical) time-duration of delay until the delivery of reward. The free parameter k is an index of the degree to which a subject discounts the delayed reward, i.e., larger k values correspond to more rapid delay discounting. In simple exponential discounting, there is no inconsistency in intertemporal choice, because a discount rate := $-(dV(D)/dD)/V = k$ is independent of D (a time-constant discount rate).

1.2 Exponential discounting with Weber-Fechner time-perception

As noted, recent studies reported that individual/group differences in temporal discounting are strongly related to those in time-perception [14,22]. Based on these findings, it has been proposed that combining psychophysical effects on time-perception such as Weber-Fechner law (i.e. logarithmic time-perception) with the exponential discounting may be capable of describing empirically observed intertemporal choice behavior such as decreasing impatience and preference reversal [15,17]. Because in logarithmic time-perception,

subjective time-duration τ (psychological time) is expressed as:

$$\tau = a \ln(1+bD), \qquad \text{(Equation 2)}$$

exponential discounting with the subjective delay τ (exponential discounting with logarithmic time-perception) is formulated as:

$$V(D) = A\exp(-k\tau) = A\exp(-ka \ln(1+bD)) = A/(1+bD)^{g} \qquad \text{(Equation 3)}$$

where A is the objective amount of the reward, D is an objective/physical delay length and b and $g=ka$ are free parameters. It should be noted that Equation 3 is the same as a general hyperbolic discount function, proposed in behavioral psychology [8], by combining psychological effects on time-perception and a (more conventional) simple hyperbolic discount function introduced below. In the general hyperbolic discount model, a discount rate = $-(dV(D)/dD)/V = bg/(1+bD)$ is a decreasing function of delay (decreasing impatience) when b and g are larger than zero, resulting in preference reversal over time. It should further be noted, that this exponential discount model with Weber-Fechner type-time perception is mathematically equivalent to the q-exponential discount model based on Tsallis' non-extensive thermostatistics in econophysics [18,19,24]. More specifically, the q-exponential discount model is

$$V_q(D) = A/\exp_q(k_q D) = A/[1+(1-q)k_q D]^{1/(1-q)} \qquad \text{(Equation 3').}$$

where q and k_q indicate time-consistency (a time-independent discount rate) and impulsivity at delay D=0 (= $-(dV(D)/dD)/V|_{D=0}$), respectively [18,19,24]. Note also that, as one can see from Equation 3, by utilizing relationships $q=(g-1)/g$ and $k_q=bg$, we obtain parameters in the q-exponential discount model from those in the Weber-Fechner discounting.

1.3 Simple hyperbolic discounting

Most studies in behavioral neuroeconomics and psychopharmacology [5,11] have been

utilizing a simpler form of the hyperbola-like discount model (a simple hyperbolic discount function) by setting *g* (in equation 3)=1 (a simple hyperbolic function):

$$V=A/(1+k_sD), \qquad \text{(Equation 4)}$$

where k_s is a simple hyperbolic discount rate, A is the objective amount of the reward, and D is an objective delay length until receipt. In the simple hyperbolic discounting, a discount rate = $-(dV(D)/dD)/V= k_s/(1+k_sD)$, which is also a decreasing function of delay (decreasing impatience) when k_s >0 (i.e. "positive time preference" in economics' sense), resulting in preference reversal over time. It is to be noted that the simple hyperbolic discount function has been introduced from the optimal foraging theory (or the application of the Holling's optimal patch use theory) with a handling time parameter but without psychophysical effects on time-perception, in behavioral ecology [7,8].

