New Smoothing Method in the Automatic Hexahedral Mesh Generator for Improving Solver Convergence Property

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Abstract— We have developed an automatic hexahedral mesh generator, and evaluated the quality of the generated mesh. It was confirmed that the ratio of the longest and the shortest diagonal lengths of the elements affected the convergence of the ICCG solver. Therefore, for improving the mesh quality, it is necessary to develop the smoothing method in the automatic hexahedral mesh generator.

I. Introduction

We have previously developed the automatic hexahedral mesh generator [1]. We have also reported about the quality evaluation of the generated hexahedral mesh [2]. As the result, it was concluded that the ratio of the longest and the shortest diagonal lengths of the elements affected the convergence of the ICCG solver.

By the way, the smoothing process in the common mesh generators finally decides the shape of the element. The Laplacian smoothing method, a popular smoothing method, often generates the distorted elements, which cause an inaccurate solution and a long computation time. In this paper, we have proposed a new smoothing method for improving the ratio of the longest and the shortest diagonal lengths.

II. Mesh Quality Evaluation

For evaluating the mesh quality, we have proposed the diagonal length ratio λ defined as [2]

$$\lambda = \sum_{i=1}^{N} \frac{\lambda_i}{N}, \quad \lambda_i = \frac{l_{i,\max}}{l_{i,\min}}, \tag{1}$$

where N is the number of elements, $l_{i,\max}$ and $l_{i,\min}$ the longest and the shortest diagonal lengths of i-th element. The smaller λ is, the more distorted the element is. In [2], it was confirmed that the diagonal length ratio strongly affected the mesh quality and the ICCG convergence.

III. New Smoothing Method

To increase the diagonal length ratio λ leads to the speed-up of the ICCG convergence. Therefore, in the smoothing process of the proposed mesh generator [1], the nodes should move so as to increase the diagonal length ratio λ.

The proposed smoothing process consists of two functions. One is to prevent the elements from turning inside out, another is to increase the diagonal length ratio. The first smoothing is

$$\boldsymbol{P}_{ij0,dst} = \sum_{j=1}^{n}\sum_{k=1}^{7} w_{ijk}\boldsymbol{P}_{ijk} \Big/ n\sum_{j=1}^{7} w_{ijk}, \tag{2}$$

$$w_{ijk} = \left|\boldsymbol{P}_{ij0} - \boldsymbol{P}_{ijk}\right|, \tag{3}$$

where $\boldsymbol{P}_{ijk}$ is the position vector of i-th node belonging to j-th element, k is the local node number in j-th element, w_{ijk} is the weight function with respect to the distance between the nodes.

The second smoothing method for increasing the diagonal length ratio λ is

$$\boldsymbol{P}_{ij0,dia} = \sum_{j=1}^{n}\left(\boldsymbol{P}_{ijk} + \frac{\left|l_{ij} + l_{j,avg}\right|}{2}\boldsymbol{e}_{ki}\right) \Big/ n, \tag{4}$$

where l_{ij} is the diagonal length connecting to i-th node on j-th element, $l_{j,avg}$ the average of four diagonal lengths on j-th element, $\boldsymbol{e}_{ki}$ the unit vector of the diagonal.

The final position of i-th node is decided from

$$\boldsymbol{P}_{ij0} = (1-w)\boldsymbol{P}_{ih0,dst} + w\boldsymbol{P}_{ih0,dia}, \tag{5}$$

where w is the weight.

The proposed smoothing method, where w = 0.3, 0.5 and 0.7, was applied to a contactor model, as shown in Fig. 1. Fig. 2 shows the property of the ICCG convergence, and Table I represents the diagonal length ratio λ. With increasing the weight w, the diagonal length ratio λ slightly increased, and the property of the ICCG convergence improved.

IV. Conclusion

It was confirmed that the diagonal length ratio λ affected the ICCG convergence property. We have proposed the new smoothing method for increasing the diagonal length ratio λ, and verified that the property of the ICCG convergence property improved.

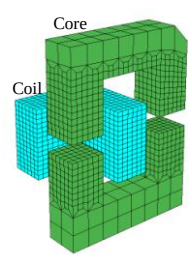

Fig. 1. Contactor Model

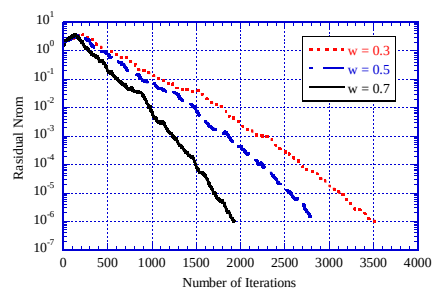

Fig. 2. ICCG convergence property.

TABLE I. Diagonal Length Ratio

Weight w	0.3	0.5	0.7
Diagonal Length Ratio λ	0.9767	0.9775	0.9789

V. References

[1] S. Nagakura, *et al.*, "Automatic Hexahedral Mesh Generation for FEM Using Shape Recognition Technique and Tree Method," *IEEE Trans. Magn.*, vol. 38, no. 2, pp. 417-420, 2002.

[2] Y. Motooka, *et al.*, "Quality Evaluation of Automatically Generated Hexahedral Mesh for FEA," *Proc.* 17*th COMPUMAG*, p. 406, 2009.