1.4 Exponential discounting with Stevens' power time-perception

Behavioral economists Daniel Read and colleagues reported that patience in intertemporal choice decreases (i.e., a discount rate increases) when the time-interval of the delay until receipt is divided into shorter time-blocks, [13]. In order to explain the subadditivity in temporal discounting exponential discounting with time-perception following Stevens' power law has been proposed [16]. Subjective time-perception (τ_s) following Stevens' power law (named after the psychophysicist Stevens) is:

$$\tau_s=cD^s, \qquad \text{(Equation 5)}$$

where *c* and *s* (>0) are psychophysical parameters [26]. When s <1, the time-derivative of psychological time decreases as delay D increases (overestimation of short time-intervals and underestimation of long time-intervals); on the contrary, when s>1 the time-derivative of psychological time increases as delay increases (underestimation of short time-interval

and overestimation of long time-interval), and there is no nonlinear psychophysical effect on time-perception when s=1. If subjects discount delayed rewards exponentially, but with subjective time-perception following the Stevens' power law (Equation 5), their temporal discounting behavior is formulated as:

$$V=Aexp(-k\tau_s)=Aexp(-k_{power}D^s), \qquad \text{(Equation 6)}$$

where A is the objective amount of the reward, and $k_{power}=kc$ and s are free parameters. In the exponential discounting with Stevens' power law-type time-perception, a discount rate $= -(dV(D)/dD)/V=k_{power}sD^{s-1}$ is also a decreasing function of delay (decreasing impatience) when $s<1$ and $k_{power}>0$ (i.e. positive time preference), resulting in preference reversal over time. More psychophysically, when s<1, decreasing impatience (i.e., impulsive in the near future but patient in the distant future) may exist and result in preference reversal, because shorter time-intervals are overestimated when s<1. It should be noted that a recent psychopharmacological study proposed that impulsivity in intertemporal choice by abusers of dopaminergic psychostimulants may be attributable to overestimation of delay with this type of nonlinear transformation of time-perception [22].

Because (i) identifying psychophysical laws of time-perception in temporal discounting is important for psychophysics and neuropsychopharmacology of impulsivity, (ii) recent biophysical and neuroimaging studies collectively suggested both the Weber-Fechner and Stevens' power laws of sensation (e.g. subjective time-duration) may result from electrical coupling via gap junctions between (dopaminergic) neurons in the caudate nucleus encoding subjective time-duration of delay in intertemporal choice [20,21], and (iii) psychopharmacological studies demonstrated that drug-dependent subjects, psychostimulant abusers and drug-withdrawn subjects have impaired time-perception [21,22], we compared explanatory powers of these discount functions with different

psychophysical effects on time-perception (note that Equation 1 and 4, i.e. simple hyperbolic and exponential discounting functions are models without psychophysical effects on temporal cognition). This point is important for establishing psychopharmacological treatment aimed to reduce impulsivity by ADHDs, drug-dependent patients and substance abusers. In the present study, the goodness of fitness of each equation to the behavioral data in intertemporal choice was quantified with AIC_c (Akaike Information Criterion with small sample correction, a second-order AIC) and compared at both group and individual levels [27].

2. Method

2.1 Participants

Twenty-six (12 male and 14 female) volunteer students from a major national university in Japan participated in the experiment. The average age was 22.07 (standard deviation= 2.3) years. They were recruited from several psychology classes. They participated in a intertemporal choice task, for their parameters of intertemporal choice models to be estimated.

2.2 Procedure

We utilized exactly the same experimental procedure as Bickel et al [5] and Ohmura et al's [12] in the intertemporal choice task. Firstly, participants were seated individually in a quiet room, and face the experimenter across a table. After that, participants received the simple instruction that the monetary reward in this experiment was hypothetical, but the experimenter wanted you to think as though it were real money. Then the participants were asked to choose between the card describing money delivered certainly and the card describing money delivered with a certain delay of receipt. The left card viewed from participants indicated the amounts of money that could be received immediately (a smaller

but immediate reward), and the right card indicated 100,000 yen that could be received with a certain delay (a larger but delayed reward).

For the intertemporal choice task, monetary rewards and the delay were printed on 3×5 index cards. The 27 monetary reward amounts were 100,000 yen (about $1,000), 99,000 yen, 96,000 yen, 92,000 yen, 85,000 yen, 80,000 yen, 75,000 yen, 70,000 yen, 65,000 yen, 60,000 yen, 55,000 yen, 50,000 yen, 45,000 yen, 40,000 yen, 35,000 yen, 30,000 yen, 25,000 yen, 20,000 yen, 15,000 yen, 10,000 yen, 8,000 yen, 6,000 yen, 4,000 yen, 2,000 yen, 1,000 yen, 500 yen, and 100 yen. The seven delays of receipt were 1 week, 2 weeks, 1 month, 6 months, 1 year, 5 years, and 25 years. The experimenter turned the 27 immediate cards sequentially. The card started with 100,000 yen, down to 100 yen, and back to 100,000 yen. For each card, the participant pointed either the immediate or delayed reward. The experimenter wrote down the last immediate reward chosen in the descending order, and the first immediate reward chosen in the ascending order, and the average of them was used as the point of subjective equality (hereafter *indifference point*) in the following analysis. This procedure was repeated at each of the seven delays (for more details, see [12]).

2.3 Data analysis

The analytical strategy of statistical procedures in the present study was exactly the same as in a previous study on probabilistic choice models [28]. We employed discounting and psychophysical parameters introduced in the discounting equations above. For estimating the parameters (i.e. k in the exponential discounting, b and g in the exponential discounting with Weber-Fecher time-perception, k_s in the simple hyperbolic discounting, and k_{power} and s in the exponential discounting with Stevens' power time-perception), we fitted the four types of the discount model equations (i.e. Equation 1, 3, 4, 6) to the

behavioral data of indifference points with the Gauss-Newton algorithm (R statistical language, non-linear modeling package), and the fitness of each equation was estimated with AIC_c (Akaike Information Criterion with small sample correction, a second order AIC) values, which is the most standard criterion for the fitness of mathematical model to observed data when sample size is small [27,28]. It should be noted that the comparison between the R-square values of equations with different numbers of free parameters are statistically irrelevant (note that an increase in the numbers of free parameters in a fitting equation always yield a larger R-square value. cf. [12]). Therefore, we compared AIC_c values of the equations with best-fit parameters because AIC_c penalizes an increase in the number of free parameters in a model equation [27,28]. It is to be noted that smaller AIC_c values correspond to better fitting, because $AICc = ln[(\text{residual sum of squares})/N] + (N+k)/(N-k-2)$, where N is the sample size and k is the number of parameters to be estimated [27]. Therefore, better fitting in terms of smaller AIC_c indicates a better tradeoff between overfitting and poor fitting.

Next, in order to examine whether the equation type has a statistically significant effect on the goodness-of-fit (indicated with AIC_c) at the individual level, we conducted a one-way repeated measure ANOVA (analysis of variance) with equation type (i.e. Equation 1, 3, 4, 6) as a four-level within-subjects factor, with respect to AIC_cs (with the F-distribution of $F(3, 25)$). We subsequently conducted *posthoc* pairwise multiple *t*-tests with Bonferoni's correction in order to compare AIC_cs, in pairwise manner, for the four discount models.

All statistical procedures were conducted with R statistical language (The R Project for Statistical Computing). Individual data are expressed as Mean ± Standard Deviation (SD). Significance level was set at 5% throughout (for pairwise multiple comparisons,

Bonferoni's correction was utilized).

3. Results

3.1 Intertemporal choice models at group level

First, we estimated the parameters in the four types of intertemporal choice models by fitting the four types of equations to group median data of the indifference points at the delays (see Fig.1). After fitting each model to the group median data, we calculated AIC_c with best-fit parameters as an index of fitness (see Table 1 for estimated parameters and AIC_cs for the group data). A group median of the psychophysical parameter of nonlinearity of time-perception in Equation 6 (i.e., *s*) was smaller than 1, suggesting that subjects are less impulsive in the distant future than in the near future. This may account for preference reversal over time and consistent with previous studies [1,4,6]. The orders of the AIC_cs for group median data were [Weber-Fechner discounting (General hyperbola) < Stevens' power law-discounting< Simple hyperbolic discounting function < Simple exponential discounting function]. The Weber-Fechner discount model (a general hyperbola, the q-exponential discounting) best fitted the behavioral data (i.e. smallest AIC_c for the general hyperbolic function). Because the inequalities on AIC_c are not due to differences in the number of free parameters in the intertemproal choice models (see 2.3), the result also indicates that the simple hyperbolic discounting function (with no psychophysical effect of time-perception) more poorly fitted the behavioral data than exponential discounting with subjective time-duration following Steven's power law, although the simple hyperbolic discounting function has more frequently been utilized in behavioral economics and psychopharmacological studies on addiction [4,5,11].

Insert Figure 1 About Here

Insert Table 1 About Here

3.2 Intertemporal choice models at individual level

Next, we examined the goodness-of-fit of the intertemporal choice models for individual data by computing AIC_cs of the intertemporal choice models for each subject's data (Table 2). The orders of the AIC_cs for medians were [Weber-Fechner discounting (General hyperbola) < Stevens' power law-discounting< Simple hyperbolic discounting function < Simple exponential discounting function]. Furthermore, we observed a significant effect of the types of intertemporal choice models on AICc at the individual levels ($F(3, 25)= 18.6488$, $p<0.0001$). The subsequent *posthoc* pairwise multiple *t*-tests revealed that all the presented inequalities (differences) in AIC_c (note that smaller AIC_cs indicate better fitting) at the individual behavior level were likewise significant ($p<.01$). This again implies that the temporal discounting models with non-linear psychophysical effects on time-perception (i.e. exponential discounting with Weber-Fechner type time-perception or with Stevens' power law-type time-perception) better fit the behavioral data than temporal discounting models without the psychophysical effects (i.e., simple hyperbolic and exponential discounting functions). Additionally, it is suggested that time-perception in intertemporal choice follows the Weber-Fechner law (logarithmic time-perception), rather than Stevens' power law. It should again be noted that the smaller

AIC_cs in psychophysical time-discounting models were not due to the larger numbers of free parameters in the intertemporal choice models incorporating psychophysical effects (see 2.3).

Insert Table 2 About Here

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4. Discussion

To our knowledge, this study is the first to systematically examine the fitness of intertemporal choice models, in terms of psychophysical laws of time-perception (including Stevens' power law), by employing AIC_c as an indicator of the fitness. A main finding of the present study is that temporal discounting models with non-linear temporal cognition due to psychophysical effects significantly better fit the behavioral data, in comparison to discounting models without psychophysical effects.

4.1 Intertemporal choice and psychophysics of time

Our analysis implies that, for group and individual data, the order of goodness-of-fit of the model is [Weber-Fechner discounting (General hyperbola) > Stevens' power law-discounting > Simple hyperbolic function (without psychophysics of time) > Simple exponential discounting function (without psychophysics of time)]. Note that the general hyperbola is equivalent to the q-exponential discount model based on Tsallis' thermostatistics [18,19,24]. It is suggested that (i) human intertemporal choice is under strong influences of psychophysics of time-perception, in line with recent findings in psychopharmacological and neurological studies on impulsivity [2,3,21,22], and (ii) human

temporal cognition in intertemporal choice (over one week-25 years) may follow the Weber-Fechner law (i.e. logarithmic time-perception), rather than Stevens' power law. Because (i) drug-dependents and psychostimulant users have impaired time-perception and exaggerated impulsivity in temporal discounting [2,3,21,22], (ii) the psychophysical laws may be attributable to electrical coupling via gap junctions [25] and a chronic administration of addictive induces overexpression of gap junction proteins (e.g. connexin 32/36) in dopaminergic neural circuits [29], and (iii) a recent neuroimaging study reported human time-perception in intertemporal choice is represented in dopaminergic brain regions [22], it should be examined how dopaminergic drugs change electrical coupling associated with psychophysical effects on temporal cognition, in order to explore neuropsychopharmacological treatment for addiction and impulsivity/inconsistency in intertemporal choice.

4.2 Relation to biophysics of psychophysical laws

Non-linear biophysical simulation studies have examined the neural dynamics underlying psychophysical laws such as Weber-Fechner law and Stevens' power law [25, 30,31]. It should be noted that the experimentally fitted value of the Stevens' exponent in the present study is close to the theoretical value (~0.4) of Copelli and colleagues' simulation study [25]. Furthermore, a biophysically plausible Stevens-type relationship (i.e., the Hill equation) has numerically similar characteristics to the Weber-Fechner law [25,30,31]. Consistent with this speculation, when the delay is short (within a year), the fitted curves of the Stevens' power discount model and the Weber-Fechner discount model were similar (see Fig. 1). Moreover, the mentioned biophysical simulation studies [25,30,31] demonstrated that the Stevens' exponent may be a critical exponent in a non-equilibrium phase transition of the neural system. Therefore, it is possible that the

collective status of time-processing neurons differ between addicts' and healthy controls' brains, because addicts have altered temporal cognition [21,22]. Tellingly, Becker and Murphy's economic theory of rational addiction [32] and subsequent mathematical analysis of the rational addiction model [33] predict that subjective values of delayed rewards and drug intake behavior by rational addicts may be chaotic and bi-stable. How the behavioral characteristics of rational addiction are related to non-linear biophysical characteristics of neural systems is an important question for future studies.

4.3 Limitations and future directions

We now discuss limitations of the present study. Because the present study was conducted in healthy control subjects, it is not completely incontestable that our present findings can be generalized for dopaminergic drug-dependent patients and substance misusers. However, because substance misusers also discount delayed rewards in a time-inconsistent manner, which is associated with their loss of self-control [1,4,5,11], it is possible for our present findings to be generalized, to some extent, for drug-dependent individuals. Future studies should extend the present findings into clinical populations such as psychostimulant abusers and dopaminergic drug-dependent patients.

4.3 Conclusions

Our present study demonstrated that (i) intertemporal choice behavior is better understood in terms of temporal cognition, (ii) human temporal cognition in intertemporal choice over one week-25 years may follow Weber-Fechner law, rather than Stevens' power law. Further studies are required in order to answer what neuropharmacological and biophysical mechanisms mediate the psychophysical effect on time-perception in

intertemporal choice.

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Table 1.

Parameters and AIC_c (Akaike Information Criterion with small sample correction) of intertemporal choice models for group data

	Simple exponential discounting	Weber-Fechner discounting (general hyperbola)		Simple hyperbolic discounting	Stevens' power law discounting	
AIC_c	156.3718	109.8369		151.3614	139.4477	
Parameter	k	b	g	k_s	k_{power}	s
	0.0001795	0.0098518	0.21025	0.0004338	0.023629	0.416527

For group median data (N=26), the order of goodness-of-fit was [Weber-Fechner discounting (General hyperbola) > Stevens' power law-discounting > Simple hyperbolic function (without psychophysics of time) > Simple exponential discounting function (without psychophysics of time)]. Note that the general hyperbola is equivalent to the q-exponential discount function and smaller AIC_c corresponds to better fitting to behavioral data.

Table 2

Parameters and AIC_cs of intertemporal choice models for individual data

	Simple exponential discounting	Weber-Fechner discounting (general hyperbola)		Simple hyperbolic discounting	Stevens' power law discounting	
AIC_c	146.836±9.90	134.699±7.64		145.1483±9.24	136.244±10.03	
Parameter	k	b	g	k_s	k_{power}	s
	0.0001602±0.00005	0.008784±0.0004	0.1997±0.06	0.0004329±0.00003	0.02373±0.0006	0.4165±0.031

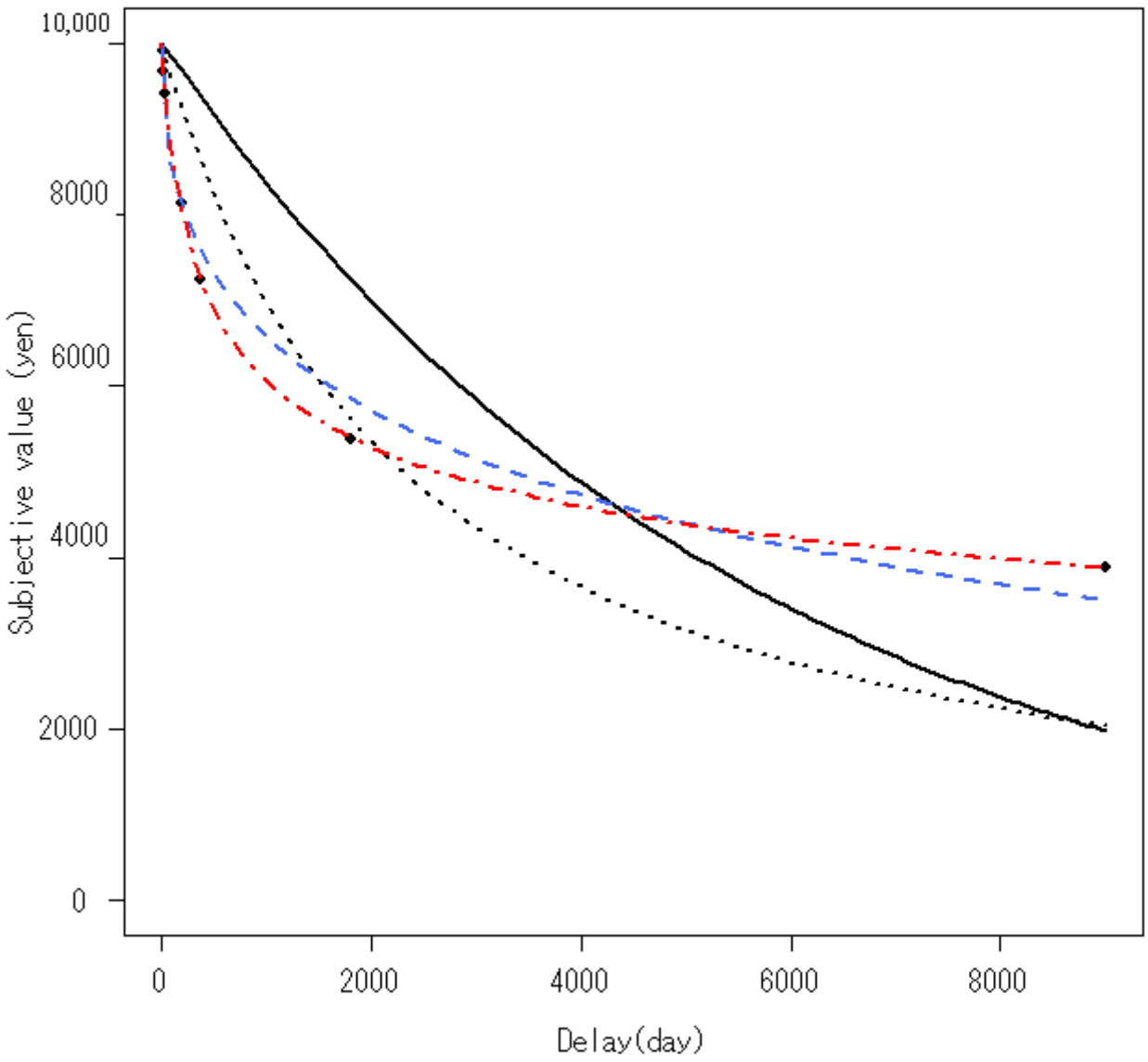
For individual data (N=26), the order of goodness-of-fit was [Weber-Fechner discounting (General hyperbola) > Stevens' power law-discounting > Simple hyperbolic function (without psychophysics of time) > Simple exponential discounting function (without psychophysics of time)]. All differences in goodness-of-fit between models were statistically significant (p<0.01). Note that the general hyperbola is equivalent to the q-exponential discount function and smaller AIC_c corresponds to better fitting to behavioral data. All values are expressed as mean±SD.

Figure legends

Fig.1

Group data of intertemporal choice and best fitted curves. Vertical axis indicates subjective value of delayed reward (yen); while horizontal axis indicates delay (day). Behavioral data are expressed as black diamonds. Best fitted curves for analyzed models are indicated; a red dashed-and-dotted curve: Weber-fechner discounting (general hyperbola, q-exponential discounting), a blue dashed curve: Stevens' power law discounting, a black dotted curve: a simple hyperbolic model, a black solid curve: a simple exponential model. We can see that the Weber-fechner discounting (equivalent to the q-exponential discount model, see text) best fitted the behavioral data, implying that time-perception in intertemporal choice follows Weber-Fechner law.

Fig.1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4000
2000
0
Subjective value (yen)
0
2000
4000
6000
8000
Delay(day)